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Voids and Adhesion Theory

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1 Introduction

Large empty regions virtually devoid of galaxies – called voids – are a distinguishing feature of the large scale structure of the Universe as revealed in redshift surveys. Sizes of individual voids are known to vary considerably and range from $5h^{-1}$ Mpc to $60h^{-1}$ Mpc [3]. The importance of voids in providing a key to understanding the large scale structure of the Universe was pointed out years ago by Zeldovich, Einasto & Shandarin [7]. In this talk I shall attempt to summarise some of the results obtained by B.S. Sathyaprakash, Sergei Shandarin and myself from a study of voids using the adhesion model of structure formation [5].

2 The Zeldovich approximation and the Adhesion model

Particle motion in a self-gravitating medium can (until shell crossing) be accurately described by the Zeldovich approximation as a mapping from Lagrangian (initial) space $\mathbf{q}$ to Eulerian (final) space $\mathbf{x}$:

$$\mathbf{x} = \mathbf{q} - a(t)\nabla\Phi \tag{1}$$

where Φ is the linear velocity potential and $a(t)$ is the scale factor of the Universe [6]. (In linear theory Φ and the gravitational potential ϕ are related: $\phi \propto \Phi$, if the Universe is flat and matter dominated.) The deformation of a unit volume element under the Zeldovich approximation (ZA) is described by the Jacobian of the transformation from $\mathbf{q}$ to $\mathbf{x}$, so that

$$dV_E = dV_L\,[1 - a(t)\lambda_1(\mathbf{q})]\,[1 - a(t)\lambda_2(\mathbf{q})]\,[1 - a(t)\lambda_3(\mathbf{q})] \tag{2}$$

where dV_E and dV_L, are the volume elements in Eulerian space and Lagrangian space respectively, and λ_1, λ_2 and λ_3 are the eigenvalues of the deformation tensor ψ_{ij} :

$$\psi_{ij} \equiv \frac{\partial^2\Phi}{\partial q_i \partial q_k}. \tag{3}$$

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Clearly, positive values of λ_i indicate collapse along at least one direction suggesting that the particle in question will ultimately end up in a caustic. Negative values of λ_i $(i = 1, 2, 3)$ on the other hand imply that the given volume element will expand rather than contract, such regions will be the progenitors of present-day voids.

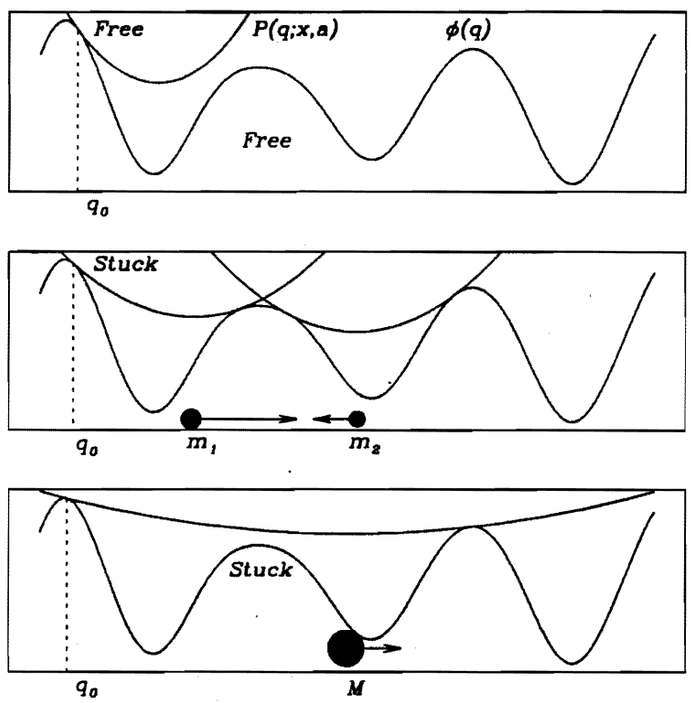

Fig. 1. A one dimensional illustration of the geometrical prescription involving a paraboloid and the linear gravitationalpotential used to demarcate regions which are *stuck* within pancakes and those that are *free*. The panels from top to bottom correspond to increasing cosmic epochs and therefore to increasing values of the radius of curvature of the paraboloid. The peaks of the potential correspond to voids at early times. A particle having Lagrangian coordinate q_0 is free in the uppermost panel, and has just entered into a caustic in the middle panel. The middle and lower panels describe the merger of clumps.

The Zeldovich approximation ceases to be valid after pancakes form and one has to use other dynamical methods to study the development of large scale structure during later epochs after shell crossing has occurred. The adhesion model (AM) provides us with a useful extension of the Zeldovich approximation within which the study of voids can be followed into the strongly non-linear

regime [1]. Like ZA the adhesion model too describes motion as a single step mapping between Lagrangian and Eulerian space. This mapping can be described geometrically by descending a paraboloid with radius of curvature proportional to the scale factor of the Universe $a(t)$, onto a three dimensional surface defined by the linear gravitational potential ϕ. If the paraboloid osculates ϕ in a given point having Lagrangian coordinate q then the Eulerian position of q can be determined by noting whether the paraboloid touches or intersects the potential at any point other than q. If it does, then the given point (q) has already entered into a caustic at a time t and is labelled *stuck*, otherwise it has not and is labelled *free* (for details see figure 1 and [5]). The location of caustics viz. pancakes, filaments and knots can be determined from the corresponding Lagrange space picture by moving the border between *stuck* and *free* regions by means of the Zeldovich approximation (1). The distribution of filaments and knots is shown superimposed on the Lagrangian picture of *stuck* and *free* regions for a two dimansional toy model Universe in figure 2.

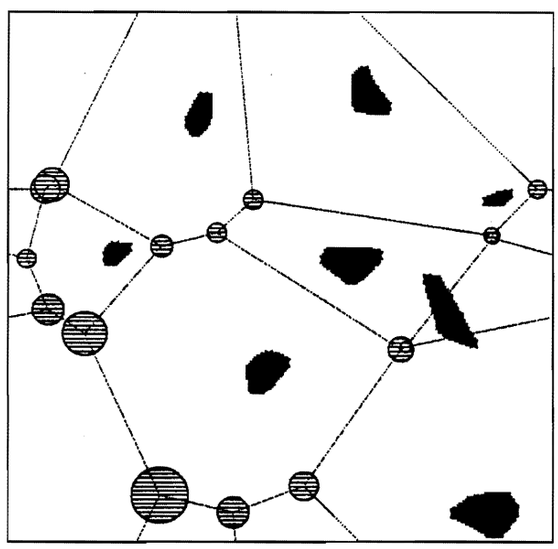

Fig. 2. Filaments (dots) and knots (filled circles) in Eulerian space are shown superimposed on *free* (shaded) and *stuck* (unshaded) Lagrangian regions.

It is easy to determine the volume of a *void* in Eulerian space by applying the Zeldovich deformation formula (2) to the corresponding *free* region in Lagrangian space. The volume of one such void is

$$V_E = \sum_{i=1}^{N} dV_L \; [1 - a(t)\lambda_1(\mathrm{q}_i)]\,[1 - a(t)\lambda_2(\mathrm{q}_i)]\,[1 - a(t)\lambda_3(\mathrm{q}_i)] \qquad (4)$$

where N denotes the number of elementary *free* volume elements in a given Lagrange space void. The above ansatz provides an elegant semi-analytical approach for determining the distribution function of voids (the void spectrum) for cosmological models in which structure forms through gravitational instability.

3 Results

Some key results obtained by applying the adhesion model to the study of voids are summarised in figures 3 - 8. Briefly these are:
(1) The dynamics of voids can be fairly complicated, some voids contract under the influence of larger voids surrounding them – ultimately disappearing, whereas others expand and grow with time. As a result the void spectrum (which describes the fractional volume contained in voids of a given diameter class) evolves considerably with cosmological epoch as is clearly demonstrated in figure 3 for a primordial power spectrum which mimicks the standard Hot Dark matter model. (Also see [2, 4, 5].)

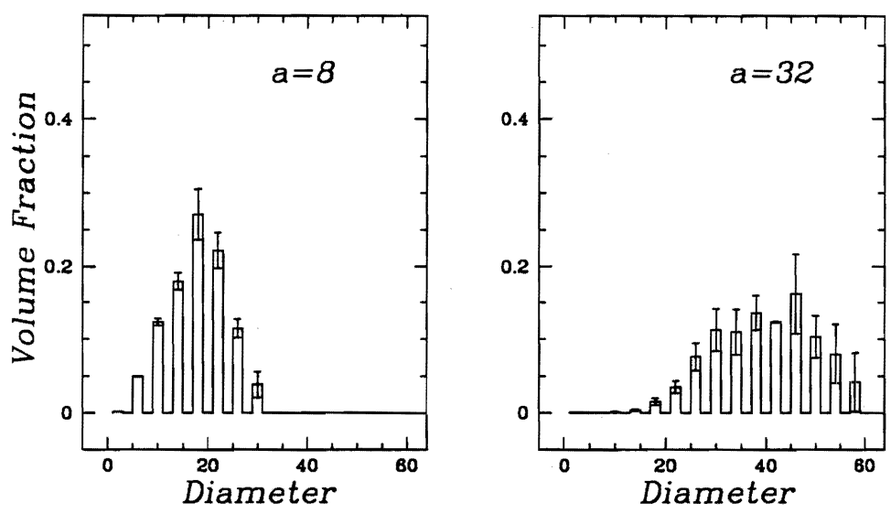

Fig. 3. The void spectrum for a cosmological model with a truncated Harrison-Zeldovich initial spectrum, is plotted for two values of the expansion factor. The x-axis is an indicator of the void diameter in units in which the box size is 128. The error bars correspond to the *rms* dispersion over three simulations.

In all our simulations we find that there are a substantially greater number of large voids at later times than there were at earlier times. A consequence of this fact is that for scale invariant power spectra $P(k) \propto k^n$ the mean void diameter grows as $D_{mean} \propto a(t)^{2/(n+3)}$ for $-1 \leq n \leq 1$. For simulations performed on a 128 Mpc box with the COBE-normalised CDM model (with

H_0 = 50km/sec/Mpc) the mean void size is $D_{mean} \simeq 11$ Mpc, the maximum void size is considerably larger $D_{max} \simeq 60$ Mpc.
(2) The sizes of voids are strongly correlated with the value of the gravitational potential at void centers – larger voids seem to be associated with higher regions in the potential. This relationship is shown in figure 4 where the value of ϕ at the (Lagrangian) center of a void is plotted against the void diameter. This correlation increases with time and can be approximately described by the empirical relation $\langle\phi\rangle/\phi_{rms} \propto \sqrt{a(t)}$ where $\langle\phi\rangle$ is the mean value of the gravitational potential averaged over all void centers at a given cosmological epoch. (Details of our results can be found in Sahni et al. (1994).)

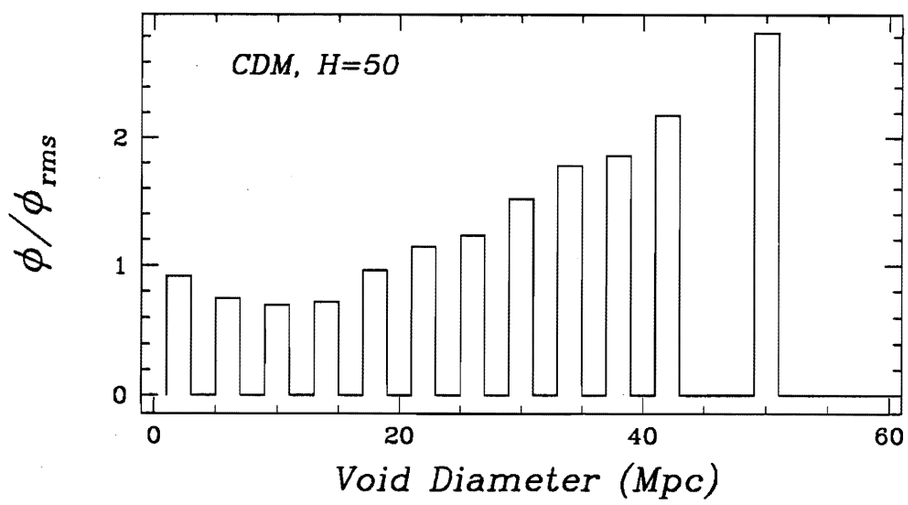

Fig. 4. The value of the primordial gravitational potential at void centers (in units of its *rms* value ϕ_{rms}) is plotted against the void diameter for one realisation of the COBE - normalised CDM model.

(3) Some features of gravitational clustering seem to be relatively insensitive to the value of the spectral index. An example is the fraction of matter in caustics $f(a)$ ($1 - f(a)$ is a measure of the underdensity in voids) shown in figure 5 for different power law spectra. We clearly see that the rate of infall of matter into caustics is virtually insensitive to values of the spectral index in the range $-1 \leq n \leq 1$. Our results show that by the epoch when $k_{nl}^{-1} \simeq R_* = \sqrt{3}\sigma_1/\sigma_2$ virtually all of the matter is already in caustics. (σ_j are the moments of the distribution of ϕ: $\sigma_j^2 = \frac{1}{2\pi^2}\int dk k^2 k^{2j} |\phi_k|^2$.) Interestingly $k_{nl}^{-1} \simeq R_*$ also marks the epoch when cellular structure in the Universe is completely formed. For $k_{nl}^{-1} < R_*$ gravitational instability proceeds via panacaking, whereas for $k_{nl}^{-1} > R_*$ hierachical clustering describes the mutual atraction and subsequent merger of pancakes, filaments and knots formed during the previous 'pancaking' epoch.

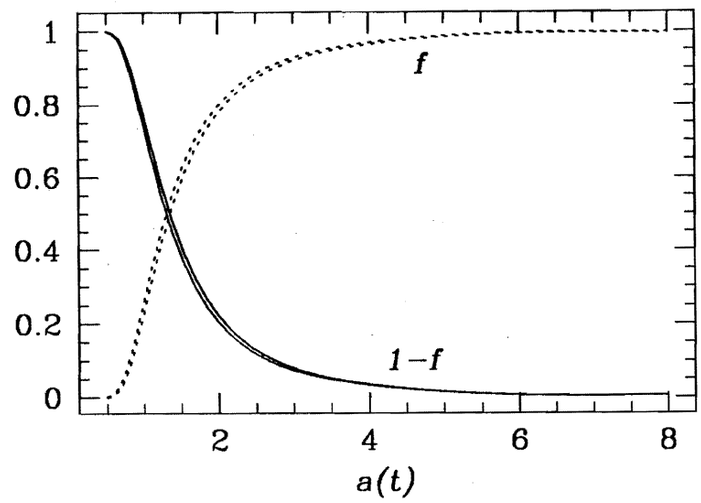

Fig. 5. The matter fraction in caustics (dotted) and the related underdensity of matter in voids (solid) are shown at different expansion epochs for power law primordial spectra. (The upper (lower) dotted curve and the lower (upper) solid curve correspond to $n = -1$ ($n = 1$).)

(4) Voids can be populated with substructure such as mini-Zeldovich pancakes and filaments which pass through a void that is bounded by major Zeldovich pancakes. In Lagrangian space such voids will be distinguished by having an *individual* topology that is not trivial: an empty void with no substructure will have the topology of a sphere, whereas a void with a single mini-pancake within it will have the topology of a torus etc. A convenient measure of the topology of a compact surface is its *genus*, a void having N units of substructure will have *genus* $= N$ when its topology is measured in Lagrangian space. In figure 6 we show a void from our simulations of the CDM model which has the topology of a torus in Lagrangian space. In figure 7 we plot the genus measure of voids belonging to a given diameter class against the void diameter.

We find that larger voids are on an average more likely to possess substructure than smaller voids. We also find that void substructure evolves with time as do the voids themselves. We demonstrate this result for the COBE-normalised CDM model in figure 8 in which the evolution of the *mean genus* averaged over the entire void ensamble is shown.

We find that approximately one void in fifteen will have some substructur at the present epoch in the CDM model. Evidence of substructure in several voids including the Bootes void and voids in the CfA survey seems to support the point of view that structure formed via gravitational instability as demonstrated by the adhesion model.

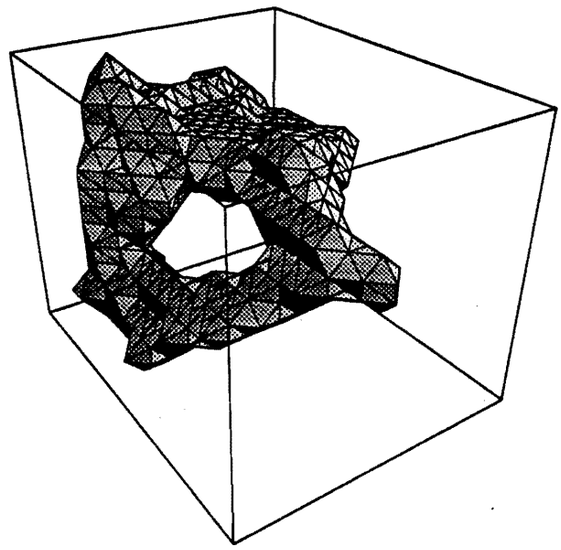

Fig. 6. A void in Lagrangian space having non-trivial topology.

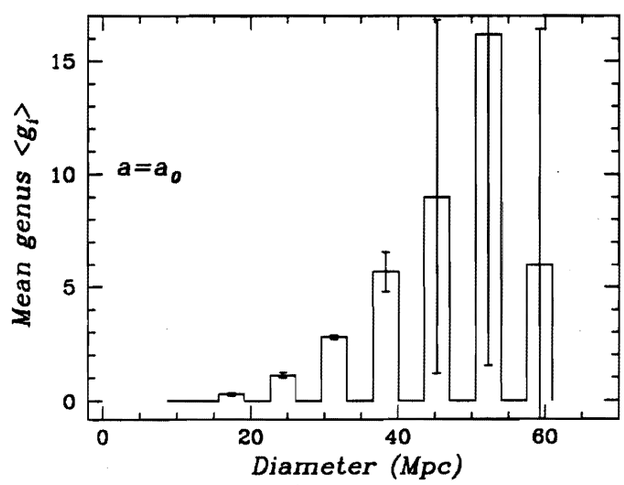

Fig. 7. The mean genus measure of voids belonging to a given diameter class $\langle g_i \rangle$, is shown plotted against the void diameter for the COBE-normalised CDM model.

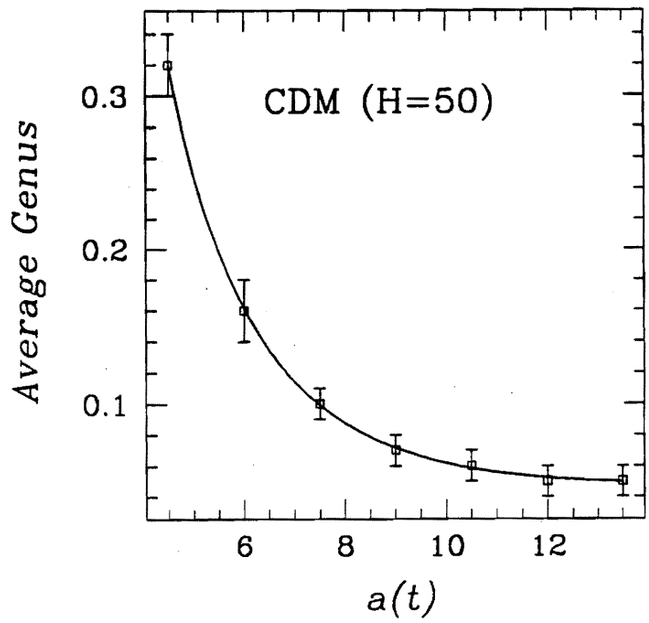

Fig. 8. The mean genus characteristic evaluated for the entire ensamble of voids in the CDM model, is shown for different values of the expansion factor a ($a = 9$ corresponds to the present epoch).

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