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Nonlinear Sigma Model for Inflation Scenarios. II

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NONLINEAR SIGMA MODEL FOR INFLATION SCENARIOS.II

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Abstract

In this paper the self-gravitating nonlinear sigma model is considered in the context of inflation scenarios as an alternative to the self-interacting scalar field theory. The complete set of new exact solutions for the two-component sigma model in the framework of spatially flat Friedmann-Robertson-Walker and de Sitter universes is obtained.

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1. Introduction

Different versions of the inflationary universe scenario have exploited a theory of a self-interacting scalar field (SSF) coupled to gravity within the context of cosmological spaces. The non-linear scalar field theory, possessing a self-interaction potential $V(\phi)$, can be regarded as the best effective theory which describes particle physics phenomenon in the early universe such as a spontaneous breakdown and restoration of the GUT gauge-symmetry [1].

On the basis of a SSF minimally coupled to Einstein's gravity in the framework of isotropic and homogeneous Friedmann-Robertson-Walker (FRW) universe the models of 'old inflation'[2], 'new inflation'[3] and chaotic inflationary scenario [4] have been analysed. Anisotropic and inhomogeneous cosmologies as well as $(N+1)$-dimentional space-times have also been investigated (see, for example,[5-7]).

Extended inflation [8] based on Brance-Dicke (BD) theory of gravity and it means in fact the existence of two scalar fields,namely BD field and 'matter' scalar field [9]. The BD type of a scalar-tensor theory of gravity as well as R^2- gravity and induced gravity models are conformally equivalent to Einstein's gravity coupled to a SSF [10]. The inflation scenarios in the presence of non-minimal coupling to gravity have been studied as well (see [11] and literature quoted therein).

Overlooking the models above, we can find, that all these inflationary models are closely connected with a gravitational field produced by a single scalar field. However the progress of great unification and supersymmetry theories shows that a gravitational field may be produced by several (effective) scalar fields in the very early universe.This approach has been used for the multicomponent inflationary model [12] and double (or multiple) inflation scenario [13]. Both of them are based on noninteracting scalar fields with different potentials and have led to better understanding of large-scale structure of the universe [14] and some other cosmological problems [12,13,15] like horizon, flatness and graceful exit ones.

The next logical step is to introduce an interaction between scalar fields. And there exist at least two possibilities [16].The first one is to use a potential term depending on all scalar fields. These type of models has been considered, for example, in references [7,17-19]. The second possibility is chiral model (or nonlinear sigma model), where an interaction is introduced by the constraint on values of scalar fields [20]. The nonlinear sigma model coupled to metric tensor of eucludean signature $(++++)$ has been proposed in ref.[24].The self-gravitating NSM, based on the space-time of lorentzian signature $(---+)$, has been introduced in [23]. In the framework of Kaluza-Klein theories the nonlinear sigma model in the context of the early universe was considered in [21].

It should be pointed out here that we have different equations and different physical sense for Kaluza-Klein theories and self-gravitating NSMs. We don't use the Einstein's equations for the target space, as has been usually done in Kaluza-Klein theory.Besides, the Einstein's equations for coordinate space-time contain the energy momentum tensor

of chiral fields, while for Kaluza-Klein theory one has a vacuum Einstein's equations.

Let us repeat again, that the SSF theory presents only the simplest example of an effective theory, describing gauge and Higgs fields (of GUT) coupled to gravity. The nonlinear sigma model (NSM) can be viewed as an effective model for GUT, as the SSF model is. Besides, the NSM is more rich than the SSF theory and bears many features corresponding to a gauge theory [20]. Moreover, the NSM can be reduced to the SSF theory [16,22].

In the present article the self-gravitating nonlinear sigma model is analysed in the framework of isotropic and homogeneous universes.As a first step of investigations of an inflation scenario, the system of Einstein's and chiral fields equations have been solved. The set of the exact solutions for the two componenet NSM in the spatially flat universes is obtained in the presence and in the absence of the cosmological constant. For the sake of completness all possibilties for the sign of the cosmological constant have been used, because of it importance in cosmology [25].

In section 2 the basic equations are presented, and the method of solving them is described.

The section 3 contains the set of exact solutions for the spatially flat FRW universe.

In section 4 and 5 the solutions for the spatially flat de Sitter universe are presented, with possitive and negative cosmological constant, respectively.

Finally, in section 6 obtained solutions are discussed and few remarks are given.

2. The model and basic equations

Let us consider the self-gravitating NSM [23] in the presence of the cosmological constant

$$S_3 = \int_{\mathcal{M}} d^m x \sqrt{g} \Big\{ \frac{R + 2\Lambda}{2\kappa} + \frac{\alpha}{2} h_{AB} \varphi^A_i \varphi^B_k g^{ik} \Big\}, \tag{1}$$

where $x = (x^1, \ldots, x^m)$ are the local coordinates of base space-time, Riemannian manifold $(\mathcal{M}, g_{ik})$; $\varphi = (\varphi^1, \ldots, \varphi^n)$ is the multiplet of scalar fields which lies in a target or chiral space, Riemannian manifold $(\mathcal{N}, h_{AB})$; α is a coupling constant; $g = |det(g_{ik})|$; $\varphi_k := \varphi_{,k} := \partial_k \varphi$; R is the scalar curvature and κ is Einstein's constant. Where repeated indices occur the summation convention is assumed. By varying the action (1) with respect to φ^A we obtain the equations of motion

$$\frac{1}{\sqrt{g}} \partial_i (\sqrt{g} g^{ik} h_{AB} \varphi^B_k) - \{ h_{CD} \Gamma^D_{AB} \varphi^B_i \varphi^C_k g^{ik} \} = 0. \tag{2}$$

Here the Γ^D_{AB} are the Christoffel symbols of the chiral manifold $(\mathcal{N}, h_{AB})$.

Einstein's equations for the model (1) with the energy momentum tensor

$$T_{ik} = \alpha \{ h_{AB} \varphi^A_i \varphi^B_k - \frac{1}{2} g_{ik} \varphi^A_j \varphi^B_l h_{AB} g^{jl} \} \tag{3}$$

can be reduced to the next form

$$R_{ik} = \kappa\alpha h_{AB}\varphi_i^A\varphi_k^B + \Lambda g_{ik}. \tag{4}$$

It should be noted here that the NSM coupled to metric tensor with signature (++++) has been proposed in ref.[24]. In [24] the instanton and meron solutions for the system of equations (2),(4) have been obtained in the framework of conformally-Weyl's spaces. The self-gravitating NSM, based on the space-times of lorentzian signature, have been introduced in [23]. This type of models stand on the Einstein's conjecture: the energy momentum tensor of matter (3) creates a gravitational field. The start point in Kaluza-Klein theories is quite different: the dynamics of matter corresponds to Einstein's vacuum equations in a space-time of more than physical (four) dimention. And the equivalence between equations of genaral relativity and Kaluza-Klein theory should be checked in every case. After that comment let us come to the concrete model.

Let us choose the metric of space-time to be a homogeneous and isotropic FRW universe

$$dS^2_{\mathcal{M}} = (dt)^2 - e^{a(t)}\{(dr)^2 + A^2(r)[(d\theta)^2 + \sin^2\theta(d\varphi)^2]\}. \tag{5}$$

Here

$$A^2(r) = \left\{ \begin{array}{c} \sin^2 r \\ (r)^2 \\ \sinh^2 r \end{array} \right\}$$

for a closed, flat and open universe, respectively.

We will use the chiral metric of the NSM, associated with the SSF theory [22], in the next form

$$dS^2_{\mathcal{N}} = d\phi^2 + 2P(\phi)d\chi^2, \ \varphi^1 = \phi, \ \varphi^2 = \chi \tag{6}$$

Here the chiral potential $P(\phi)$ corresponds, in some sence, to the potential of self-interaction $V(\phi)$. Namely, $P(\phi)$ should be equal to minus $V(\phi)$, if we want to get the same equation of motion for a nonlinear sigma model as for a scalar field one. Substituting (5) and (6) into (2) and (4), we can find that chiral fields ϕ and χ should be dependent on the cosmological time t only. Therefore, doing some simplification, we can obtain the system of equations

$$e^a[\frac{1}{2}a_{tt} + \frac{3}{4}a_t^2] = \Lambda e^a - 2k, \tag{7.1}$$

$$\frac{3}{2}[a_{tt} + \frac{1}{2}a_t^2] = -\kappa\alpha[\phi_t^2 + 2P\chi_t^2] + \Lambda, \tag{7.2}$$

$$\frac{3}{2}a_t\phi_t + \phi_{tt} - \frac{dP}{d\phi}\chi_t^2 = 0 \tag{7.3}$$

$$\frac{3}{2}a_t\chi_t + \chi_{tt} + \frac{dlnP}{d\phi}\phi_t\chi_t = 0 \tag{7.4}$$

where $k = +1, 0, -1$ for closed, flat and open universes, respectively.

Let us turn our attention to the method of solving the system of equations (7). First of all, the gravitational field can be calculated from the equation (7.1). Using this result one can obtain from the equation (7.2) the relation (ansatz) between chiral fields' derivatives and chiral potential term $P(\phi)$. The ansatz (7.2) gives us the possibility to find the energy momentum tensor and to reduce the equation (7.3) to the ordinary differential equation of the first order. This reduced equation can be, in general, formally solved. But the result will depend on our suggestions about the chiral potential and first derivatives from chiral fields. Therefore we will present a set of possible solutions for each case. There is no need to solve the equation (7.4), because it is differential consequence of (7.2)-(7.3).

It should be noticed here that the equation (7.1) can also be reduced to the first order ordinary differential equation

$$zp\frac{dp}{dz} + 2p^2 - \Lambda z^2 + 2k = 0, \quad (p = z_t, z = e^{a/2}), \tag{8}$$

which gives the standard gravitational fields corresponding to the extremely stiff matter [26] or massless scalar field.

Thus the equation (8) gives us the scalar factor $e^{a(t)}$ for the FRW and de Sitter open, flat and closed universes for the two-component NSM (6). In the case of spatially flat universes we have got the complete set of exact solutions for the model under consideration.

3. Exact solutions for the spatially flat FRW universe ($k = 0, \Lambda = 0$)

Taking into account $\Lambda = 0, k = 0$ in equations (7) we immediatly obtain from (7.1)

$$K^2(t) = e^{a(t)} = a_0(t + t_0)^{2/3}, a_0 = const.$$

Thus the metric of space-time (5) takes the form of the power law inflation

$$dS^2_{\mathcal{M}} = (dt)^2 - a_0(t + t_0)^{2/3}\{(dr)^2 + r^2[(d\theta)^2 + \sin^2\theta(d\varphi)^2]\}. \tag{9}$$

This gravitational field is independent from a chiral potential term $P(\phi)$. As well as the energy momentum tensor is. Non-vanishing components of the energy momentum tensor can be obtained now from (7.2) and read

$$\varepsilon = T_4^4 = \frac{1}{3\kappa t^2} = -T_3^3 = -T_2^2 = -T_1^1 = p. \tag{10}$$

It is obvious from (10) that the matter, corresponding to this solution, gets an extremely stiff state. And there exists a physical singularity for the solutions presented in this section as $t \to 0$.

3.1 Solution A.

Let $\phi_t^2 = h(t)$ and $P = P(t)$. The dependence P from t may be suggested because during the phase transition there exists a dependence of the potential $V(\phi)$ from temperature [1] and, consequently, from time. Thus, obtaining $\phi_t(t)$ from the equations (7), we can find $\frac{dP}{d\phi}$ and then, by integrating, P as a function from ϕ.

The solution can be written as

$$\phi = \int \sqrt{\beta^2 - \frac{\mu^2}{P(t)}} \frac{dt}{t},$$

$$\chi = \frac{\mu}{\sqrt{2}} \int \frac{1}{P(t)} \frac{dt}{t}, \tag{11}$$

$$\mu^2 = const, \beta^2 = 2(3\kappa\alpha)^{-1}, P < \frac{\mu^2}{\beta^2}.$$

We should remember that $P < 0$ when $V > 0$. Therefore the chiral metric (6) has to have a lorentzian signature $(+-)$.

Introducing the time dependence for the potential "by hand", we can obtain some analytical solutions from (11). For example, when $P = -\nu^2 t^2, \nu^2 = const$

$$\phi = -f(t) + \frac{\beta}{2} \ln \left| \frac{\beta + f(t)}{\beta - f(t)} \right| + \phi_0,$$

$$f(t) = \sqrt{\beta^2 + \frac{\mu^2}{\nu^2 t^2}}, \tag{11a}$$

$$\chi = \frac{\mu}{2\sqrt{2}\nu^2} t^{-2} + \chi_0.$$

3.2 Solution B.

Let us suppose that $\phi_t = \frac{\psi(\phi)}{t}$ and $P = P(\phi)$. Then we can find the solution

$$\ln t = \int \frac{d\phi}{\sqrt{\beta^2 - \frac{\mu^2}{P(\phi)}}},$$

$$\chi = \frac{\mu}{\sqrt{2}} \int \frac{1}{P(\phi)} \frac{dt}{t}, \tag{12}$$

$$\mu^2 = const, P < \frac{\mu^2}{\beta^2}.$$

In this case the chiral potential P depends on the chiral field ϕ. Substituting the concrete potential $P(\phi)$ into the first integral in (12), one can find the dependence ϕ from t. And then to define the chiral field χ from the second integral in (12).

3.3 Solution C

Let us suppose that $P(\phi) = P_0 = const$ as in the case of 'slow rolling' regime [1]. This assumption denotes that we investigate the theory of two non-interacting scalar fields. The solution in this case is

$$\phi = c_1 \ln t + \phi_0,$$

$$\chi = c_2 \ln t + \chi_0; \tag{13a}$$

$$P = P_0, \; c_1^2 + 2P_0 c_2^2 = \beta^2.$$

Note that there exists an exceptional solution when $c_1^2 = 0$

$$\phi = \phi_0 \neq 0, \quad P = P_0,$$

$$\chi = \chi_0 + \beta(|2P_0|)^{-1/2} \ln t. \tag{13b}$$

In this case the potential does not equal to zero and coupling constant α should be negative when $P_0 < 0$. The chiral metric (6) is degenerate for the solution (13b).

3.4 Solution D

Let $\chi_t = 0$ or $\chi_i \chi^i = 0$, that is χ is an isotropic field. Then we have the standard solution for massless scalar field for any type of the chiral potential $P(\phi)$

$$\phi = \phi_0 + \beta \ln t, \quad \chi = \chi_0 = const. \tag{14}$$

Note that if $P = P_0$, the solution (14) is equivalent to (13a) with $c_2^2 = 0$. The chiral metric (6) is also degenerate for this solution.

3.5 Solution E

Let $\chi_t = 1$. This case, corresponding to the SSF theory [22], is described by relations

$$\phi = 2f_1(t) + \beta \ln \left| \frac{\beta - f_1(t)}{\beta + f_1(t)} \right| + \phi_0,$$

$$f_1(t) = \sqrt{\beta^2 - \mu^2 t}, \ \mu^2 = const, \tag{15}$$

$$\chi = t + t_0, \quad P(\phi(t)) = \frac{\mu}{\sqrt{2}} \frac{1}{t}.$$

The relations (15) may be obtained from (11) by choosing $P(t)$.

3.6 Solution F

Let us consider the case of the exponential potential

$$2P(\phi) = me^{\lambda\phi}, \ m = const, \ \lambda = const. \tag{16}$$

The solution for the case we can write:

$$\phi = -\beta \ln t + 2\lambda^{-1} \ln(1 + c_3 t^{\lambda\beta}), \ c_3 = const,$$

$$\chi = 2\beta m^{-1} \int t^{(3/2)\lambda\beta - 1}[1 + c_3 t^{\lambda\beta}]^{-3} dt \tag{17}$$

It is clear, that for some relations between λ and β (for example $\lambda\beta = n \in \mathbf{Z}$) the last integral in (17) leads to the analytical function for χ.

It should be noted here, that the energy momentum tensor for all solutions, presented in this section, are determined by (10).

4. Exact solutions for the spatially flat de Sitter universe ($k = 0, \Lambda \neq 0, \Lambda > 0$)

Solving the equation (7.1) with $\Lambda > 0$, we can obtain the exponentially expanding universe (5) in the form

$$dS^2_{\mathcal{M}} = (dt)^2 - b_0\{\cosh(\tau)\}^{2/3}\{(dr)^2 + r^2[(d\theta)^2 + \sin^2\theta(d\varphi)^2]\}. \tag{18}$$

Here $\tau = t\sqrt{3\Lambda}, b_0 = const$. We have got again the gravitational field which is independent from the metric of a target space (6). Non-vanishing components of the energy momentum tensor are

$$\varepsilon = T^4_4 = -\frac{\Lambda}{\kappa \cosh^2(\tau)} = -T^3_3 = -T^2_2 = -T^1_1 = p. \tag{19}$$

The relations (19) correspond to the extremely stiff matter again. Moreover, we have got a non-singular solution, which belong to the oscillating universe of the second kind

[26]. When $t = 0$, the energy density does not equal to zero, and the gravitational field (18) becomes the Minkovskian space-time. Let us now take into account some special cases corresponding to the solutions in the section 3.

4.1 Solution A

Let $\phi_t^2 = h_1(t)$ and $P = P(t)$. The solution can be written as

$$\begin{aligned} \phi &= \int \sqrt{-[\frac{\mu^2}{P} + \beta^2]} \frac{d\tau}{\cosh \tau}, \\ \chi &= \frac{\mu}{\sqrt{2}} \int P^{-1} \frac{d\tau}{\cosh \tau}, \\ -\frac{\mu^2}{\beta^2} &< P < 0. \end{aligned} \tag{20}$$

It is clear that we have to introduce the time dependence for P(t) "by hand".

4.2 Solution B

Let $\phi_\tau = g_1(\phi) \cosh^{-1} \tau$ and $P = P(\phi)$. Then (7.2) implies that $\chi_\tau \cosh \tau = g_2(\phi)$. The solutions can be obtained from the relations

$$\begin{aligned} \tau &= arcsinh \tan I_1(\phi), \\ I_1(\phi) &= \int \{-[\nu^2 P^{-1} + \beta^2]\}^{-1/2} d\phi, \\ \chi &= \frac{\nu}{\sqrt{2}} \int P(\phi)^{-1} \frac{d\tau}{\cosh \tau}, \\ \nu^2 &= const, -\nu^2 \beta^{-2} < P < 0. \end{aligned} \tag{21}$$

The rule of obtaining solutions is the same as in the item 3.2.

4.3 Solution C

Let $P(\phi) = P_0 = const$. Under this assumption we have got the theory of two non-interacting scalar fields. The solution is

$$\begin{aligned} \phi &= c_1 \arctan(\sinh \tau) + \phi_0, \\ \chi &= c_2 \arctan(\sinh \tau) + \chi_0, \end{aligned} \tag{22}$$

$$c_1^2 + 2P_0 c_2^2 = -\beta^2, \; P_0 < -\frac{c_1^2}{2c_2^2}.$$

4.4 Solution D

Let $\chi_t = 0, (c_2^2 = 0)$. Then, for any type of the potential $P(\phi)$, we can find the solution

$$\phi = \beta \arctan(\sinh \tau) + \phi_0,$$
$$\chi = \chi_0 = const. \tag{23}$$

The coupling constant α should be less then zero.

4.5 Solution E

Let $\chi_\tau = 1$. This is the special case of the solution 4.1 A (20).

$$\phi = \int \sqrt{-[\beta^2 + \sqrt{2}\mu \cosh \tau]} \frac{d\tau}{\cosh \tau},$$
$$\chi = \tau + \chi_0, P = \frac{\mu}{\sqrt{2}} \cosh^{-1} \tau. \tag{24}$$

4.6 Solution F

Considering the case of the exponential potential (16), one can obtain the next solution

$$\phi = \phi_0 - \frac{4}{\lambda \kappa \alpha} \ln \cos(\frac{v}{4}\sqrt{2\kappa\alpha/3}),$$
$$v = \frac{\lambda}{2} \arctan(e^\tau), \quad c_3 = const, \tag{25}$$
$$\chi = c_3 \int \left\{ \cos(v\sqrt{2\kappa\alpha/3}) \right\}^{2/\kappa\alpha} \frac{dv}{\lambda}.$$

The energy momentum tensor for all solutions, presented here, is defined by (19).

5. Exact solutions for the spatially flat de Sitter universe ($k = 0, \Lambda \neq 0, \Lambda < 0$)

When $\Lambda < 0$ the solution of the equation (7.1) leads to the next metric of the space-time

$$dS^2_{\mathcal{M}} = (dt)^2 - d_0|\cos\eta|^{2/3}\{(dr)^2 + r^2[(d\theta)^2 + \sin^2\theta(d\varphi)^2]\}. \tag{26}$$

In (26) $\eta = -\sqrt{-3\Lambda}t, d_0 = const$. Non-vanishing components of the energy momentum tensor are

$$\varepsilon = T^4_4 = -\frac{\Lambda}{\kappa\cos^2\eta} = -T^3_3 = -T^2_2 = -T^1_1 = p. \tag{27}$$

According to (26),(27) there exists some "pulsation" of the universe (26) and the energy density ε. When $t = 0$,we have Minkovskian space-time with the energy density $\varepsilon = -\Lambda\kappa^{-1} > 0$. Then space-time will collapse into the singularity with the infinite energy density when $t = -\frac{\pi}{\sqrt{-3\Lambda}}(\frac{1}{2}+n), n = -1, -2, -3, \ldots$. In this way, we have got an oscillating universe of the first kind [26].

Now we can obtain the solutions for chiral fields in the same special cases as in sections 3 and 4.

5.1 Solution A

Let $\phi_t = \phi_t(t)$ and $P = P(t)$. The solution can be written as

$$\phi = \int\sqrt{\mu^2 P^{-1} + \beta^2}\frac{d\eta}{\cos\eta}, \quad P < -\mu^2\beta^{-2}, \tag{28}$$

$$\chi = \frac{\mu}{\sqrt{2}}\int P^{-1}\frac{d\eta}{\cos\eta}, \eta = -t\sqrt{-3\Lambda}.$$

5.2 Solution B

Let $\phi_\eta = h(\phi)\cos^{-1}\eta$ and $P = P(\phi)$. The solution can be obtained from the formulas

$$\eta = \arcsin\tanh I_2(\phi),$$

$$I_2(\phi) = \int\sqrt{\beta^2 - \frac{\nu^2}{P(\phi)}}d\phi, \tag{29}$$

$$\chi = \frac{\nu}{\sqrt{2}}\int P(\phi)^{-1}\frac{d\eta}{\cos\eta},$$

$$\nu = const, P < -\nu^2\beta^{-2} < 0.$$

5.3 Solution C

Let $P(\phi) = P_0 = const.$ Then we can write the solution

$$\phi = c_1 \ln\left|\frac{1+\sin\eta}{1-\sin\eta}\right| + \phi_0,$$

$$\chi = c_2 \ln\left|\frac{1+\sin\eta}{1-\sin\eta}\right| + \chi_0, \tag{30}$$

$$P = P_0, \quad c_1^2 + 2P_0 c_2^2 = \beta^2, \quad -\frac{c_1^2}{2c_2^2} < P_0 < 0.$$

5.4 Solution D

Let $\chi_\eta = 0$, i.e. $c_2^2 = 0$ in the solution (30). Then for any type of $P(\phi)$ we can find the solution

$$\phi = \beta \ln\left|\frac{1+\sin\eta}{1-\sin\eta}\right| + \phi_0, \tag{31}$$

$$\chi = \chi_0 = const.$$

5.5 Solution E

Let $\chi_\eta = 1$. The solution is

$$\phi = \int \sqrt{\beta^2 - 2c_3\cos\eta}\,\frac{d\eta}{\cos\eta}, \tag{32}$$

$$\chi = \eta + \chi_0, \quad P(\eta) = \frac{c_3}{\cos\eta}, \quad c_3 = const.$$

5.6 Solution F

When the chiral potential takes an exponential form (16) we can find

$$\phi = \sqrt{-3\Lambda}\ln\cosh\{\frac{\beta\lambda}{4}\ln\left|\frac{1+\sin\eta}{1-\sin\eta}\right|\} + \phi_0. \tag{33}$$

The other field χ can be found from "ansatz" (7.2).

6. Discussion and remarks

The concept of inflation is based on the proposition, that the inflation is a temporary stage of evolution of the universe, posessing the next features:

(i) exponentially (or power law) expansion of the universe;

(ii) vacuum-like state of matter $p = -\epsilon$ with the absence of particles;

(iii) the spontaneous breakdown of the GUT gauge-symmetry, effectively described by a SSF coupled to gravity;

(iv) nonvanishing cosmological constant $\Lambda : \Lambda \neq 0$.

In the present paper, it is proposed to use massless nonlinear sigma model as an effective theory (section 2) instead of the self-interacting scalar field theory (see the item (iii)). It is found, that in the case of the two-component NSM there exists , in accordance with the item (i), an exponentionally and power law expansion of the universe (sections 3 and 4). Besides , it is obtained the oscillating type of the universe (section 5) when the cosmological constant is negative. As for the statement (ii) it seems impossible to get the vacuum-like state of matter in the framework of massless NSM. By introducing the massive term (or the potential of interaction for chiral fields) it is , clearly, possible to reach the condition (ii), because of the strong correspondence to the SSF theory. Therefore in our investigation, based on massless NSM , we have introduced the cosmological constant "by hand". Nevertheless, it is possible to think about the reason of arising the cosmological constant in the same way, as for the SSF theory. Namely, Λ may arise from an additional term of the NSM like the mass or the potential of self-interaction.

It is shown, that all solutions of the two-component NSM lead to the extremely stiff matter , whith the equation of state $p = \varepsilon$. It is found, that gravitational fields for the two-component NSM are independent from chiral metric and coincide with the gravitational fields of the extremely stiff matter or massless scalar field.

It seems, that solutions for the chiral field equations as well as the exact solutions for the gravitational field will be useful for further physical analysis of an inflationary scenario. If we will do the identification between the potential $V(\phi)$ and $P(\phi)$, one can insert $P(\phi)$ as given function from ϕ into solutions presented in sections 3-5. It is also possible to use an appropriate approximation, corresponding to the version of the scenario, in general equations (7), presented in section 2.

It is interesting to note, that solutions E (in the sections 3-5), corresponding to the SSF theory , show the physical sense of an auxiliary chiral field χ as the time-like coordinate.

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