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ON THE COSMOLOGICAL CONSTANT PROBLEM

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ABSTRACT

A simple quantum mechanical model of a closed interacting system is studied following the intrinsic time formalism developed recently,on the basis of the modified Born-Oppenheimer approximation.Apart from shedding further insights into the recent results on a possible nongravitating vacuum energy in the universe, the study also offers potentially interesting possibilities even in the atomic/molecular physics.

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In this paper we analyze an interesting question: Can the global vacuum energy in our universe gravitate? The question stated in this form already eliminates the local vacuum polarization energy around a black hole event horizon from our discussion.One notes however that the definition of a vacuum state and consequently the associated vacuum energy in a cosmological background is not very well defined[1].An well known example is the Rindler's accelerated frames in an otherwise flat Minkowski spacetime.Studies in the quantum field theory in a curved spacetime during the last two decades reveal however that the concept of an adiabatic vacuum is quite general, and powerful enough to tackle a number of delicate issues concerning the quantized fields in a cosmological model.In the course of the following discussion on the gravitational properties of the vacuum energy we will therefore assume, whenever necessary, the existence of a suitably defined adibatic vacuum in a cosmological background. It turns out that even then our discussion remains sufficiently general to consider physically interesting situations.

We start by listing the odds favouring the view that the vacuum energy should be gravitating. Not surprisingly, the list is not at all long. First and appears to be the most powerful, is the argument based on the equivalence principle. A field theoretic vacuum consists of an infinite number of harmonic oscillator modes each contributing a ground state energy $O(\frac{1}{2}\hbar)$. Because of the global Poincare invariance of the Minkowski space, the infinite vacuum

energy, being unobservable, can be normal ordered to zero. However, in the presence of an high energy phase transition, e.g., the GUT phase transition, the vacuum field configuration can in principle make transitions from a metastable state to the real vacuum state, thus releasing a huge amount [$O(10^{56} Gev^4)$ for the GUT phase transition and $O(10^{76} Gev^4)$ for the Planck scale transition] of vacuum energy. In the presence of a nonzero curvature in the spacetime, this nonzero vacuum energy also gets a small curvature contribution. However, one does not expect the curvature contribution to be large in the ordinary field theoretic vacuum unless one could probe distance scales of the order of the Planck scale.

Now, the equivalence principle tells us that every bit of inertial mass is a source of the gravitational field with a universal coupling strength G, the Newton's constant. Further, this gravitational field can only be neutralized by moving to an accelerated frame defined locally in the neighbourhood of that mass. In other words, in the presence of gravity, local accelerated frames are equivalent to the inertial frames. The total vacuum energy of the universe, by virtue of the mass-energy relationship, must therefore act like a huge source of the gravitational field. Any relativistic theory of gravity e.g., the general relativity theory, must take into account this huge source by accommodating in the RHS of the field equations a contribution from the Vacuum energy Λ. The general covariance demands that the contribution must be of the form $\Lambda g_{\mu\nu}$, where $g_{\mu\nu}$ is the metric of the space time. The vacuum energy

thus acts like a large positive cosmological constant in the gravitational action.Because of the large value of the vacuum cosmological constant one expects that its effects should be detectable in the large scale structure of the universe.Surprisingly, however, the observational limit on Λ is incredibly small: $|\Lambda| << 10^{-47}$ GeV^4.Thus, emerges a deep paradox in modern physics:What makes this huge source of the gravitational energy dynamically inert?In otherwords, why does the vacuum energy not gravitate?[2].

The second argument favouring the gravitating vacuum energy is based on the semiclassical Einstein equations (SCEE). Though,a variant of the above, the argument is formulated on the basis of an ansatz of the form [1]

$$G_{ij}=8\pi G \langle\chi|T_{ij}|\chi\rangle \qquad (1)$$

where G_{ij} is the Einstein's tensor and the RHS involves the stress-energy tensor expectation value in a suitably defined quantized matter (vacuum) state.One expects the above ansatz should yield reasonble predictions near the Planck scale.Far from the Planck energy, the expectation value,on the ground of physical consistency, should also approach the stress-energy tensor for the classical field configurations in the local Minkowski space quantum field theory.In any case, one again ends up facing a dark wall: the cosmological constant problem.Worse still, in this framework, one faces as well numerous other issues relating to the nature and validity of the ansatz,an invariant concept of the vacuum and the

particle states, issues on renormalizability and so on[3-8].

On little reflections one notes however, that the arguments supporting the gravitating vacuum energy lacks sound theoretical/empirical support. Even the equivalence principle can at best claim a local validity: the experiments establishing the equality of the inertial and the gravitational mass involve only locally defined conepts. One should remember that a local concept may not always allow a global extension. The local Lorentz invariance, for instance, does not have a global meaning in the presence of gravity. Similarly, the gravitational Casimir effect can also claim a local meaning, given in the form of the boundary conditions on the plates. A more elaborate framework is nevertheless necessary to support/disprove these plausible suggestions. In the following, we describe a simple mathematical model in favour of a nongravitating vacuum energy in the context of quantum cosmology(QC).

Let us consider an interacting quantum system consisting of two classes of degrees of freedom {Q}and {q}; widely separated in the mass scales. The Schrodinger equation describing the evolution of the total system is given by

$$H_T\Psi \equiv (H(P,Q) + h(p,q,Q)) \Psi(Q,q) = i\hbar\frac{\partial}{\partial t} \Psi(Q,q) \qquad (2)$$

where the Hamiltonian of the heavy system Q is of the form

$$H(P,Q) = \frac{1}{2M} G^{ij}P_iP_j + M\ V(Q) \qquad (3)$$

Here G^{ij} is the (super-)metric of the configuration space of Q_i variables, $P_i(=i\hbar\ \partial/\partial Q_i)$ $(i=1,2,...,N)$ the corresponding conjugate

momentum, V(Q) the (super-)potential, h the Hamiltonian of the lighter system q and M is a mass scale much heavier than that of the q-system. Ψ (Q,q) denotes the quantum state of the total system.The explicit time(t) derivetive in eq.(2) refers to an externally defined frame, as in the ordinary quantum mechanics/field theory.In QC there is no external frame;rendering eq.(2) a constraint on the Hilbert space of physical states,the Wheeler-Dewitt(WD) equation.

In the presence of an external time, the definition of the Hilbert space H of the states Ψ follows the standard formalism of quantum mechanics.Let $H_0 \subset H$ be the space of unit state vectors Ψ : $\langle\Psi|\Psi\rangle = 1$ and P the set of rays in H_0,the projective space.Then H_0 may be considered as a U(1) principal bundle over P [9,10]. The tangent space of H_0 at a point Ψ ,$TH_0(\Psi)$ can thus be decomposed into orthogonal complementary subspaces $TH_0(\Psi) = H_v \oplus H_h$,where H_v and H_h are known as the vertical and horizontal subspaces.Recall that this is a standard decomposition essential in defining a connection in a principal fibre bundle[8].A given state Ψ when evolved along a horizontal curve in H ,satisfies ,by definition ,the condition[9,10] $\langle\Psi_h|\frac{d}{dt}\Psi_h\rangle = 0$, $\Psi_h \in H_h$. This means that eq.(2) when constrained to the horizontal subspace ,reduces precisely to the WD equation of a gravity -matter system

$$H_T\Psi_h = 0 \qquad (4)$$

This amounts essentially to removing the dynamical phase from Ψ ,thus introducing a redefinition of the potential $V \to (V-\tilde{E}/M)$.*

Conversely, by retracing the above steps backward one may

obtain an interesting reinterpretation of the WD equation. Instead of defining the physical Hilbert space as the normalizable solutions of the WD equation, one may as well view the WD equation as a costraint along the horizontal subspace of *an* extended Hilbert space. The corresponding verticle subspace then appears to define an 'external' time frame even for the universe quantum states.A possible physical interpretation of this observation along with a remark on the inner-product in the space of states in QC will be offered later.

When H_T is not explicitly t-dependent, eq.(4) is the zero energy stationary state of the interacting system. At this level there is as such no concept of (an external) time in the dynamics of the total system.We now show that even in such a case an intrinsic concept of time can in principle be obtained provided the fast(light) degrees of freedom acquires a nontrivial geometric phase.In other words,*the existence of an internally defined microscopic time can be associated with a nontrivial geometric phase in the model*.This is seen easily by using an improved Born Oppenheimer(BO) adiabatic approximation to eq.(4) following the standard approach in semiclassical gravity [3-9](We drop the subscipt $\hbar$ in eq.(4) henceforth).

Let us write

$$\Psi\ (Q,q) = \psi\ (Q)\ \chi\ (Q,q) \qquad (5)$$

and project eq.(4) on the q-state χ ($\langle\chi|\chi\rangle=1$).(The following results are independent of any factor ordering ambiguity).We get the

effective WD equation for the Q - system

$$[\frac{1}{2M} G^{ij}(P_i - A_i)(P_j - A_j) + MV + \langle h\rangle + \rho\ ]\ \psi(Q) = 0 \qquad (6)$$

where $\langle h\rangle = \langle\chi|h|\chi\rangle$ and the adiabatic U(1) vector connection A_i and the 'electric' scalar potential ρ (non-abelian gauge groups are considered in ref[7]) are given by

$$A_i = i\hbar\langle\chi|\partial_i\chi\rangle,\ \partial_i = \partial/\partial Q_i$$

$$\rho = \frac{\hbar^2}{2M} G^{ij}\ \langle\chi|(P_i - A_i)(P_j - A_j)|\chi\rangle \qquad (7)$$

Note that eq.(6) does not have an explicit time dependence. One can therefore imagine a situation analogous to the WD equation in QC where the evolution of the total system ought to be described by an internally generated time parameter. Assuming the motion of the Q-system quasiclassical(e.g.the position variables of a heavy molecule), the effective wave function $\psi(Q)$can be approximated by an oscillatory WKB state $\psi \simeq \exp(\frac{i}{\hbar} MS_o(Q))$. The quasiclassical evolution of the Q system is thus given by

$$\frac{1}{2M} G^{ij}\ (P_i - A_i)(P_j - A_j) + MV + \langle h\rangle + \rho = 0 \qquad (8)$$

where the internal WKB timelike parameter τ and the classical effective momentum are defined by

$$\frac{d}{d\tau} = G^{ij}\frac{\partial\ So}{\partial Q_i}\frac{\partial}{\partial Q_j} \Rightarrow \frac{dQ_i}{d\tau} = G^{ij}\frac{\partial\ So}{\partial Q_j} = G^{ij}\frac{P_j}{M} \qquad (9)$$

A justification of this WKB ansatz is offered in ref[6] following the decoherence approach.

Eq.(8) is the standard zeroth order semiclassical equation

obtained from the BO expansion in the powers of M^{-1} with higher order corrections from the adiabatic gauge potentials. Note that A_i and $\langle h \rangle$ are $O(M^0)$ and ρ is $O(M^{-1})$. In addition, the state χ appearing in eq.(8) can be shown [11] to be determined by the $O(M^0)$ correction to the WKB phase S of the total system and satisfies the Schrodinger equation

$$i\hbar \frac{d}{d\tau} |\chi\rangle = h |\chi\rangle \tag{10}$$

So the $O(M^{-1})$ potential ρ asides, eqs.(8) and (10) are obtained selfconsistently at $O(M^0)$.

The definition (9) for the internal time τ and eqs.(7) and (10) now yield the important relation

$$A \equiv A_i dQ^i = \langle h \rangle \, d\tau \tag{11}$$

Here A denotes the U(1) adibatic connection 1-form. Eq.(11) encodes the fact that in an intrinsic description of the dynamics the total phase acquired by the internal quantum states must be geometric. The dynamical phase asks for an external definition of time. We note that the expectation values are highly gauge dependent. In fact, $\langle h \rangle$ is actually identified with a magnetic potential in 1-dimension only. The corresponding gauge invariant field 2-form F thus vanishes identically:

$$\begin{aligned} F &= dA = d^2\alpha = 0, \\ \alpha &= \int^{\tau(Q)} \langle h \rangle \, d\tau \end{aligned} \tag{12}$$

for any (super)metric G^{ij}. Here, $d = dQ^i \partial_i$ denotes the coordinate independent exterior derivative in the Q-configuration space. Thus the connection A is in general, a flat connection. A is globally

exact for a simply connected Q space.Note that the exactness of the 1-form breaks down only in the presence of an external time t. In such a case ,$d/d\tau$ in eq.(9) is replaced by $\partial/\partial t + (d\tau/dt)d/d\tau$ .But in that case the description of the dynamics will become *extrinsic* instead of an *intrinsic* one that we are interested in here.An example of an extrinsic description in semiclassical gravity is that of a black hole evaporation.The external time t then refers to an asymptotic observer's time at infinity.

We note that the back reactions of the fast system on the slow one must be gauge invariant.The vanishing of the field tensor F tells us that the corresponding induced adiabatic bundle is trivial.Thus *in an intrinsic description, an expectation value of the fast system can not back react on the slow one*[8].An interpretation of this null effect may be offered thus : an 'observer' stationed in the internal parameter space of the interacting system is unable to ascertain the instantaneous energy eigenstate where she/he is in.The energy flux (difference) is a more relevant concept in that case; the *internal* observer's state being adjusted unambiguousely to the zero energy level.Consequently,the physical time in the q-sector can not be defined cosistently at the zeroth adiabatic order $O(M^0)$[5,6].The internal time appears only with the gauge invariant 'electric' potential in eq.(8),which is essentially a *nonadiabatic* effect[6].

Mathematically,one is free to introduce gauge rotations in the adiabatic bundle without spoiling the inherent gauge invariance of

the formalism.One such choice is [6]

$$\frac{d}{d\tilde{\tau}} = e^{-i\hbar^{-1}\int E\, dt} \frac{d}{d\tau} e^{i\hbar^{-1}\int E\, dt} \quad , \quad E = \langle h \rangle \tag{13}$$

The Schrodinger equation (10) then assumes the form

$$i\hbar \frac{d}{d\tilde{\tau}} |\chi\rangle = \tilde{h} |\chi\rangle \quad , \quad \tilde{h} = h - E \tag{14}$$

We now show that the timelike parameter $\tilde{\tau}$ does relate to the *physical* time in the intrinsic description.To make this identification consistent the q-states $|\chi\rangle$ should be interpreted as U(1) fibres on the projective space P_q of the q-states., instead of being considered simply as the adibatic U(1) fibres.The parameter $\tilde{\tau}$ thus keeps the record of the true nonadiabatic evolution of the q-states along a horizontal curve in the Aharonov-Anandans' U(1) nonadiabatic bundle [6,12].The corresponding connection 1-form A_P related to the fluctuations of the evolving states is obtained as[6,12]

$$A_P = i\langle\chi|d\chi\rangle = \hbar^{-1}\langle\Delta\tilde{h}\rangle\, d\tilde{\tau} \tag{15}$$

Here,d stands for the Fubini-Study exterior derivative in P_q and $\langle\Delta\tilde{h}\rangle$ should be interpreted as the root-mean-square energy fluctuations around the instantaneous eigenstate $|\chi\rangle$.As shown explicitly in ref.[6], the holonomy of A_P around a closed loop in P_q is directly related to the electric potential ρ.We note that $d\tilde{\tau}$,in eq.(15),denotes the Fubini-Study arc-length joining the infinitesimally separated rays $\chi(\tilde{\tau})$ and $\chi(\tilde{\tau}+d\tilde{\tau})$ in P_q.As shown by Aharonov and Anandan [12],the actual time recorded by a physical

clock is then given by $d\tilde{\tau}/\langle\Delta\tilde{h}\rangle$. Thus the origin of the intrinsic physical time is traced to the nonadiabatic evolution of the q-states leading to level transitions/particle production. We note incidentally that the unitary transformation (13) offers a possiblity of obtaning a gauge invariant definition of the normal ordering in the framework of an intrinsic description, even in the presence of nonzero curvature in the space-time.

Let us remark here that the inner-product in the Hilbert space of the total wave function Ψ in QC is defined by the semiclassical ansatz: $\langle\Psi|\Psi\rangle = \langle\chi|\chi\rangle$, where χ is a solution of eq.(10) and interpreted as a horizontal vector in the adiabatic limit. The corresponding verticle subspace at Ψ can thus be defined *formally* as the space of the orthogonal states Φ : $\langle\Psi|\Phi\rangle = 0$, in this inner-product. It appears plausible that, in relation to the extended Hilbert space $\{\Psi,\Phi\}$, a dynamical phase of the form $e^{-iE\tau/\hbar}$ *maybe* associated with the total state vector Ψ. This in turn may lend an extension of the physical time $\tilde{\tau}$ to an external 'observer's' adiabatic time t. But in that case, $E=\langle h\rangle$ should be evaluated in the external asymptotically flat space-time, instead of an internal cosmological space-time.

An application of the above results in the matter vacuum sector in QC gives the novel result : *in a semiclassical adiabatic regime the vacuum energy in a universe is gauge equivalent to zero and hence nongravitating.* From the general discussions of the above paragraphs, the non-gravitating nature of the vacuum energy does not

appear surprising.Instead it is a special consequence of the *intrinsic* description of the dynamics of an interacting system.Further, at an elementary level the concept of a force e.g.,the gravitational force,is related to the availability of an apriori definition of time.Since,the concept of time is found to be related to an intrinsic dynamical gauge structure, the non-gravitating vacuum energy turns out to be a natural consequence of the global gauge triviality of the induced adiabatic bundle.In this sense, the equivalence principle is violated for a globally defined object viz:the vacuum energy in the universe.We note further that there seems to exist a special complementarity relationship here:what seems to be a local event for an external observer, may actually be global in the intrinsic description.

Note that the renormalization of the vacuum energy in a curved space-time involves higher derivative terms in the gravitational action[1,3,13].For simplicity, the renormalized higher derivative coupling constants are suitably adjusted to zero [3] in the present discussion.The vacuum energy referred to here may therefore be interpreted as the renormalized cosmological constant in the model.The terms involving quantized matter fields in eq.(8),in particular, should therefore be considered formally as the corresponding renormalized values. Some explicit forms of the vacuum energy in simple cosmological models are discussed in Ref.[3,6].The vanishing of the vacuum energy may therefore be interpreted as a natural resolution of the cosmological constant problem in the

present context[5-8].We note that in the absence of a gravitating vacuum energy there is as such no empirical reason to keep a bare cosmological constant in the original gravitational action [2].Moreover the present discussion can be generalized formally to a higher derivative model [13] as well.Note that there is no $O(M^0)$ contribution from the particle production because of the vanishing of the corresponding field 2-form ,eq.(12).The lower order potential ρ plays the role of a tiny cosmological constant ,which may indeed be necessary to meet the demands of the observational cosmology[2].

To sum up, we have discussed some novel physical predictions of an intrinsic gauge invariant description of an interacting dynamical system.Can the null effects of the expectation values on the slow system,as observed by an 'internal observer', be experimentally tested in the context of the laboratory atomic/molecular physics [8]?-this remains to see.We note that although the present discussion is designed to be directly applicable to the quantum general relativity theory, the main results obtained here are expected to survive even as one considers the semiclassical limit of a *genuine* quantum gravity theory. Finally,assuming the WD equation as a constraint along a horizontal subspace and following the converse arguments alluded already(c.f.remarks after eq.(4)),one may imagine a possible extension of the internal concept of time in the universe to an external asymptotic time.In that case,a closed expanding universe will appear as a white hole to an 'external' asymptotic observer (collapsing universe →black hole).Consequently,a

Planck size space-time bubble (for an external observer), is perceived instead by an 'internal observer' inside the bubble as almost a quasiclassical object, as we do in the case of our universe.In otherwords, the internal description of the dynamics of a(n) (expanding) Planck-size bubble turns out, in the above extended framework, to be approximately similar to the universe as a whole.Doesn't this self-similarity of the intrinsic physical descriptions of both the universe and a Planck-size region inside the universe, indicate a finer (fractal-like) structure in the space-time? This particularly intriguing possibility will, however, be taken up in a future paper.We rather close the present discussion on a note of musing:Is space-time fractal,finer structures revealing at the Planck scale?

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Footnote in page 6.

* $\tilde{E}$ denotes the instantaneous energy eigenvalue of the total system.