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COMMENTS ON

'NEW ASYMPTOTIC EXPANSION METHOD FOR THE WHEELER-DEWITT EQUATION'

(Phys.Rev.D 52,3382,(1995))

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ABSTRACT

It is pointed out that the approach of Ref.[1] fails to give the semiclassical Einstein equations with quantized matter back reaction in the form of an energy expectation value.

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In Ref.[1](denoted I henceforth),Kim claims to obtain both the semiclassical Einstein equations (SECC's) with back reaction and the matter Schrodinger equation self-consistently,by using a 'new asymptotic expansion method for the Wheeler-Dewitt(WD) equation'.However,the claim doesn't seem to be justified.The approach of the author does not differ much from the conventional methods [4] in an essential way.In fact,the classicality of the gravitational degrees of freedom is assumed,as in the standard approaches,at a WKB level.However,the quantum matter state $\Phi(\phi, h_a)$[we follow the notations of I] is expanded in terms of a complete orthogonal set of states [eq.(2.8) of I]

$$\Phi(\phi, h_a) = \Sigma c_k(h_a) \mid \Phi_k(\phi, h_a) > \tag{1}$$

Here,$\mid \Phi_k(\phi, h_a) >$ is an eigenstate [eq.(2.15) of I]

$$\hat{I}(h_a) \mid \Phi_k >= \lambda_k \mid \Phi_k > \tag{2}$$

of a generalized invariant operator $\hat{I}$ [eq.(2.13) of I]

$$\frac{d}{d\tau}\hat{I}(h_a) = \frac{\partial}{\partial\tau}\hat{I}(h_a) - \frac{i}{\hbar}[\hat{I}, \hat{H}_m] = 0 \tag{3}$$

The parameter τ denotes the WKB time introduced by [eq.(2.4) of I]

$$\frac{\delta}{\delta\tau} = m_p^{-2} G_{ab} \frac{\delta S}{\delta h_a} \frac{\delta}{\delta h_b} \tag{4}$$

where S is supposed to satisfy the EHJ equation with the matter back reaction [eq.(2.19) of I]

$$\frac{1}{2m_p^2} G_{ab} \frac{\delta S}{\delta h_a} \frac{\delta S}{\delta h_b} - 2m_p^2 c^2 \sqrt{h}\, {}^3R(h_a) + \langle \Phi_k | \hat{H}_m | \Phi_k \rangle = 0 \tag{5}$$

The main motivation of the paper rests on the derivation of the above EHJ equation from the diagonal matrix equation for the matter states [eq.(2.16) of I]

$$i\hbar \frac{\delta}{\delta\tau} c_k(h_a) + [\Omega^{(0)}(h_a) - \Omega^{(1)}_{kk}(h_a) + \Omega^{(2)}_{kk}(h_a)] c_k(h_a) = 0 \tag{6}$$

However,eq.(5) is obtained from eq.(6) by a *deliberate* choice [eq.(2.17) of I]

$$\Omega^{(0)} - \Omega^{(1)}_{kk} = 0$$

$$i\hbar \frac{\delta}{\delta \tau} c_k + \Omega^{(2)}_{kk} c_k = 0 \tag{7}$$

without giving sufficient justification for the ansatz.Note that,this chioce is not unique.One could as well have made either of the following two choices:

$$\Omega^{(0)} = 0 \tag{8}$$

$$i\hbar \frac{\delta}{\delta \tau} c_k + [-\Omega^{(1)}_{kk} + \Omega^{(2)}_{kk}] c_k = 0$$

and/or

$$\Omega^{(0)} - \Omega^{(1)}_{kk} + \Omega^{(2)}_{kk} = 0 \tag{9}$$

$$i\hbar \frac{\delta}{\delta \tau} c_k = 0$$

Moreover,the ansatz (7) fails to satisfy the author's demand.As noted in I,the definition of τ depends on the mode number k and hence on the mode depended gravitational action S_k.In other words,for each eigenmode $\mid \Phi_k >$ of the invariant operator I,the WKB time τ_k is defined on the basis of the gravitational action S_k, a solution of the EHJ equation (5).In that case the matter states get decoupled

$$\Phi(\phi, h_a) = \exp(\frac{i}{\hbar} \int \Omega^{(2)}_{kk} d\tau_k) |\Phi_k(\phi, h_a)\rangle \tag{10}$$

instead of the superposed states,eq.(2.21) of I.[This point seems to be got corrected subsequently,viz.eq.(22) of Ref.[2]].Consequently,one gets from the matter Schrodinger equation (2.6) of I, $< \Phi_k \mid \hat{H}_m \mid \Phi_k >= 0$.This indicates that one doesn't really obtain the EHJ equation with back reaction as claimed.

A more accurate choice in the author's approach would have been eq.(8).Even in this case one gets the sourceless EHJ equation only.However,the matter state Φ does satisfy the time dependent Schrodinger equation [eq.(2.6) of I].Note also that the choice (9) corresponds to the adiabatic evolution:$\mid \Phi_k >$ being the eigenstate of the matter Hamiltonian $\hat{H}_m$.

The apparent ambiguites in the above three possible choices could be traced to the phase ambiguity noted previousely[5,6] in the semiclassical reduction of the WD equation.This phase ambiguity is now known [5] to be due to the induced geometric $U(1)$ gauge freedom associated with the WD equation.In Ref.[5],we have obtained a "geometric gauge invariant"form of the SCEE's.Besides resolving the above mentioned ambiguities,these equations seem to shed interesting insights into the gravitational properties of the cosmological vacuum energy,particle production and concept of time etc.,in the context of quantum cosmology.Further,the decoupling of the matter states,eq.(10),can also be understood in terms of the decoherence in the mode dependent gravitational branches [5].

We conclude with a remark on a related previous work of the author[3].It was claimed that the symmetry in the cosmological time might be broken by the mode dependent geometric phases in the WD wave function.Unfortunately,this claim is also not very well founded.Note that the total WD wave function is a geometric gauge invariant scalar quantity.Both the matter and the effective gravitational wave functions may pick up equal geometric phases separately,but with a change in sign so as to keep the total wave function phase independent.One can not therefore draw any inference on the time asymmetry on the basis of the total WD wave function,in contrary to the suggestion made in Ref.[3].One may,however,try to offer an argument in favour of the author's view using instead the effective gravitational wave function.

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