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Extended Robertson-Walker Cosmology

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Abstract - An account on the origins of cosmic chaos and its physical impact in an open and multiply connected universe is given. A new type of cosmic evolution by topological deformations, unpredicted by Einstein's equations, is pointed out. The chaoticity of the galactic world-lines provides a mechanism to create the galactic equidistribution. Global metrical deformations of the open and multiply connected spacelike slices induce angular fluctuations in the temperature of the microwave background. They cause backscattering of electromagnetic fields and particle creation in quantum fields. The topological micro structure of space-time provides a mechanism for CP violation by self-interference.

I. INTRODUCTION

Our contemporary understanding of the global structure of the Universe is based on the assumption of a space-time continuum, a curved four-dimensional world. The observational evidence for this is actually scarce, but it offers at any rate a very simple qualitative explanation of the cosmic red-shifts. The second motivation to model the Universe as a Riemannian four-manifold is the successful application of Riemannian geometry in explaining planetary perihelion shifts, and the deflection of light by gravitating bodies. These are local phenomena, which can be explained by introducing a Riemannian metric on a single co-ordinate chart. In cosmology, however, we are concerned with the global structure, with the topology of the four-manifold [1-3].

In local problems of general relativity, on a single co-ordinate chart, Einstein's equations completely determine the evolution of the metric, once the energy-momentum tensor of the local gravitational sources is known. In cosmology we have to consider the evolution of the metric as well as the *evolution of the topology* of space-time. Einstein's equations certainly do not give any hint on the topology, and the evolution of the metric is likewise undetermined, because we do not really

know the energy-momentum tensor of the matter distribution in the Universe, nor the boundary conditions to be imposed. It is in my opinion pointless to hunt for elusive *laws* of cosmic evolution, the modeling employed in such endeavors is always copied from finite classical or quantum systems, and there is no reason to assume that the Universe is a finite Hamiltonian system like any other. A much more promising approach to cosmology is to figure out *possibilities* of cosmic evolution, and to think over which local physical manifestations they can have.

The actual question is not so much what is the topology, but rather how does it evolve, because otherwise it is difficult to motivate why the 3-space should have acquired, once and for all, a particular type of topology and metric. What we will advocate here is a cosmology with an open three-space that evolves by global metrical deformations, which can give rise to transitions from one topology to another (Extended Robertson-Walker (RW) Cosmology [4-7]). We do not know the present topology of the Universe, and so we have to content ourselves to figure out possible physical effects of a multiply connected topology. In Sec. III we will review some local manifestations of the global 3-space topology, but at first I outline a cosmic evolution which goes far beyond the usual scaling of the length unit in traditional Robertson-Walker cosmology.

II. TOPOLOGICAL EVOLUTION BY GLOBAL METRICAL DEFORMATIONS OF THE 3-SPACE

In Refs. [4-7] a cosmology was introduced in which the 3-space is *negatively curved, open,* and *topologically multiply connected.* These three conditions allow topological evolution by global deformations. The negatively curved spacelike slices must be open and multiply connected, otherwise they cannot be deformed without destroying the constant curvature. The negative curvature (local hyperbolicity) is necessary to make the world-lines unstable, and the multiple connectivity can trap them and confine to a finite region, the *chaotic nucleus* of the open 3-space. This confinement, and the local hyperbolicity renders world-lines chaotic. The size of the nucleus depends on the expansion factor and, above all, on the topology of the 3-slices.

Chaoticity is an efficient mechanism to create an equidistribution, but the actual problem is to explain the inhomogeneities in the galactic distribution. The time evolution of the world-lines depends on the expansion and on the global deformations that the 3-space undergoes. Concepts like mixing and ergodicity, which are commonly used in Hamiltonian dynamics to describe the degree of

chaoticity [8,9], do not reflect the time evolution of the system. They are based on the geometric shapes of trajectories of infinite length. Here the question is what happens in finite times, and with world-lines of finite length [4,7]. Regular trajectories entering the chaotic center of the three-space are shadowed by chaotic ones, before they ultimately diffuse out of the center into the open 3-space. The mixing in the center tends to create a uniform distribution, but inhomogeneities will always remain in a finite time [10].

The center of the multiply connected three-space is geometrically determined by the convex hull of the limit set of the covering group, its explicit construction is a matter of elementary geometry [20]. Regular trajectories are shadowed by chaotic ones whenever their covering geodesics have initial or terminal points close to the limit set.

The multiple connectivity of the open 3-space provides a new type of *cosmic evolution by global metrical deformations*. 'Global' means here a deformation which does not locally change the curvature of the 3-space. Two spacelike sections which have the same topology and constant curvature (apart from a trivial rescaling by the expansion factor) may be globally, as 3-manifolds, non-isometric. If the 3-space is open and multiply connected, it can be endowed with a variety of metrics, all giving rise to the same constant curvature. These metrics can be labeled by a finite number of independent parameters, the dimension of this parameter space depends on the topology of the spacelike slices. The line element of the extended RW cosmology is $ds^2 = -c^2dt^2 + a^2(t)g_{ij}(y(t),\mathbf{x})dx^idx^j$. Here $g_{ij}(y_1,\ldots,y_n,\mathbf{x})$ denotes a continuous family of hyperbolic metrics of constant negative curvature -1. The parameters y_k may be chosen to vary in a compact domain of R^n . The evolution of the spacelike slices is characterized by a time dependent path $y(t)$ in this n-dimensional deformation space, and by the expansion factor $a(t)$. If the path $y(t)$ reaches the boundary of the deformation space, a topology change occurs, the connectivity of the 3-space can change, or it may even disintegrate, leaving behind cusp singularities on its newly emerging components. The curvature of the 3-space stays constant (apart of the mentioned rescaling) during the whole deformation process. For an exactly solvable example of a global deformation see Refs. [11,6].

Remark : In Ref. [12] we considered local distortions of the constantly curved 3-space geometry by gravitational waves. Constant curvature is not really necessary for the evolution mechanism described here, it just ensures that this topological evolution does not occur at the expense of creating large local inhomogeneities in the curvature. If the 3-space metric is uniformly close to a metric of constant

negative curvature it will stay so during a global deformation.

The topology, the dynamics induced by it via global deformations, and the resulting topology changes are not predictable by Einstein's equations, which nevertheless impose restrictions on the speed and size of global deformations, because the energy and pressure densities *defined* by the Einstein tensor must stay positive.

III. PHYSICAL MANIFESTATIONS OF THE TOPOLOGICAL 3-SPACE STRUCTURE

Objections against open cosmologies are sometimes based on the argument that we will not be able to look at infinity and to verify what is happening there. What is overlooked here is that the topological structure of the 3-space can manifest itself locally. We will give now some examples. I also mention here that an entirely different approach to the problem of "how to look at infinity" in an open universe was suggested in Ref. [10], where tachyonic (i.e. superluminal [13,14]) signal transfer is considered in the comoving galactic frame of reference.

Angular fluctuations in the temperature of the microwave background radiation [15,16]: If one studies the impact of global deformations of the open 3-space on the free photon gas [4], one has to take two time scales into account: cosmic time as defined by the expansion factor, and the time scale on which the deformations take place. The latter is determined by the time parametrization of the path $y(t)$, which determines the metric on the spacelike sections. Only if the scale on which noticeable variations of $g_{ij}(y(t), \mathbf{x})$ occur is much larger than the time scale determined by the expansion factor, one has a really uniform expansion.

An adiabatic variation of $g_{ij}(y(t), \mathbf{x})$ on the time scale of the cosmic expansion leads to a small analytic distortion of the free photon wave vectors, which results in slightly angular dependent frequency shifts. These shifts can be absorbed in the Planck distribution by introducing an angular dependent temperature variable.

Particle creation in quantum fields and backscattering of electromagnetic radiation [4,6]: In simply connected Robertson-Walker cosmologies the variation of the expansion factor leads to particle creation. However, this cannot happen in neutrino or electromagnetic fields, because in the solutions of the corresponding wave equations the expansion factor scales out with a simple power law. But global metrical deformations of the 3-space do create particles and backscattering even in conformally coupled fields. The point here is that during a deformation the wave equation is not time separable, and a wave initially composed of positive frequencies

will acquire negative frequency modes. So antiparticles emerge in quantum fields, and backscattered wave trains in classical fields.

Topological CP violation by self-interference: The reasons for CP violation are as yet not really understood. This is so despite, or perhaps, because this extraordinary phenomenon could be easily incorporated into particle physics by adding symmetry breaking interactions to the Lagrangians. The space-reflection symmetry is of course always present in classical mechanics, its violation is a pure quantum effect [17]. In Ref. [4] it was therefore suggested to explain parity violation as a topological interference phenomenon, by adopting Weyl's idea to associate elementary particles with topological excitations of the 3-space [18,19]. A space-reflected wave can wrap around a microscopic geodesic loop, and this causes self-interference [11], which destroys the anti-unitarity of the parity operator. Thus parity is a broken symmetry already on the level of free wave equations. Some open research problems in extended RW cosmology are outlined in Ref. [10].

I have tried to show here that the topology and the topological dynamics of an open universe may have very concrete physical manifestations, and that the topology of space-time is a dynamical object, able to evolve like the metric in local problems of general relativity.

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