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Ether in a Cosmic World View

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Abstract

A theory of superluminal motion is proposed in the context of a Robertson-Walker cosmology, based on a pre-relativistic approach to electromagnetism. Cosmic space itself is supposed to have physical substance. This substance, the ether, manifests by its permeability, which determines the speed of light. Massive particles cannot exceed this speed, but there is no bound on the speed of light itself. Locally, in Minkowski frames, the ether determines the constants of nature, which get so functions of cosmic time. Scaling laws for mass, charge, and the vacuum speed of light in Minkowski space are derived.

The dynamics of point particles, electromagnetic waves, and spinor fields in the ether is defined, and the cosmic time dependence of energy, frequency, and wave length is studied. It is pointed out how the ether influences red-shifts.

1. Introduction

The Gallilean and the special principle of relativity are associated with spaces that are isotropic, homogeneous, and essentially void. Minkowski space simply means the void, the vacuum, in which events are labeled by co-ordinate axes in space and time. In cosmology, however, space is generated by the galactic grid, this is the essence of cosmic space, and this grid provides a distinguished frame of reference. We may define an observer at rest with respect to the galactic background, if he sees the galaxies around him uniformly receding. This uniform galactic recession is easiest described by a Robertson-Walker (RW) geometry with the line element $ds^2 = -c^2 d\tau^2 + a^2(\tau)\sigma_{ij}dx^i dx^j$. Here σ_{ij} denotes a metric of constant curvature on the 3-space, and the expansion factor $a(\tau)$ defines the length unit at a given instant of cosmic time τ. Observers at rest, like galaxies, have constant space co-ordinates. Their mutual recession is a mere consequence of the expansion of the 3-space, determined by $a(\tau)$.

Unlike in Minkowski space, in a RW cosmology with a generic expansion factor there do not exist global symmetry transformations which mix space and time co-ordinates. Rest frames of geodesically moving observers can only infinitesimally be defined and linked to each other by the introduction of locally geodesic co-ordinates. In such infinitesimal neighborhoods uniformly moving co-ordinate frames are still connected by Lorentz boosts.

Not only that the cosmic grid determines an absolute space, it also defines an absolute cosmic time. Any co-ordinate transformation involving time will change the form of the RW line element, and then the time separation of infinitesimally neighboring events would not any more be the differential $d\tau$. Observers moving in the galactic frame of reference will see an anisotropic galactic background. Accordingly, rest as defined above and uniform motion are physically distinguishable states, and this is the

reason that there are no global space-time mixing symmetry transformations relating uniformly moving observers.

In Minkowski space the vacuum speed of light must be regarded as the upper bound on the speed of signal transfer, unless one is willing to give up our traditional conception of causality [1-3]. If, for an observer in Minkowski space, emission and absorption events are separated by intervals dx, $dt > 0$, and if $dx / dt > c$, then it is always possible to choose a Lorentz boost so that in the new frame $dt' < 0$. This means that for an observer in the new frame (t', x') the time order of cause and effect is inverted (absorption prior to emission), or, if one prefers, that cause and effect are interchanged (what appears to be emission in the first frame, is absorption in the second, and vice versa). This certainly violates our conception of causality, namely, that every effect has a cause, that this cause precedes the effect, and that the decision on what is cause and effect is unambiguous and observer independent [4].

In this article we present a theory of superluminal motion without giving up this traditional causality principle. We assume that cosmic space as generated by the expanding galactic grid is not a mere geometric construct, but has itself substance. This substance, the ether, manifests by its permeability. Electromagnetic waves propagate through, or better, by means of the ether. Their speed is determined like in a dielectric medium by a permeability tensor which, with the usual appeal to cosmic homogeneity and isotropy, takes the form $ds^2 = -c^2 h^{B^2} d\tau^2 + b^2(\tau)\sigma_{ij} dx^i dx^j$, with two scale factors $h^B(\tau)$ and $b(\tau)$. In this article we do not make any assumptions on the micro-structure of the ether.

There are, however, two important differences compared to electrodynamics in a dielectric medium. The speed of light, a function of cosmic time, is not bounded from above in the cosmic evolution. Secondly, the ether does not only affect electromagnetic

waves but also massive particles. Though particles cannot exceed the speed of light, they can come arbitrarily close to it. In fact, nothing can surpass the speed of light, but there is no universal bound on the speed of light itself. "Superluminal" does not mean here "faster than light", but rather that there is no uniform limit on the speed of signal transfer in the cosmic evolution. There is no *cosmic* vacuum speed of light, since there is no vacuum, this is a geometric idealization, as is Minkowski space.

All that refers to the dynamics of particles and waves in cosmic time, in the context of the expanding galactic grid. One must also ask how the ether effects local physical processes, which happen at a given instant τ_0 of cosmic time. We can then scale the factors $a(\tau_0)$, $b(\tau_0)$, and $h^B(\tau_0)$ into the constants of nature, which get so functions of cosmic time, but they are true constants in the local Hamiltonians. If we introduce locally geodesic co-ordinates, we fully recover Minkowski space, Lorentz invariance, causality, and relativity, with constants of nature depending on the cosmic time parameter, which is unrelated to the local system time in the Hamiltonian. The possibility of unbounded, superluminal signal transfer emerges only on a cosmic level, there where one needs it in an infinite universe.

In Sec. 2 we define the propagation of electromagnetic waves in the ether, assuming that the determinant of the permeability tensor and of the RW metric coincide. (This technical condition is removed in Sec. 6.) We derive the field equations, at first manifestly covariant, and then in three-dimensional form, and discuss the eikonal equation and the refraction index of the ether. We derive the wave equation for the vector potentials, the analogue to the Lorentz condition, and the electromagnetic field energy in the ether. Finally an explicit example for all that in an open universe is given.

In Sec. 3 we define the dynamics of classical point particles in the cosmic ether by extending the eikonal equation to a Hamilton-Jacobi equation, and give some examples of particle motion for special choices of the scale factors in the metric and the

permeability tensor. In Sec. 4 we couple the Dirac equation to the permeability tensor, and study the propagation of spinor fields in the ether. At hand of the examples of Sec. 3 we discuss phase and group velocity, as well as the semiclassical limit. In particular we study the evolution of wave length, frequency, and energy in cosmic time, and how the ether manifests in absorption spectra. At the end of Sec. 4 we explain how the ether determines the dependence of the constants of nature on cosmic time, and we derive scaling laws for mass and vacuum speed of light in local Minkowski frames.

In Sec. 5 the coupling of scalar particles to the permeability tensor is defined, as a preparation to Sec. 6. Spinors are actually more straightforward to couple to the permeability tensor then scalar particles, since the Dirac Lagrangian is linear in the field derivatives. In Sec. 6 we again review the electromagnetic field, without the determinant condition of. Sec. 2 on the permeability tensor. Gauge symmetry in the ether is defined, a gauge invariant coupling of spinor fields to the electromagnetic potentials is introduced, and a cosmic scaling law for electric charge is derived. In the Conclusion, Sec. 7, we summarize the principles and consequences of classical and wave mechanics in the ether.

2. Electromagnetism and ether

In this Section we discuss electrodynamics in the ether, the hypothetical material substance of space [5], which macroscopically manifests by a symmetric permeability tensor $g^B_{\mu\nu}$. Quite analogously to a dielectric medium, the following formalism is based on two symmetric tensor fields, the space-time metric $g_{\mu\nu}$ (inverse $g^{\mu\nu}$), and the permeability tensor $g^B_{\mu\nu}$ (inverse $g^{B^{-1}\mu\nu}$). Action and Lagrangian for the free electromagnetic potentials we define as

$$S = \frac{1}{c}\int L\sqrt{-g}\,dxdt\,,\ L = -\tfrac{1}{4}F_{\mu\nu}F_{\tilde{\mu}\tilde{\nu}}g^{B^{-1}\mu\tilde{\mu}}g^{B^{-1}\nu\tilde{\nu}}\,, \tag{2.1}$$

and $F_{\mu\nu} = A_{\nu,\mu} - A_{\mu,\nu}$. We assume in this Section that $g^{II} = g$, i. e., the determinants of permeability tensor and space-time metric coincide. (In the next Sections we will drop this constraint, but it makes things simpler here.) Covariant differentiation with respect to the space-time metric $g_{\mu\nu}$ we denote by a semicolon, and with respect to the permeability tensor $g^{II}_{\mu\nu}$ by a double semicolon (;;). We raise and lower indices exclusively by means of the space-time metric $g_{\mu\nu}$.

The interaction with a current is defined as

$$S_{\text{int}} = \frac{1}{c}\int A_\mu j^\mu dxdt\,. \tag{2.2}$$

If we introduce the tensor [6,7]

$$H^{\mu\nu} := g^{II^{-1}\mu\tilde{\mu}} g^{II^{-1}\nu\tilde{\nu}} F_{\tilde{\mu}\tilde{\nu}}\,, \tag{2.3}$$

we may write the Lagrange equations as

$$H^{\mu\nu}{}_{;\nu} \equiv \frac{1}{\sqrt{-g}}\frac{\partial\left(\sqrt{-g}H^{\mu\nu}\right)}{\partial x^\nu} = \frac{1}{c}j^\mu\,. \tag{2.4}$$

From the potential representation of $F_{\mu\nu}$ we immediately obtain the homogeneous Maxwell equations

$$\frac{1}{\sqrt{-g}}\varepsilon^{\lambda\alpha\beta\gamma}F_{\alpha\beta;\gamma} = 0\,. \tag{2.5}$$

It is very instructive to write the field equations in 3-d form. To avoid technical complications we assume that the flanks of $g_{\mu\nu}$ and $g^{II}_{\mu\nu}$ vanish, $g^{II}_{m0} = g_{m0} = 0$ (Greek indices run from 0 to 3, Latin ones from 1 to 3), and that $g_{\mu\nu}$ and $g^{II}_{\mu\nu}$ are defined via the line elements

$$ds^2 = -c^2 d\tau^2 + \gamma_{ij}dx^i dx^j\,;\quad ds^2_{II} = -c^2 h^{II^2} d\tau^2 + \gamma^{II}_{ij}dx^i dx^j\,, \tag{2.6}$$

respectively. (γ^{ij} and $\gamma^{\prime\prime -1 ij}$ denote the inverse, γ and $\gamma^{\prime\prime}$ the determinants of the 3-tensors γ_{mn} and $\gamma^{\prime\prime}_{mn}$, and $g^{\prime\prime}_{00} = -c^2 h^{\prime\prime 2}(\tau, x)$.) Because of the determinant condition $g^{\prime\prime} = g$ we have $\gamma = \gamma^{\prime\prime} h^{\prime\prime 2}$. We define the 3-vectors of electric and magnetic field strengths E_i, B^i and displacements D_i, H^i as

$$E_i = c^{-1} F_{i0}, \quad D^i = c H^{0i}, \quad B^i = \tfrac{1}{2}\gamma^{-1/2}\varepsilon^{imn} F_{mn}, \quad H_i = \tfrac{1}{2}\gamma^{1/2}\varepsilon_{imn} H^{mn}, \tag{2.7}$$

and inversely, $F_{mn} = \gamma^{1/2}\varepsilon_{mnk} B^k$, $H^{mn} = \gamma^{-1/2}\varepsilon^{mnl} H_l$. Charge density and 3-current are defined as $\rho = j^0$ and $j^{(3)k} = j^k$. The continuity equation, $j^{\mu}{}_{;\mu} = 0$, obtained from (2.4), reads as

$$\frac{1}{\sqrt{\gamma}}\frac{\partial}{\partial \tau}\left(\sqrt{\gamma}\,\rho\right) + j^{(3)k}{}_{\|k} = 0. \tag{2.8}$$

The double stroke (||) denotes covariant differentiation with respect to the 3-metric γ_{ij}.

The homogeneous equations (2.5) are equivalent to

$$B^l{}_{\|l} = 0, \quad \frac{1}{\sqrt{\gamma}}\varepsilon^{klm} E_{m\|l} + \frac{1}{c}\frac{1}{\sqrt{\gamma}}\frac{\partial\left(\sqrt{\gamma}\,B^k\right)}{\partial \tau} = 0. \tag{2.9}$$

The inhomogeneous Maxwell equations read as

$$D^l{}_{\|l} = \rho, \quad \frac{1}{\sqrt{\gamma}}\varepsilon^{mkl} H_{l\|k} - \frac{1}{c}\frac{1}{\sqrt{\gamma}}\frac{\partial\left(\sqrt{\gamma}\,D^m\right)}{\partial \tau} = \frac{1}{c} j^{(3)m}. \tag{2.10}$$

From (2.7) we obtain the relations between the field strengths and displacements as

$$D^m = \varepsilon^{mn} E_n, \quad \mu^{kl} H_l = B^k, \tag{2.11}$$

with the permeability tensors $\varepsilon^{mn} = \mu^{mn} := h^{\prime\prime -2}\gamma^{\prime\prime -1 mn}$.

Note that in deriving (2.9)-(2.11) we used the determinant condition $g = g^{\prime\prime}$. The whole formalism developed so far is of course just that of electrodynamics in an anisotropic dielectric medium, cf., e. g., Refs.[6-8].

To relate the permeability tensors in (2.11) with the refraction index and the speed of light, we start with the eikonal equation

$$g^{\prime\prime -1\mu\nu}\psi_{,\mu}\psi_{,\nu} = 0. \tag{2.12}$$

So the differential equation for rays is

$$-c^2 h^{\prime\prime 2} d\tau^2 + \gamma^{\prime\prime}_{ij} dx^i dx^j = 0, \tag{2.13}$$

and the speed of light $v_c(e^i)$ in direction e^i $(\gamma_{ij} e^i e^j = 1)$ we obtain accordingly as

$$v_c(e^i) = c h^{\prime\prime}\left(\gamma^{\prime\prime}_{ij} e^i e^j\right)^{-1/2} =: \frac{c}{n(e^i)}. \tag{2.14}$$

$n(e^i)$ is the index of refraction in direction e^i.

Remark: If we assume that $h^{\prime\prime}$ depends only on cosmic time τ, and that γ_{ij} and $\gamma^{\prime\prime}_{ij}$ are 3-metrics of constant curvature,

$$\gamma_{ij} = a^2(\tau)\hat{\gamma}_{ij},\ \gamma^{\prime\prime}_{ij} = b^2(\tau)\hat{\gamma}_{ij}, \tag{2.15}$$

with $\hat{\gamma}_{ij}$ a metric of constant curvature independent of cosmic time, then we have $n = b/(h^{\prime\prime} a)$ as refraction index. If we define $\varepsilon^{mn} =: \varepsilon\delta^{mn}$, the familiar formula $n = \sqrt{\varepsilon\mu}$ is recovered, since $\varepsilon = \mu$, cf. (2.11).

Next we derive the wave equation for the vector potential A_μ. Note that in (2.4) we may replace $H^{\mu\nu}{}_{;\nu}$ by $H^{\mu\nu}{}_{;;\nu}$ because of the determinant condition. (The double semicolon denotes covariant differentiation with respect to $g^{\prime\prime}_{\mu\nu}$). Likewise, in Eq. (2.5) the space-time metric can be replaced by the permeability tensor, and the Lorentz condition we may write as

$$g^{\prime\prime -1\mu\nu} A_{\mu;;\nu} = 0. \tag{2.16}$$

Commuting covariant $g^{\prime\prime}_{\mu\nu}$-derivatives, and applying (2.16), we obtain from (2.4) the wave equation

$$g^{B^{-1}\lambda\kappa}\left(A_{\mu;;\lambda;;\kappa} - R^{B}_{\mu\lambda} A_{\kappa}\right) = -\frac{1}{c} j^{\nu} g^{B}_{\nu\mu} . \tag{2.17}$$

Here $R^{B}_{\mu\lambda}$ denotes the Ricci tensor of metric $g^{B}_{\mu\nu}$ with sign conventions as in Ref. [6].

The energy-momentum tensor we define as

$$T_{\nu}{}^{\mu} := -F_{\nu\alpha} H^{\mu\alpha} + \tfrac{1}{4} \delta^{\mu}_{\nu} F_{\alpha\beta} H^{\alpha\beta} , \tag{2.18}$$

which is the usual definition of electromagnetic energy in a dielectric medium, cf., e. g., Ref. [9]. With the 3-vectors defined in (2.7) the energy density reads as

$$T_{0}{}^{0} = \tfrac{1}{2}\left(E_k D^k + B^k H_k\right) , \tag{2.19}$$

its positivity follows from (2.11). Likewise, we may write in (2.1)

$$L = \tfrac{1}{2}\left(E_i D^i - H_i B^i\right) . \tag{2.20}$$

Using the field equations (2.4) with $H^{\mu\nu}{}_{;\nu} = H^{\mu\nu}{}_{;;\nu}$, we readily derive

$$T_{\nu}{}^{\mu}{}_{;;\mu} = \frac{1}{c} F_{\nu\alpha} j^{\alpha} . \tag{2.21}$$

Because $g^{B} = g$, we may write

$$T_{\nu}{}^{\mu}{}_{;\mu} = T_{\nu}{}^{\mu}{}_{;;\mu} + \Delta^{\kappa}{}_{\mu\nu} T_{\kappa}{}^{\mu} , \quad \Delta^{\kappa}{}_{\mu\nu} := \Gamma^{B\kappa}_{\mu\nu} - \Gamma^{\kappa}_{\mu\nu} , \tag{2.22}$$

where $\Gamma^{(B)\kappa}_{\mu\nu}$ denote the Christoffel 3-indices for the tensors $g^{(B)}_{\mu\nu}$. Inserting here (2.21) for $T_{\nu}{}^{\mu}{}_{;;\mu}$, we obtain a differential conservation law with respect to the space-time metric. (For the explicit time evolution of electromagnetic energy see the following example.) Note that $T_{\nu\mu}$ is not symmetric, only $T_{\nu}{}^{\alpha} g^{B}_{\alpha\mu}$ is.

We conclude this Section with an explicit example, electromagnetic waves freely propagating in a perfectly isotropic and homogeneous cosmic ether, in the context of an open Robertson-Walker universe with line elements (2.6) and constantly curved 3-space metrics (2.15). We assume that h^{B} is a function of cosmic time only, and that the 3-space is open and negatively curved. We use the Poincaré half-space representation of

hyperbolic geometry; $\hat{\gamma}_{ij}$ in (2.15) is then defined by the line element $ds^2 = R^2 t^{-2}\left(dt^2 + |dz|^2\right)$ in the half-space H^3 : $(z,t), z \in C, t > 0$, cf. Ref. [10]. (H^3 has constant sectional curvature $-1/R^2$). The line elements (2.6) then read as

$$ds^2 = -c^2 d\tau^2 + a^2(\tau) R^2 t^{-2}\left(dt^2 + |dz|^2\right), \tag{2.23}$$

$$ds_B^2 = -c^2 h^{B^2}(\tau) d\tau^2 + b^2(\tau) R^2 t^{-2}\left(dt^2 + |dz|^2\right), \tag{2.24}$$

and the determinant condition is $a^3 = b^3 h^B$.

We have to solve Eqs. (2.4) with $j^\mu = 0$, under the Lorentz condition (2.16) and the Coulomb gauge $A_0 = 0$. Using the separation ansatz $A_n = \varphi(\tau)\hat{A}_n(x)$, $x^i = (z,t)$, we may write

$$H^{ab} = \varphi(\tau) b^{-4}(\tau) \hat{\gamma}^{am}\hat{\gamma}^{bn}\left(\hat{A}_{n,m} - \hat{A}_{m,n}\right), \quad H^{a0} = c^{-2} h^{B^{-2}} b^{-2} \varphi'(\tau) \hat{\gamma}^{am} \hat{A}_n, \tag{2.25}$$

($\hat{\gamma}^{mn}$ denotes the inverse of $\hat{\gamma}_{mn}$). The $\mu = 0$ component of Eq. (2.4) is then equivalent to the Lorentz condition (2.16). The $\mu = m$ components give for $\hat{A}_i(x)$

$$\frac{1}{\sqrt{\hat{\gamma}}} \frac{\partial\left[\sqrt{\hat{\gamma}}\, \hat{\gamma}^{ma} \hat{\gamma}^{kb}\left(\hat{A}_{b,a} - \hat{A}_{a,b}\right)\right]}{\partial x^k} = \frac{s^2}{R^2} \hat{A}_k \hat{\gamma}^{km}, \tag{2.26}$$

with a real separation parameter s. This is solved by the vector fields $\hat{A}_n = (t/R)^{is}(1,0,0)$ or $\hat{A}_n = (t/R)^{is}(0,1,0)$. For φ we obtain

$$\frac{b}{h^B}\frac{d}{d\tau}\left(\frac{b}{h^B}\frac{d}{d\tau}\varphi\right) + \frac{c^2 s^2}{R^2}\varphi = 0, \quad \varphi = \exp\left[\mp is \frac{c}{R}\int h^B b^{-1} d\tau\right]. \tag{2.27}$$

The indicated solutions $\varphi(\tau)\hat{A}_n(t)$ of (2.4) satisfy the Lorentz condition, and constitute a complete set of plane waves propagating along the t-axis. Because H^3 is a homogeneous space, we obtain a complete set of transversal plane waves propagating in any other direction by applying symmetry transformations of H^3 to the vector fields $\hat{A}_n$, cf. Ref. [10]. For reasons of technical simplicity, and without loss of generality, we focus

in this article on waves and particles propagating along the t-axis of H^3. If we calculate with the plane waves $\varphi(\tau)\hat{A}_{\mu}(t)$ the energy density $T_0{}^0$ in (2.19), we obtain the time dependence $T_0{}^0\sqrt{\gamma} \sim h^B(\tau)b^{-1}(\tau)$. This is the same time behavior as of the frequency $\omega = |s|cR^{-1}h^B b^{-1}$ in (2.27), and suggests for the photon energy the Einstein relation $E(\tau) = \hbar\omega(\tau)$, with $\hbar$ independent of cosmic time; we will return to that point in (4.17). Also note that the phase of the spectral elementary waves is linear in the spectral variable s. Like in vacuum electrodynamics there is no dispersion in wave trains composed (by a Gaussian average over s) of plane waves propagating in the same direction.

3. Classical particles in the ether

From the eikonal equation (2.12) it is clear how to define the mechanics of classical particles in the ether, namely by the Hamilton-Jacobi equation

$$g^{B^{-1}\mu\nu} S_{,\mu} S_{,\nu} = -m^2 c^2 . \tag{3.1}$$

This corresponds to the Lagrange function

$$L_p(s) = -mc^2 \sqrt{-g^B_{\mu\nu} \dot{x}^\mu \dot{x}^\nu} , \tag{3.2}$$

with the action $S = \frac{1}{c} \int L_p ds$. Note that in the action the space-time metric does not enter at all.

In this Section we discuss geodesic motion in the context of the line elements (2.23), (2.24). We drop from now on the determinant condition $g^B = g$ which we assumed throughout Sec. 2.

Like in the discussion of electromagnetic waves at the end of Sec. 2, we study geodesic motion along the t-axis ($z=0$) of H^3. This is technically simple, and does not mean any restriction, since H^3 is homogeneous. From the Lagrange equations we have immediately two integrals of motion along the t-axis,

$$c^2 h^{B^2} \dot{\tau}^2(s) - b^2 R^2 \left(\dot{t}(s)/t\right)^2 = \varepsilon , \quad b^2 R \dot{t} / t = \nu , \tag{3.3}$$

with ν a real integration constant, and $\varepsilon = 1$ for particles and $\varepsilon = 0$ for rays. We introduce the cyclic variable $\hat{t} = \log(t/R)$, and write $\hat{\tau} := \Lambda\tau$, $\Lambda := c/R$.

As solutions of (3.3) ($\varepsilon = 1$) we obtain

$$d\hat{t} = \nu h^B b^{-1} \left(b^2 + \nu^2\right)^{-1/2} d\hat{\tau} , \quad ds = R h^B b \left(b^2 + \nu^2\right)^{-1/2} d\hat{\tau} . \tag{3.4}$$

We specify now the functions $b(\tau)$ and $h^B(\tau)$ as power laws,

$$b(\tau) = B(\Lambda\tau)^\beta , \quad h^B(\tau) = H^B (\Lambda\tau)^\lambda , \tag{3.5}$$

with positive constants B, H^B, and real exponents β, λ. Defining $\hat{\nu} := \nu / B$, we have

$$d\hat{t} / d\hat{\tau} = H^B B^{-1} \hat{\nu} \hat{\tau}^{\lambda-\beta} \left(\hat{\tau}^{2\beta} + \hat{\nu}^2\right)^{-1/2}, \tag{3.6}$$

$$ds / R = H^B \hat{\tau}^{\lambda+\beta} \left(\hat{\tau}^{2\beta} + \hat{\nu}^2\right)^{-1/2} d\hat{\tau}. \tag{3.7}$$

Example I: $\beta = 0$, λ arbitrary (but $\lambda \neq -1$). We have as solution of (3.6)

$$t / R = \kappa \exp\left[\frac{H^B}{B} \frac{\hat{\nu}}{\sqrt{1+\hat{\nu}^2}} \frac{1}{1+\lambda} (\Lambda\tau)^{1+\lambda}\right], \tag{3.8}$$

with a positive integration constant κ.

Example II: $\beta = 0$, $\lambda = -1$. The solution of (3.6) is

$$t / R = \kappa (\Lambda\tau)^{\delta}, \; \delta := H^B B^{-1} \hat{\nu} (1+\hat{\nu}^2)^{-1/2}. \tag{3.9}$$

Example III : $\lambda = -1$, β arbitrary ($\beta \neq 0$),

$$t / R = \kappa \exp\left[\frac{-H^B}{B\hat{\nu}\beta} \frac{1}{\hat{\tau}^{\beta}} \sqrt{\hat{\tau}^{2\beta} + \hat{\nu}^2}\right]. \tag{3.10}$$

Example IV: $\beta = (1+\lambda)/2$, λ arbitrary ($\lambda \neq -1$),

$$t / R = \kappa \left[\hat{\tau}^{(1+\lambda)/2} + \sqrt{\hat{\tau}^{(1+\lambda)} + \hat{\nu}^2}\right]^{\delta}, \; \delta := 2H^B B^{-1} \hat{\nu} (1+\lambda)^{-1}. \tag{3.11}$$

We will discuss these examples extensively in the following sections.

Next we turn to the asymptotics of Eq. (3.6). We ask whether the particle can reach the boundary at infinity ($t = \infty$ or $t = 0$) of the 3-space H^3 for $\tau \to \infty$, cf. Ref. [4] for the physical implications of that in a multiply connected 3-space. A short inspection gives: If $\beta \leq 0$, then $\lambda - \beta + 1 \geq 0$ must be satisfied in order to reach infinity. If $\beta > 0$, then $\lambda - 2\beta + 1 \geq 0$ is needed to reach infinity. In particular, if $\lambda = 0$, we must have $\beta \leq 1/2$ to reach infinity.

Remark: We may introduce a new time variable T via $\Lambda\tau = \exp(T/2)$, so that the origin of cosmic time, $\tau = 0$, is moved to $T = -\infty$. The line element (2.24), (3.5) of the permeability tensor then reads

$$ds_B^2 = -\tfrac{1}{4} R^2 H^{B^2} \exp[(1+\lambda)T]\, dT^2 + B^2 \exp(\beta T) R^2 t^{-2} \left(dt^2 + |dz|^2\right). \tag{3.12}$$

In the preceding examples we have only to replace $\hat{\tau}$ by $\exp(T/2)$. Note that in this Section we did not make, up to now, any use of the space-time metric (2.23). So we may define, without referring to this co-ordinate transformation and (2.23), a space-time metric as

$$ds^2 = -R^2 dT^2 + \exp(\alpha T) R^2 t^{-2} \left(dt^2 + |dz|^2\right), \tag{3.13}$$

with some constant α. The choice of Examples I-III is illuminated by (3.12), but the really nice thing is that T is without beginning and end. Throughout this article we use τ as cosmic time, assuming a universe with a finite past.

We define the generalized 4-momentum as

$$p_\nu := mc\dot{x}^\mu(s) g^B_{\mu\nu}, \tag{3.14}$$

$x^\mu := (\tau, z, t)$, and $E = -p_0$ as energy. From (3.3) and (3.4) we obtain

$$E = \frac{mc^2 h^B}{\sqrt{1 - h^{B^{-2}} c^{-2} \gamma^B_{ij} x'^i x'^j}} = mc^2 h^B \sqrt{1 + \nu^2 b^{-2}}, \tag{3.15}$$

$$p_t = \frac{mb^2 R^2 t'}{h^B t^2 \sqrt{1 - h^{B^{-2}} c^{-2} \gamma^B_{ij} x'^i x'^j}} = mc \frac{R\nu}{t}, \tag{3.16}$$

with $x'^i = dx^i / d\tau$, $t' = dt / d\tau$, so that $E^2 g^{B^{-1}00} + p_t^2 g^{B^{-1}tt} = -m^2 c^2$. In the global cosmic evolution energy is positive definite but not conserved, because of the explicit time dependence of the Lagrangian (3.2) via the scale factors $b(\tau)$ and $h^B(\tau)$ in the permeability tensor. Energy and momentum as defined in (3.14) emerge, however, as the usual conserved interaction parameters in local Minkowski frames, cf. the end of Sec. 4. The

vector p_v also appears in the phases of quantum mechanical elementary waves in the semiclassical approximation, cf. (4.16)-(4.18), so that $E(\tau)$ is proportional to their frequency.

For a particle moving with velocity v^i, $\gamma_{ij}v^i v^j := |\mathbf{v}|^2$, in the direction of a unit vector e^i, we have by definition $x'^i = |\mathbf{v}|e^i$. Accordingly, cf. (2.14),

$$\frac{\gamma^B_{ij} x'^i x'^j}{h^{B2}c^2} = |\mathbf{v}|^2 \frac{\gamma^B_{ij} e^i e^j}{h^{B2}c^2} = \frac{|\mathbf{v}|^2}{|\mathbf{v}_c|^2}, \tag{3.17}$$

with $\mathbf{v}_c$ the speed of light in the direction of $\mathbf{v}$. If we substitute (3.17) into (3.15), we see that no massive particle can exceed the speed of light, but $\mathbf{v}_c$ itself need not be bounded. Explicitly we have with (2.23) and (3.4),

$$|\mathbf{v}| = aRt^{-1}dt / d\tau = ch^B \frac{a}{b} \frac{\nu}{\sqrt{b^2 + \nu^2}}. \tag{3.18}$$

For $\nu \to \infty$ we recover the speed of light $|\mathbf{v}_c|$.

4. Dirac equation coupled to the permeability tensor, and some cosmic scaling laws

In Eqs (3.1), (3.2) we defined classical particle motion in the ether. By analogy it is straightforward now to couple spinors to the permeability tensor. We just replace the metric $g_{\mu\nu}$ in the Dirac Lagrangian by $g^B_{\mu\nu}$,

$$L = i\left[\tfrac{1}{2}\overline{\psi}\gamma^{B\alpha}\nabla^B_\alpha\psi - \tfrac{1}{2}\left(\nabla^B_\alpha\overline{\psi}\right)\gamma^{B\alpha}\psi + \hat{m}\psi\overline{\psi}\right], \tag{4.1}$$

($\hat{m} := mc/\hbar$) and $S = c^{-1}\int L\sqrt{-g}dxdt$. This is essentially also what we have done in the case of the electromagnetic field in Sec. 2, namely to replace $g_{\mu\nu}$ by $g^B_{\mu\nu}$ in the Lagrangian. ∇^B_μ in Eq. (4.1) denotes covariant spinor differentiation with respect to the metric $g^B_{\mu\nu}$, and $\gamma^{B\mu}\gamma^{B\nu} + \gamma^{B\nu}\gamma^{B\mu} = 2g^{B^{-1}\mu\nu}$. Note, however, that the volume element in the action is that of the space-time metric $g_{\mu\nu}$, and we do not assume the determinant condition, $g^B = g$, of Sec. 2.

The action we may also write as

$$S = c^{-1}\int L^B\sqrt{-g^B}dxdt, \tag{4.2}$$

with $L^B := L\kappa$, and a scalar field $\kappa := \sqrt{g/g^B}$. Euler variation of (4.2) quickly leads to the Lagrange equations

$$\gamma^{B\alpha}\nabla^B_\alpha\psi + \tfrac{1}{2}\frac{\kappa_{,\alpha}}{\kappa}\gamma^{B\alpha}\psi + \hat{m}\psi = 0, \tag{4.3}$$

$$\left(\nabla^B_\alpha\overline{\psi}\right)\gamma^{B\alpha} + \tfrac{1}{2}\overline{\psi}\gamma^{B\alpha}\frac{\kappa_{,\alpha}}{\kappa} - \hat{m}\overline{\psi} = 0. \tag{4.4}$$

We sketch here very shortly the spectral theory of Eq. (4.3) in the context of the line element (2.24), a very similar spectral problem we have dealt with in Ref. [11]. We indicate here only the modifications to be carried out in some of the formulas there. In the Appendix

of Ref. [11] we have to replace $g_{\mu\nu}$ by $g^B_{\mu\nu}$. More explicitly, in the Christoffel indices in Eq. (A4) of Ref. [11] we have to replace $a(\tau)$ by $b(\tau)$, c by $ch^B(\tau)$, and $\Gamma^0_{00} = \dot{h}^B / h^B$. Eq. (A7) reads as $\gamma_0 = h^B \tilde{\gamma}_0$, and (A11) as $b^0_0 = 1/a^0_0 = h^B$. In the formulas for the spinor connection (A12) and (A13) we have likewise to replace $a(\tau)$ by $b(\tau)$, $\tilde{\gamma}_0$ by $\tilde{\gamma}_0 h^B$, and c by ch^B. The $\tilde{\gamma}_\mu$ are Dirac matrices in Minkowski space defined in (A6) of Ref. [11]. We may then write Eq. (4.3) explicitly as

$$\left[-\frac{1}{c^2 h^B}\tilde{\gamma}_0 \frac{\partial}{\partial\tau} + \frac{x^3}{Rb}\tilde{\gamma}_i \frac{\partial}{\partial x^i} - \frac{1}{c^2 h^B}\tilde{\gamma}_0\left(\frac{3}{2}\frac{\dot{b}}{b} + \frac{1}{2}\frac{\dot{\kappa}}{\kappa}\right) - \frac{1}{Rb}\tilde{\gamma}_3 + \hat{m}\right]\psi = 0, \quad (4.5)$$

which replaces Eq. (2.2) of Ref.[11]. In deriving Eq. (4.5) we assume that $\kappa(\tau)$ depends only on cosmic time. $x^i = (z,t)$.

Using the separation ansatz (3.1) of Ref. [11], we obtain

$$\left[D^+_\tau D^-_\tau - s^2 c^2 R^{-2}\right]\varphi(\tau) = 0, \quad (4.6)$$

where s is the real spectral variable (separation parameter) and $\varphi(\tau)$ determines the time dependence of the spectral elementary waves, in particular their frequencies. The operators $D^\pm_\tau$ are defined as

$$D^\pm_\tau := \frac{b}{h^B}\left[i\frac{d}{d\tau} + i\left(\frac{3}{2}\frac{\dot{b}}{b} + \frac{1}{2}\frac{\dot{\kappa}}{\kappa}\right) \pm \hat{m}ch^B\right], \quad (4.7)$$

which replaces Eq. (3.3) of Ref. [11]. For $\kappa = 1$, $h^B = 1$, and $a(\tau) = b(\tau)$, formulas (4.5)-(4.7) reduce to those of Ref. [11]. If we introduce in Eq. (4.6) a new dependent variable, $\varphi =: \chi(\tau) b^{-2} h^{B^{1/2}} \kappa^{-1/2}$, we obtain

$$\ddot{\chi} + \left[-\frac{1}{2}\frac{\ddot{b}}{b} + \frac{1}{2}\frac{\ddot{h}^B}{h^B} + \frac{1}{2}\frac{\dot{h}^B}{h^B}\frac{\dot{b}}{b} + \frac{1}{4}\frac{\dot{b}^2}{b^2} - \frac{3}{4}\frac{\dot{h}^{B^2}}{h^{B^2}} + ic\frac{\dot{b}}{b}h^B\hat{m} + \hat{m}^2 c^2 h^{B^2} + \frac{s^2 c^2 h^{B^2}}{R^2 b^2}\right]\chi = 0. \quad (4.8)$$

Note that κ does not any more appear in this equation, it enters only as a scale factor in $\varphi(\tau)$, and does not affect its phase. This is because the Lagrangian is linear in the spinor derivatives. In fact, we may scale κ into the wave field, writing $L^B(\psi) = L(\hat{\psi})$, $\hat{\psi} = \kappa^{1/2}\psi$, in the action (4.2).

In the following we discuss the quantum mechanical analogue to Examples I-IV of Sec. 3. We specify b and h^B as in (3.5), and write again $\hat{\tau} = \Lambda\tau$, and $\hat{M} := H^B R\hat{m}$. Then Eq. (4.8) reads as

$$d^2\chi/d\hat{\tau}^2 + \left[-\tfrac{1}{4}(\beta-\lambda)(\beta-\lambda-2)\hat{\tau}^{-2} + i\hat{M}\beta\hat{\tau}^{\lambda-1} + \hat{M}^2\hat{\tau}^{2\lambda} + s^2 H^{B^2} B^{-2}\hat{\tau}^{2(\lambda-\beta)}\right]\chi = 0. \quad (4.9)$$

Example I: ($\beta = 0, \lambda \neq -1$). As fundamental solutions of Eq. (4.9) we obtain

$$\chi = \hat{\tau}^{-\lambda/2}\exp\left[\mp i\hat{A}|\lambda+1|^{-1}\hat{\tau}^{\lambda+1}\right], \quad (4.10)$$

$\hat{A} := \sqrt{\hat{M}^2 + s^2 H^{B^2} B^{-2}}$, with frequency $\omega = \hat{A}\Lambda\hat{\tau}^{\lambda}$.

Example II: ($\beta = 0, \lambda = -1$).

$$\chi = \hat{\tau}^{1/2\mp i\hat{A}}, \quad (4.11)$$

with $\hat{A}$ as in (4.10), and frequency $\omega = \hat{A}\Lambda\hat{\tau}^{-1}$.

Example III: ($\lambda = -1, \beta \neq 0$). Two independent solutions are

$$\chi^{(1),(2)} = \hat{\tau}^{1/2} H_{\hat{\alpha}}^{(2),(1)}\left(\hat{B}|\beta|^{-1}\hat{\tau}^{-\beta}\right), \ \hat{B} := |s| H^B B^{-1}, \ \hat{\alpha} := \tfrac{1}{2} - i\hat{M}\beta^{-1}; \quad (4.12)$$

$H_{\hat{\alpha}}^{(2),(1)}$ are the usual Hankel functions. If $\beta < 0$ and $\hat{\tau} \to \infty$, we have asymptotically

$$\chi^{(1),(2)} \sim \text{const.}\hat{\tau}^{(1+\beta)/2}\exp\left[\mp i\hat{B}|\beta|^{-1}\hat{\tau}^{-\beta}\right], \quad (4.13)$$

and thus $\omega \sim \hat{B}\Lambda\hat{\tau}^{-\beta-1}$.

Remark: (Particle creation.) For $\beta < 0$ and $\hat{\tau} \to 0$, we have in this Example $\chi^{(1)} \approx c_1 \hat{\tau}^{1/2+\beta\hat{\alpha}} + c_2 \hat{\tau}^{1/2-\beta\hat{\alpha}}$, i. e., a mixture of positive and negative frequency modes $\exp\left(\mp i\hat{M}\log\hat{\tau}\right)$ with $\omega \sim \Lambda\hat{M}\hat{\tau}^{-1}$.

Example IV: $\beta = (1+\lambda)/2$, $\lambda \neq -1$.

$$\chi = \hat{\tau}^{-\lambda/2} W_{\pm\hat{\alpha},1/4}\left(\pm 2i\hat{M}(1+\lambda)^{-1}\hat{\tau}^{1+\lambda}\right), \tag{4.14}$$

$W_{\pm\hat{\alpha},1/4}$ are Whittaker functions with $\hat{\alpha} := \frac{1}{4}\left[1 - 2i(1+\lambda)^{-1} s^2 H^{B^2} B^{-2} \hat{M}^{-1}\right]$. If $1+\lambda > 0$, we obtain, for $\hat{\tau} \to \infty$,

$$\chi \approx \text{const.}\hat{\tau}^{-\lambda/2 \pm (1+\lambda)/4} \exp \mp i\left[(1+\lambda)^{-1}\hat{M}\hat{\tau}^{1+\lambda} + \tfrac{1}{2}s^2 H^{B^2} B^{-2}\hat{M}^{-1}\log\hat{\tau}\right], \tag{4.15}$$

and thus $\omega \approx \Lambda\hat{M}\hat{\tau}^{\lambda} + \frac{1}{2}s^2\Lambda H^{B^2} B^{-2}\hat{M}^{-1}\hat{\tau}^{-1}$.

We identify the integration constant ν in (3.3) with the spectral parameter s via

$$\nu = \hbar s (Rmc)^{-1}. \tag{4.16}$$

(ν is an integration constant, and should not be mixed up with frequency.) Then we have in all these cases asymptotic (or, in Examples I and II, exact) equivalence of the classical energy (3.15) with the quantum mechanical $\hbar\omega$.

If $m = 0$ in Eqs. (4.3) and (4.8), we obtain without specification of h^B and b the fundamental solutions

$$\chi = \sqrt{b/h^B}\exp\left[\mp is\Lambda\int h^B b^{-1} d\tau\right], \tag{4.17}$$

because of the conformal coupling of the massless wave equation. By analogy we define then the energy of massless particles by means of the Einstein relation, $E = \hbar\omega$, with $\omega = |s|\Lambda h^B b^{-1}$. Note that the phase in Eq. (4.17) coincides with that of electromagnetic waves, cf. (2.27).

Up to now we studied only the time dependence of the spectral elementary waves. The spatial part of the separation problem of the Dirac equation in hyperbolic space H^3 was discussed in detail in Sec. III of Ref. [11]. Considering plane waves propagating along the t-axis of H^3, we found that the spatial part of the phase is $s\log(t/R)$, the same as we obtained for the electromagnetic field, cf. (2.26). For the wave vector we therefore have $k_t = s/t$ (along the t-axis), for photons and particles alike. If we identify ν in Eq. (3.16) with s according to (4.16), we obtain $p_t = \hbar k_t$. For the wave length we obtain, with the 3-space metric as defined in (2.23),

$$\lambda\!\!\!^{-}\, = |k_t|^{-1} = aR|s|^{-1}. \tag{4.18}$$

With ω as in (4.17) we have so for the speed of light in the ether $|\mathbf{v}_c| = \lambda\!\!\!^{-}\,\omega = ch^B ab^{-1}$, cf. also (2.14) and (3.18).

To obtain the group velocity of the elementary waves, we differentiate the phase with respect to s, and equate then the (τ, t)-differential to zero. The phase velocity is likewise calculated in this way, but without prior s-differentiation. We discuss that shortly for the previous Examples I-IV.

Example I, cf. (4.10): The phase is $\hat{A}(1+\lambda)^{-1}\hat{\tau}^{1+\lambda} + s\log(t/R)$, and therefore

$$|\mathbf{v}_g| = \frac{a(\tau)c|s|H^{B^2}\hat{\tau}^{\lambda}}{B^2\sqrt{\hat{M}^2 + s^2 H^{B^2} B^{-2}}}, \quad |\mathbf{v}_{ph}| = \lambda\!\!\!^{-}\,\omega = ca(\tau)|s|^{-1}\sqrt{\hat{M}^2 + s^2 H^{B^2} B^{-2}}\;\hat{\tau}^{\lambda}. \tag{4.19}$$

The group velocity $|\mathbf{v}_g|$ coincides with the classical particle speed (3.18) via the identification (4.16). For the product of group and phase velocity we have $|\mathbf{v}_g||\mathbf{v}_{ph}| = |\mathbf{v}_c|^2$, with the speed of light $|\mathbf{v}_c| = caH^B B^{-1}\hat{\tau}^{\lambda}$. The formulas obtained here for the velocities also hold true for $\lambda = -1$, cf. Example II, (4.11).

Example III, cf. (4.13): For $\beta < 0$ and $\hat{\tau} \to \infty$ we have asymptotically

$$|\mathbf{v}_g| \sim |\mathbf{v}_{ph}| \sim |\mathbf{v}_c| = ca(\tau)H^B B^{-1}\hat{\tau}^{-\beta-1}, \tag{4.20}$$

and likewise, via (4.16), asymptotic equivalence with the classical particle speed (3.18), which approaches the speed of light.

Example IV, cf. (4.15):

$$|\mathbf{v}_g| \sim ca(\tau)|s|H^{B^2}B^{-2}\hat{M}^{-1}\hat{\tau}^{-1}, \ |\mathbf{v}_{ph}| \sim ca(\tau)|s|^{-1}\hat{M}\hat{\tau}^{\lambda}. \tag{4.21}$$

$|\mathbf{v}_g|$ is again asymptotically equivalent to the classical particle speed, and $|\mathbf{v}_g|^{1/2}|\mathbf{v}_{ph}|^{1/2} \sim |\mathbf{v}_c| = ca(\tau)H^B B^{-1}\hat{\tau}^{(\lambda-1)/2}$.

Examples I-IV deal with global cosmic evolution, relating the behavior of free particles to the cosmic expansion and the scale factors in the permeability tensor. If we study local interactions based on energy-momentum conservation, we will of course assume $a(\tau)$, $b(\tau)$ and $h^B(\tau)$ in (2.23) and (2.24) at a given instant τ_0 of cosmic time. Then these factors can be scaled into the constants of nature which get so functions of cosmic time. If we replace in Eq. (4.8) $b(\tau)$, $h^B(\tau)$ by $b(\tau_0)$, $h^B(\tau_0)$, we obtain as a pair of fundamental solutions

$$\chi = \exp[\mp i\omega(\tau - \tau_0)], \ \omega := \tilde{c}\sqrt{\tilde{R}^{-2}s^2 + \tilde{m}^2\tilde{c}^2\tilde{\hbar}^{-2}}, \tag{4.22}$$

with $\tilde{R} = Ra(\tau_0)$,

$$\tilde{c} = ch^B(\tau_0)a(\tau_0)b^{-1}(\tau_0), \tag{4.23}$$

$$\tilde{m}/\tilde{\hbar} = (m/\hbar)b^2a^{-2}h^{B^{-1}}. \tag{4.24}$$

$\tilde{R}$ we could of course put equal to one, measuring length in units of $\tilde{R}$ in the 3-space of curvature $-1/\tilde{R}^2$. In (4.24) we obtain only a scaling law for the ratio $m/\hbar$; the scaling of m we read off from the Hamilton-Jacobi equation (3.1) and the line element (2.24). We

consider again particle motion along the t-axis of H^3. At a given instant τ_0 of cosmic time we may then write Eq. (3.1) as

$$-\tilde{c}^{-2}S_{,\tau}^2 + t^2\tilde{R}^{-2}S_{,t}^2 = -\tilde{m}^2\tilde{c}^2, \tag{4.25}$$

with

$$\tilde{m} = mb^2a^{-2}h^{B^{-1}}. \tag{4.26}$$

Therefore $\tilde{\hbar} = \hbar$ in (4.24), which means that $\hbar$ is independent of cosmic time.

For energy and momentum as defined in (3.15) and (3.16) we recover with (4.23) and (4.26) the familiar local expressions $E = \tilde{m}\tilde{c}^2\left(1-|\mathbf{v}|^2/\tilde{c}^2\right)^{-1/2}$ and $|p_t| = \tilde{m}|\mathbf{v}|\left(1-|\mathbf{v}|^2/\tilde{c}^2\right)^{-1/2}$.

The most important prediction of this theory is, that the photon frequency does *not* scale inversely proportional to its wave length. The wave length scales, as in traditional RW cosmology, with the expansion factor, $\lambda\!\!\!^{-} \sim a(\tau)$. The scaling of frequency , however, is completely determined by the scale factors in the permeability tensor, $\omega \sim h^B(\tau)b^{-1}(\tau)$, cf. (4.17). At the end of Sec. 2 and in (4.16) we argued that the Einstein relation $E(\tau) = \hbar\omega(\tau)$ remains valid in the cosmic evolution with the Planck constant independent of cosmic time. So the photon energy scales as $E \sim h^B(\tau)b^{-1}(\tau)$. (In traditional RW cosmology we have instead $E \sim a^{-1}(\tau)$.) Accordingly, red-shifts of absorption spectra should differ from those obtained by comparing diffraction patterns, since I do not see any reason to assume the relation $h^B b^{-1} = a^{-1}$ among the scale factors.

5. Scalar particles in the ether

We define here the interaction of scalar fields with the permeability tensor $g^B_{\mu\nu}$. The same coupling mechanism will be used in the next Section, when we discuss the electromagnetic field in the ether without the determinant condition $g^B = g$ of Sec. 2.

In Sec. 4 we found that the solutions of the Dirac equation scale with $\kappa^{-1/2}$, $\kappa := \sqrt{g/g^B}$, cf. Eq. (4.8). In particular κ does not enter the phase of the spectral elementary waves. Likewise, we found the usual conformal scaling in the massless case, cf. (4.17). These properties we also want to retain for scalar fields. We replace at first in the Lagrangian $L_0 = -\frac{1}{2}\left(g^{\mu\nu}\psi_{,\mu}\overline{\psi}_{,\nu} + \hat{m}^2\psi\overline{\psi}\right)$ the tensor $g^{\mu\nu}$ by $g^{B^{-1}\mu\nu}$, as we did in (4.1) with the Dirac Lagrangian ($\hat{m} := mc/\hbar$). But to ensure the mentioned scaling properties, we have to add some further terms, and the scalar Lagrangian corresponding to Eq. (4.1) is

$$L = -\tfrac{1}{2}g^{B^{-1}\mu\nu}\psi_{,\mu}\overline{\psi}_{,\nu} - \tfrac{1}{2}\left[\hat{m} + \left(\tfrac{1}{6}R^B_{\mu\nu} + \tfrac{1}{4}\kappa^{-2}\kappa_{,\mu}\kappa_{,\nu} - \tfrac{1}{2}\kappa^{-1}\kappa_{,\mu;\nu}\right)g^{B^{-1}\mu\nu}\right]\psi\overline{\psi}. \qquad (5.1)$$

$R^B_{\mu\nu}$ is the Ricci tensor of metric $g^B_{\mu\nu}$, as in Eq. (2.17); it ensures the conformal scaling. The κ – terms are needed for the $\kappa^{-1/2}$ – scaling of the wave fields. (The double semicolon denotes covariant differentiation with respect to $g^B_{\mu\nu}$.) In fact, we may define now, cf. Eq. (4.2), $L^B = L\kappa$, $\hat{\psi} = \kappa^{1/2}\psi$, so that

$$S = \frac{1}{c}\int L\sqrt{-g}\,dx\,dt = \frac{1}{c}\int L^B\sqrt{-g^B}\,dx\,dt, \qquad (5.2)$$

$$L^B = -\tfrac{1}{2}\left[g^{B^{-1}\mu\nu}\hat{\psi}_{,\mu}\overline{\hat{\psi}}_{,\nu} + \left(\hat{m} + \tfrac{1}{6}R^B\right)\hat{\psi}\overline{\hat{\psi}}\right], \qquad (5.3)$$

($R^B := R^B_{\mu\nu}g^{B^{-1}\mu\nu}$) which explains the choice of the Lagrangian (5.1). Instead of imposing the determinant condition $g^B = g$ of Sec. 2, we simply scale κ into the wave fields. This ensures the right semiclassical limit, cf. also Sec. 6.

The field equation reads as

$$\left(\psi_{,\mu;;\nu} + \kappa^{-1}\kappa_{,\nu}\psi_{,\mu}\right)g^{B^{-1}\mu\nu} - \left[\hat{m} + \tfrac{1}{6}R^{B} + \left(\tfrac{1}{4}\kappa^{-2}\kappa_{,\mu}\kappa_{,\nu} - \tfrac{1}{2}\kappa^{-1}\kappa_{,\mu;;\nu}\right)g^{B^{-1}\mu\nu}\right]\psi = 0. \qquad (5.4)$$

To compare with the Dirac equation, we shortly indicate the spectral resolution of this wave equation in the context of the line elements (2.23) and (2.24). Note that the space-time metric (2.23) enters only via κ in (5.4); $\kappa(\tau) = a^3/(b^3 h^B)$. The curvature scalar of metric (2.24) reads as

$$\tfrac{1}{6}R^{B} = \frac{-1}{b^2R^2} + \frac{1}{c^2}\frac{1}{h^{B2}}\left(\frac{\ddot{b}}{b} + \frac{\dot{b}^2}{b^2} - \frac{\dot{b}}{b}\frac{\dot{h}^B}{h^B}\right). \qquad (5.5)$$

Applying the d'Alembertian of metric $g^{B}_{\mu\nu}$ to $\kappa(\tau)$, we obtain

$$\frac{\kappa_{,\mu;;\nu}g^{B^{-1}\mu\nu}}{\kappa} = \frac{-1}{c^2h^{B2}}\left[\frac{\ddot{\kappa}}{\kappa} + \left(3\frac{\dot{b}}{b} - \frac{\dot{h}^B}{h^B}\right)\frac{\dot{\kappa}}{\kappa}\right]. \qquad (5.6)$$

We consider a plane wave propagating along the t-axis of H^3, and perform the variable separation $\psi = \varphi(\tau)t^{1+is}$ in Eq. (5.4), cf. Ref. [12]. If we define $\varphi(\tau) =: \chi(\tau)b^{-3/2}h^{B^{1/2}}\kappa^{-1/2}$, we arrive at a differential equation for $\chi(\tau)$ which is identical to (4.8), but with the imaginary mass term $ic\dot{b}b^{-1}h^{B}\hat{m}$ dropped. Accordingly, all the considerations of Sec. 4, from Eq. (4.8) onwards, apply with minor modifications. Solution (4.12) gets elementary with $\hat{\alpha} = 1/2$, and in Eq. (4.14) the real part of $\hat{\alpha}$ has to be dropped which, however, does not affect the phase in (4.15).

6. Electromagnetism revisited

In this Section we consider electrodynamics in the ether without the determinant condition $g^B = g$ on the permeability tensor, which we assumed in Sec. 2 (but dropped already in Secs. 3-5). The solutions of the field equations should scale with $\kappa^{-1/2}$, $\kappa = \sqrt{g/g^B}$, as is the case with spinor and scalar fields, cf. Secs. 4 and 5. This scaling assures that the phase of the wave fields does not depend on the space-time metric, which enters in the action via the volume element, cf. (4.1) and (5.2). This is necessary since both eikonal (2.12) and Hamilton-Jacobi equation (3.1) are independent of the space-time metric, and we want the right semiclassical limit. To achieve this scaling, we modify the Lagrangian (2.1), (2.2) of the electromagnetic field,

$$L = -\tfrac{1}{4} G_{\mu\nu} K^{\mu\nu} + \tfrac{1}{c} A_\mu j^\mu , \tag{6.1}$$

$$G_{\mu\nu} := F_{\mu\nu} + \frac{1}{2}\left(\frac{\kappa_{,\mu}}{\kappa} A_\nu - \frac{\kappa_{,\nu}}{\kappa} A_\mu \right) , \quad K^{\mu\nu} := g^{B^{-1}\mu\tilde{\mu}} g^{B^{-1}\nu\tilde{\nu}} G_{\tilde{\mu}\tilde{\nu}} . \tag{6.2}$$

Defining $L^B := L\kappa$, $\hat{A}_\mu := \kappa^{1/2} A_\mu$, and $\hat{j}^\mu := \kappa^{1/2} j^\mu$, we may write, cf. (5.2), $S = \frac{1}{c}\int L^B \sqrt{-g^B}\, dxdt$, with

$$L^B = -\tfrac{1}{4} \hat{F}_{\mu\nu} \hat{H}^{\mu\nu} + \tfrac{1}{c} \hat{A}_\mu \hat{j}^\mu , \tag{6.3}$$

$$\hat{F}_{\mu\nu} := \hat{A}_{\nu;;\mu} - \hat{A}_{\mu;;\nu} = \kappa^{1/2} G_{\mu\nu} , \quad \hat{H}^{\mu\nu} := g^{B^{-1}\mu\tilde{\mu}} g^{B^{-1}\nu\tilde{\nu}} \hat{F}_{\tilde{\mu}\tilde{\nu}} . \tag{6.4}$$

(The double semicolon denotes covariant differentiation with respect to $g^B_{\mu\nu}$). From (6.3) we immediately derive the field equations

$$\hat{H}^{\mu\nu}{}_{;;\nu} = \tfrac{1}{c} \hat{j}^\mu , \tag{6.5}$$

and from the potential representation (6.4) follow the homogeneous equations

$$(-g^B)^{-1/2} \varepsilon^{\lambda\alpha\beta\gamma} \hat{F}_{\alpha\beta;;\gamma} = 0 . \tag{6.6}$$

Expressed in $G_{\mu\nu}$ and $K^{\mu\nu}$ tensors, we may write the field equations as

$$K^{\mu\lambda}{}_{;;\lambda} + \tfrac{1}{2}\kappa^{-1}\kappa_{,\lambda}K^{\mu\lambda} = \tfrac{1}{c}j^{\mu}, \tag{6.7}$$

$$(-g^{B})^{-1/2}\varepsilon^{\lambda\alpha\beta\gamma}\left(G_{\alpha\beta;;\gamma} + \tfrac{1}{2}\kappa^{-1}\kappa_{,\gamma}G_{\alpha\beta}\right) = 0. \tag{6.8}$$

Eqs. (6.5)-(6.8) are entirely written in terms of the permeability tensor $g^{B}_{\mu\nu}$, the space-time metric enters only via its determinant g in the scalar field κ. From (6.5) and (6.6) it is clear that we may impose the Lorentz condition $g^{B^{-1}\mu\nu}\hat{A}_{\mu;;\nu} = 0$, or, equivalently,

$$g^{B^{-1}\mu\nu}\left(A_{\mu;;\nu} + \tfrac{1}{2}\kappa^{-1}\kappa_{,\mu}A_{\nu}\right) = 0, \tag{6.9}$$

reflecting the gauge invariance $A_{\mu} \to A_{\mu} + \kappa^{-1/2}\varphi_{,\mu}$ of (6.7) and (6.8) with an arbitrary scalar φ. By means of this gauge condition we write Eq. (6.7) as a wave equation for the vector potential, cf. (2.17),

$$g^{B^{-1}\mu\nu}\left[A_{\lambda;;\mu;;\nu} + A_{\lambda;;\mu}\frac{\kappa_{,\nu}}{\kappa} - A_{\mu}R^{B}_{\nu\lambda} + A_{\lambda}\left(\frac{1}{2}\frac{\kappa_{,\mu;;\nu}}{\kappa} - \frac{1}{4}\frac{\kappa_{,\mu}\kappa_{,\nu}}{\kappa^{2}}\right)\right] = -\frac{1}{c}j^{\mu}g^{B}_{\mu\lambda}. \tag{6.10}$$

(Note that we raise and lower indices of tensors always with the space-time metric, never with the permeability tensor.) $R^{B}_{\nu\lambda}$ denotes the Ricci tensor of metric $g^{B}_{\mu\nu}$ as in (2.17) and (5.1).

In the case that $j^{\mu} = 0$, we may impose on the solutions of the field equations (6.7), (6.8) in addition to the Lorentz gauge (6.9) the covariant Coulomb condition

$$g^{B^{-1}\mu\nu}\kappa_{,\mu}A_{\nu} = 0. \tag{6.11}$$

Using the gauge conditions (6.9) and (6.11) (differentiated), we write Eq. (6.7) ($j^{\mu} = 0$) as

$$H^{\mu\lambda}{}_{;;\lambda} + \kappa^{-1}\kappa_{,\lambda}H^{\mu\lambda} + M^{\mu\nu}A_{\nu} = 0, \tag{6.12}$$

$$M^{\mu\nu} := g^{B^{-1}\mu\nu}g^{B^{-1}\alpha\beta}\left(\frac{1}{4}\frac{\kappa_{,\alpha}\kappa_{,\beta}}{\kappa^{2}} - \frac{1}{2}\frac{\kappa_{,\alpha;;\beta}}{\kappa}\right) + g^{B^{-1}\mu\alpha}g^{B^{-1}\nu\beta}\frac{\kappa_{,\alpha;;\beta}}{\kappa}, \tag{6.13}$$

to the Lagrangian (3.2) the interaction term $e\kappa^{1/2}A_\mu \dot{x}^\mu$. We have then for the current of a point charge

$$\frac{1}{c}\hat{j}^\mu(x) = e\kappa \int ds(-g)^{-1/2}\delta\big(x(s)-x\big)\dot{x}(s) = \kappa^{1/2}\frac{1}{c}j^\mu . \tag{6.16}$$

This means a variation of charge in cosmic time,

$$\tilde{e} = e\kappa^{1/2} = ea^{3/2}b^{-3/2}h^{B^{-1/2}}, \tag{6.17}$$

which complements the scaling laws (4.23) and (4.26).

The considerations on electromagnetic energy in (2.18)-(2.22) can easily be adapted to the case $\kappa_{,\mu} \neq 0$. Instead of (2.18) we start with the tensor

$$\hat{T}_\nu{}^\mu := -\hat{F}_{\nu\alpha}\hat{H}^{\mu\alpha} + \tfrac{1}{4}\delta^\mu_\nu \hat{F}_{\alpha\beta}\hat{H}^{\alpha\beta} . \tag{6.18}$$

From Eq. (6.5) we have $\hat{T}_\nu{}^\mu{}_{;;\mu} = c^{-1}\hat{F}_{\nu\mu}\hat{j}^\mu$, and

$$\left(\kappa^{-1}\hat{T}_\nu{}^\mu\right)_{;\mu} = \kappa^{-1}\left(\tfrac{1}{c}\hat{F}_{\nu\alpha}\hat{j}^\alpha + \Delta^\alpha_{\beta\nu}\hat{T}_\alpha{}^\beta\right), \tag{6.19}$$

with $\Delta^\alpha_{\beta\nu}$ as in (2.22). We define the energy-momentum tensor as

$$T_\nu{}^\mu := \kappa^{-1}\hat{T}_\nu{}^\mu = -G_{\nu\alpha}K^{\mu\alpha} + \tfrac{1}{4}\delta^\mu_\nu G_{\alpha\beta}K^{\alpha\beta}, \tag{6.20}$$

so that

$$T_\nu{}^\mu{}_{;\mu} = \tfrac{1}{c}G_{\nu\alpha}j^\alpha + \Delta^\alpha_{\beta\nu}T_\alpha{}^\beta . \tag{6.21}$$

That $T_\nu{}^\mu$ is indeed the right choice for the energy-momentum tensor gets clear if we use 3-d formalism. We only indicate here the very minor modifications that formulas (2.7)-(2.10) undergo if we drop the determinant condition. In (2.7) we have to replace $F_{\mu\nu}$ and $H^{\mu\nu}$ by $G_{\mu\nu}$ and $K^{\mu\nu}$, respectively. The definition of charge density and 3-current remains unaltered. In the continuity equation (2.8) and the Maxwell equations (2.9) and (2.10) the only modification is to replace $\sqrt{\gamma}$ by $\sqrt{\kappa\gamma}$ in the terms containing the time derivative, i.e.,

$\gamma^{-1/2}\partial(\gamma^{1/2}\cdot)/\partial\tau$ is replaced by $(\kappa\gamma)^{-1/2}\partial((\kappa\gamma)^{1/2}\cdot)/\partial\tau$. The material equations (2.11) remain unaltered. The energy density $T_0{}^0$ as defined now in (6.20) relates to field strengths and displacement vectors as in (2.19), and formula (2.20) holds true for the free part of the Lagrangian (6.1).

7. Conclusion

The cosmology developed here is based on two symmetric tensor fields, a space-time metric, and a symmetric permeability tensor representing the world ether. This permeability tensor we assume as homogeneous and isotropic. It is determined by two scale factors $h^B(\tau)$ and $b(\tau)$, both functions of cosmic time like the expansion factor $a(\tau)$ in the Robertson-Walker metric.

Electromagnetic fields are coupled to the permeability tensor like in a dielectric medium, cf. Sec. 2, apart from some technical modifications described in Sec. 6. Classical mechanics in the ether is defined by replacing in the Hamilton-Jacobi equation the space-time metric by the permeability tensor, cf. Sec. 3. The propagation of spinor fields in the ether is likewise defined by this substitution in the Dirac Lagrangian, cf. Sec. 4.

We studied the evolution of free particles and wave fields in the ether, and obtained explicit formulas for the variation of their wave length, frequency, velocity, and energy in cosmic time. The speed of light is a function of the three cosmic scale factors, and need not be, as a function of cosmic time, bounded from above. Massive particles may come arbitrarily close to the speed of light, but cannot really reach it. (However, at two different instants of cosmic time the particle speed may well exceed the speed of light; if $c(\tau_1) > c(\tau_2)$ we may have $v_p(\tau_1) > c(\tau_2)$, of course.)

We do not suggest evolution equations for the space-time metric, nor for the permeability tensor. What we do in this article is to try various choices for the three scale factors and to see what makes the difference in the resulting particle dynamics in an open universe, cf. Examples I-IV in Secs. 3 and 4.

The fact that the speed of light is unbounded in cosmic time does not result in an unbounded speed of signal transfer in local Minkowski frames. Whenever we consider some local physical process we can neglect the cosmic time variation, and take the space-time metric as well as the permeability tensor during the whole process at a given instant of cosmic time. Then the three cosmic scale factors can be scaled into the constants of nature in the Hamiltonian of the local system, cf. Sec. 4. In the Minkowskian limit cosmic time parametrizes the constants of nature, and must be distinguished from the local system time that labels the evolution of the Hamiltonian system. So, locally, the special relativistic vacuum dynamics in Minkowski space is recovered, with constants of nature depending on the cosmic time parameter. The concept of local Minkowski frames and relativistic mechanics works because we can accommodate, formally, the effects of the ether in the constants of nature. But Minkowski space is only an abstraction based on a purely geometric space-time view.

This theory is unrelated to the ideas of Eddington, Milne, and Dirac on the variation of the constants of nature; in particular it is not a scaling theory for 'large ratios', cf. Ref. [13]. The variation of the constants is a consequence of the cosmic ether.

Because the speed of light itself is varying in cosmic time, the energy of photons does not scale inversely to their wave length, cf. Sec. 4. So the prediction is that red shifts of absorption spectra will differ from those based on wave length measurements by diffraction, via grating spectrographs, for example.

Acknowledgments

The author acknowledges the support of the *Japan Society for the Promotion of Science*, contract no. P-95378. The stimulating atmosphere of the Institute of Physics, Bhubaneswar, where the final stages of this work were completed, is likewise gratefully acknowledged.

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