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INCLUSIVE AND EXCLUSIVE PRODUCTION OF η MESONS BY PIONS

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ABSTRACT

Inclusive and exclusive (π, η) reactions are studied in the framework of a momentum space DWIA model. It is assumed that both reactions are dominated by formation of an $N(1535)$ resonance. Medium corrections for the self energy of the intermediate N^* are taken into account. Cross sections are calculated for (π^+, η) inclusive reactions on ^{12}C and compared with experiment and with existing theoretical calculations. We show that quantitative agreement is obtained with experimental data, under the assumption that this reaction is dominated by quasifree single–nucleon knockout. Differential and total cross sections are calculated for the exclusive reaction $^{13}C(\pi^+, \eta)^{13}N$.

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1. Introduction

The past few years have seen considerable interest in nuclear production of η mesons. First, the Saclay threshold measurements of $p + d \rightarrow \eta +^3 He$ revealed cross sections much larger than expected [1]. Theoretical calculations by Liu and collaborators [2] suggested that bound nuclear states of the η may exist. A new generation of machines being constructed at Jülich and Uppsala, plus proposed accelerators such as PILAC at Los Alamos [3] and various ϕ factories [4], promise excellent energy resolution and the possibility of efficient detection of η mesons. These developments may herald the beginning of a series of experiments involving production and detection of η mesons in a variety of nuclear interactions.

Nuclear interactions of the η meson have been the subject of several recent investigations [2,5-7]. At present, data is sufficiently scarce that our understanding of η reactions with nuclei is incomplete. However, as a good starting point we can assume that the η meson is produced predominantly through coupling to the $N(1535)$, an S_{11} π–N resonance [8]. Consequently, observation of η production in nuclei can provide information on production and propagation of the N(1535) in the nucleus. Since the N(1535) is an isospin 1/2 baryon, observation of the (π, η) process in nuclei allows us to look at formation of an intermediate baryon resonance without the reaction being completely determined by the [generally dominant] phenomenon of intermediate $\Delta(1232)$ production.

In this paper we focus on two possible reactions arising in pion–induced production of η mesons. First, we consider inclusive production via the (π, η) reaction on nuclei. This follows the methods outlined in a previous shorter communication [9]. Next, we consider the exclusive process $A(\pi, \eta)B$, where A and B are bound nuclear states. Our paper is organized as follows. In Sec. 2, we outline our formalism and review our results for inclusive (π, η) reactions. In Sec. 3 we apply this same formalism to exclusive reactions in certain light nuclei, and we present predictions for cross sections there. In Sec. 4 we summarize the main results from our calculations and review the advantages and drawbacks of our present model. We conclude with suggestions for future improved calculations of these processes.

2. Inclusive (π, η) Reactions

There is now data on the inclusive (π^+, η) process on nuclear targets [7]. There have been two previous models applied to this process. The first was an intranuclear cascade calculation by Gibbs and Kaufman [10]. Such a semiclassical calculation could be expected to give the gross features of the reaction, but not a quantitative explanation; this will be seen to be the case. The second type of calculation is a DWIA approach to the reaction. Two existing calculations are by Kohno and Tanabe [6], and by the present authors [9]. Here we will briefly review the results of the two DWIA models.

Both DWIA calculations assume that η formation occurs through excitation of a nucleon by a pion to a $N(1535)$, which propagates through the nucleus and then decays to an η plus a nucleon. In paper I it was shown that the inclusive cross sections can be written as the square of a transition amplitude $T^{\pi\eta}$ with the general form

$$T^{\pi\eta}(\vec{p}, \vec{k_\pi}, \vec{q_\eta}) = V^{\pi\eta}(\vec{p}, \vec{k_\pi}, \vec{q_\eta}) + \int \frac{d^3k'}{(2\pi)^3} \frac{V^{\pi\eta}(\vec{p}, \vec{k'}, \vec{q_\eta}) T^{\pi\pi'}(\vec{k'}, \vec{k_\pi})}{\varepsilon(k') - \varepsilon(k_\pi) + i\epsilon} \tag{1}$$

In Eq. (1), k_π denotes the incident pion momentum, q_η the outgoing η momentum, and p the momentum of the [unseen] final proton. The first term $V^{\pi\eta}$ corresponds to pion–induced η production with no distortion of the incident pion, and the second term includes the effects of incident pion rescattering followed by η production. The initial–state rescattering between the incident pion and nucleus is accounted for in both models; the Kohno–Tanabe calculation uses a local pion optical potential assumed to be directly proportional to the one–body nuclear density, while the calculation of paper I uses a Glauber model calculation for the pion rescattering together with an assumed separable off–shell vertex function.

Propagation of the $N(1535)$ in the nuclear medium, and the final–state nuclear interactions of the η, are accounted for through inclusion of the N^* self–energy in the $\pi + N \rightarrow \eta + N$ t–matrix. In both calculations this elementary t–matrix was taken from the form proposed by Bhalerao and Liu [5]. In the Kohno–Tanabe calculation a constant imaginary part is assumed for the self–energy [roughly $-50i \rightarrow -100i$ in strength]; in paper I, the imaginary part of the self energy is calculated by estimating

the effects shown in Fig. 1. The first term, shown in Fig. 1(a) represents Pauli blocking of the N^* decay, and tends to reduce the N^* width below its free value. The other terms, shown diagrammatically in Fig. 1(b) and (c), represent isobar decays to a single meson (π or η), and two pions plus a particle–hole pair, respectively. The imaginary self–energy contribution from these terms can be evaluated using the Cutkosky rules [11], as was done by Chiang *et al.* [12]. The final nuclear contribution from these terms is obtained by evaluating the resulting Lindhard functions in the Fermi Gas approximation [14]. The contribution from Fig. 1(b) and (c) tends to increase the N^* width in the nucleus. As we shall show, overall these terms dominate and we find the isobar width in the nuclear medium is increased over its free value.

Calculation of inclusive cross sections involves a sum over all unobserved final states populated in the reaction. Both of the existing DWIA calculations assume that the dominant reaction process in (π^+, η) reactions is resonant excitation of a neutron by a π^+ to a N(1535) state followed by quasifree decay of the nucleon resonance to a free η and proton. Therefore only one particle–one hole final states of the nuclear system are considered. The final state nucleon–nucleus interactions are treated as in quasifree (p, p') inclusive reactions [15]. In the calculation of paper I, the small kinetic energy of the recoil nucleus in this reaction is neglected. The particle–hole nuclear states are represented through the standard single–particle shell model.

In Fig. 2 we show (π, η) inclusive cross sections on a ^{12}C target, in μb/MeV–sr, vs. the η kinetic energy in MeV, for pion lab momentum 680 MeV/c. Experimental points are those of Peng *et al.* [7]; the solid curve is the result of the full DWIA calculation of paper I, while the dashed curve shows the result without the isobar self–energy contribution, i.e. assuming the free width for the N*. The data includes angles from $0 \leq \theta_\eta \leq 30°$, so the theoretical calculation has been averaged over the same angular region. As can be seen, the full theoretical curve of paper I gives good quantitative agreement with the experimental data. The full curve is less than the result using the free isobar propagator because the N* width is increased in the nuclear medium. A similar result was obtained by Kohno and Tanabe in their DWIA calculation [6].

The dominant effect on the self–energy comes from the two pion processes shown

diagrammatically in Fig. 1(c). A plausible reason for this would be because N^* decays in medium release a substantial amount of energy. Energy sharing in two pion decays can lead to processes where one of the pions has moderately low energy; such a low energy pion can have a substantial coupling to particle–hole nuclear states and hence produce a significant contribution to the isobar width in the nucleus.

Fig. 3 shows the results of earlier theoretical calculations. The histogram is the intranuclear cascade [INC] calculation of Gibbs and Kaufman [10], and the solid curve is the DWIA calculation of Kohno and Tanabe [6]. The INC result is good at large η energies but too low at small η KE; as expected, it gives reasonable qualitative fit but fails to predict experiment quantitatively. The DWIA calculation of Kohno and Tanabe gives good agreement with the shape of the experimental spectrum but is roughly a factor of two below the data.

We have previously argued [9] that the discrepancy between the two DWIA calculations is probably due to differences in the description of the π–^{12}C initial state rescattering. Since the transition amplitude $\pi + N \rightarrow \eta + N$ is small while the pion–nucleus scattering is comparatively very large, the multiple scattering of the incident pion with the nucleus must be accounted for. Kohno and Tanabe describe the initial state rescattering through an optical potential of the form $t\rho$. For $k_\pi = 800$ MeV/c a similar calculation [16] for π scattering on ^{12}C produces results which underestimate forward pion scattering data [17] by about 40%. Therefore, we suggested that the Kohno–Tanabe calculation might underpredict the inclusive cross sections [which are averaged over forward angles] by about this amount. The Glauber model used for pion rescattering in paper I gives a better prediction for pion scattering data [17] on ^{12}C for angles less than 25°. Such a model appears to better describe the multiple scattering of the pion from the nucleus in the forward direction.

In Fig. 4 we show predicted cross sections for the (π, η) reaction on a ^{12}C target, for a pion lab momentum 620 MeV/c. In our previous calculations off–shell effects in pion rescattering were found to be small. For this calculation we therefore included only on–shell pion rescattering; therefore we expect small corrections to this estimate due to off–shell effects not included in Fig. 4.

3. Exclusive (π, η) Reactions

The same DWIA model which we used to calculate inclusive η production can also be utilized to predict cross sections for exclusive reactions, such as $\pi + A \rightarrow \eta + B$, where A and B are bound nuclear states. For this process the CM differential cross section for η production has the form

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega_\eta} = \frac{q_\eta}{(2J+1)k_\pi} \overline{\sum} |F^{\pi\eta}(q_\eta, k_\pi)|^2 \tag{2}$$

In Eq. (2), $\frac{1}{(2J+1)}\overline{\sum}$ represents an average over the initial spins and sum over final spin states in this transition. In Eq. (2), k_π and q_η are the incident pion and outgoing η momenta, respectively.

The amplitude $F^{\pi\eta}$ in Eq. (2) is related to the T–matrix for the elementary reaction by the relativistic expression

$$\left\langle \vec{Q}' \left| F(E) \right| \vec{Q} \right\rangle = -\frac{1}{2\pi} \left[\mathcal{M}(Q')\mathcal{M}(Q) \right]^{1/2} \left\langle \vec{Q}' \left| T(E) \right| \vec{Q} \right\rangle$$

where

$$\mathcal{M}_{ij}(q) = \frac{E_i(q)\, E_j(q)}{E_i(q) + E_j(q)}$$

We want to calculate the amplitude for a charged pion incident on a nuclear target to produce an η meson, leaving the final nucleus in a bound state. In DWIA this amplitude can be written as the sum of two terms,

$$F^{\pi\eta}(\vec{k_\pi}, \vec{q_\eta}) = U^{\pi\eta}(\vec{k_\pi}, \vec{q_\eta}) - \frac{1}{(2\pi)^2} \sum_{\pi'} \int \frac{dk'}{\mathcal{M}(k')} \frac{F^{\pi\pi'}(\vec{k_\pi}, \vec{k}')U^{\pi'\eta}(\vec{k}', \vec{q_\eta})}{\varepsilon(k') - \varepsilon(k_\pi) + i\epsilon} \tag{3}$$

In Eq. (3), the first term represents production of an η by a single hard collision without initial–state nuclear scattering by the pion. The second term includes a sum over all initial–state pion interactions, followed by η production. In the second term we include both elastic and single charge–exchange scattering of the pion. The final–state nuclear interactions of the η are accounted for in the self–energy of the N^* in the nuclear medium. This calculation is identical to the self–energy calculation for the inclusive reaction and is given in paper I.

The transition amplitude $U^{\pi\eta}$ in Eq. (3) is related to the T–matrix for the process $\pi + N \rightarrow \eta + N$ through the relation

$$U^{\pi\eta} = -\frac{\left(\mathcal{M}_{\eta B}(q_\eta)\mathcal{M}_{\pi A}(k_\pi)\right)^{1/2}}{2\pi} \langle B; q_\eta \,|t(\omega)|\, A; k_\pi \rangle \tag{4}$$

We confine ourselves to nuclear transitions involving only a single nucleon. In this case the nuclear matrix elements in Eq. (4) can be approximated by

$$\langle B; q_\eta \,|t(\omega)|\, A; k_\pi \rangle \simeq F_{CM}^{\frac{1}{2}}\, t(\omega, q_\eta, k_\pi, < p >) \int d^3(r)\phi_f^*(\vec{r})\phi_i(\vec{r}) \exp\left[i\frac{\mathcal{A}-1}{\mathcal{A}}\vec{Q}\cdot\vec{r}\right] \tag{5}$$

In Eq. (5), ϕ_i and ϕ_f are respectively single–particle wave functions in initial and final states, the momentum transfer $\vec{Q} = \vec{k_\pi} - \vec{q_\eta}$, and the factor $F_{CM}^{\frac{1}{2}} = \exp\left[Q^2/4\mathcal{A}\alpha\right]$ is the standard correction which compensates for the lack of translation invariance in the single particle shell model [18], where $\mathcal{A}$ is the nuclear mass number and α the harmonic oscillator parameter. To evaluate Eq. (5) we have made a factorization approximation in which the elementary amplitude is evaluated at some effective Fermi momentum $< \vec{p} >= -(2\vec{k_\pi} + (\mathcal{A}-1)\vec{Q})/2\mathcal{A}$.

In Fig. 5 we show cross sections for the exclusive reaction $^{13}C(\pi^+,\eta)^{13}N$ leading to the ground state of ^{13}N. The cross sections in μb/sr are shown vs. η CM angle in degrees. In Fig. 5, the dashed curve is for incident pion lab momentum 680 MeV/c, the solid curve for 650 MeV/c, and the long–dashed curve for 620 MeV/c. The cross sections show a typical diffractive behavior, with a first minimum which moves to lower angles as the incident pion momentum increases. The overall cross section falls off rapidly with increasing momentum transfer [scattering angle], reflecting the falloff in the single–particle momentum density. We predict a maximum cross section of roughly 100 μb/sr in the forward direction.

In Fig. 6 we show the dependence of the exclusive cross sections on various parameters in our calculation. The dashed curve represents our full results including both pion single charge–exchange and isobar self–energy corrections. The solid curve represents the same calculation with pion charge–exchange excluded. The long–dashed curve includes elastic and charge–exchange pion scattering but without N^* self–energy

corrections, e.g. assuming only the free width for the isobar. From Fig. 6 we see that inclusion of pion single charge exchange tends to decrease the cross sections by about 25%. The isobar self–energy tends to be a more important ingredient in this calculation; assuming the free width of the isobar increases the cross section by a factor of roughly 2.5. As we mentioned in the inclusive reaction Section, the dominant process which increases the width of the isobar in the nuclear medium is the two–pion decay of the isobar, shown diagrammatically in Fig. 1(c). The total exclusive cross sections for this reaction are listed in Table I, for the different incident pion momenta considered here. The total cross sections increase with increasing incident pion momentum. Over this range the elementary cross section $\pi + N \rightarrow \eta + N$ varies slowly with energy, so the increasing total cross section is due to an increase in phase space for the reaction.

4. Conclusions

We have previously shown that the inclusive (π, η) reaction on nuclei can be described quite well in a DWIA calculation. In this work we have extended this calculation to predict also the differential and total cross sections for exclusive nuclear (π, η) reactions. We used the same DWIA formalism to predict cross sections for the reaction $^{13}C(\pi^+, \eta)^{13}N_{gs}$. This model assumes that the incident pion excites a nucleon to an intermediate N(1535) resonance, which then undergoes decay to $\eta + N$. A Glauber calculation was used to approximate the initial state interactions of the incident pion, and we made estimates of the imaginary part of the N^* self energy.

We obtained quantitative agreement with experimental data for the inclusive reaction $^{12}C(\pi^+, \eta)^{12}N$. We have made qualitative predictions for the exclusive reaction $^{13}C(\pi^+, \eta)^{13}N_{gs}$, for which reaction no measurements exist at present. of the exclusive (π, η) cross sections, There were a number of areas in which these calculations could be improved. First, we assumed that η formation occurred solely through the $N(1535)$ resonance. Near threshold the reaction $\pi + N \rightarrow \eta + N$ is dominated to a large extent by the S_{11} resonance at 1535 MeV; however, higher resonances should contribute substantially to this process at higher pion incident energies. For example, phenomenological fits to η production cross sections induced by pions assumed

contributions from three different nucleon resonances [19]. Additional precise data on the basic production reaction $\pi + N \rightarrow \eta + N$ over a wide energy band would still be quite useful.

We have estimated the imaginary part of the N^* self energy, i.e. the change in width of the isobar in the nucleus. To be consistent we should also calculate the nuclear shift in the real part of the self energy. In addition, calculation of the N^* self energies was also carried out using a Fermi Gas approximation for the relevant nuclear densities; more realistic results would be obtained using a local density approximation and realistic nuclear wavefunctions.

The reactions we are studying involve momentum transfers of at least 1.5 fm^{-1}; we have used a rather simple model for the nuclear transition density matrix. These should be replaced with more realistic nuclear densities. When these improvements have been carried out one can calculate η production cross sections for a wide variety of energies and nuclear targets. Finally, for simplicity we have made a factorization approximation for the exclusive reaction calculation. This involved evaluating the transition operator at some averaged value of the momentum rather than integrating over it. The validity of this approximation for such calculations needs to be checked.

In the future there are several possible applications of these DWIA calculations. First, for certain exclusive reactions one may be able to isolate different parts of the $\pi + N \rightarrow \eta + N$ transition amplitude; for example, one may be able to isolate the spin-dependent part of this transition amplitude for certain nuclear transitions. One can also use this same formalism to examine inelastic (π, η) transitions leading to excited states of the residual nucleus. This model could also be extended to treat quasifree reactions involving three bodies [η, nucleon and residual nucleus] in the final state. We might expect these states to be the dominant final states resulting from production of an N^* in the nuclear medium. Finally, a similar model could be constructed to examine proton–induced η production, e.g. (p, η) inclusive reactions, or (p, N^*) reactions [20].

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Table 1

k_π (MeV/c)	σ_{tot} (μb)
620	20.3
650	28.0
680	34.1

Total Cross Sections for $^{13}C(\pi^+,\eta)^{13}N$ Reaction

Figure Captions

1. Diagrammatic representation of self–energy contributions from various processes in the nucleus. (a) Pauli blocking contribution to N^* width; (b) one–meson (π or η) decay of N^* in nucleus; (c) two–pion decay of N^*.

2. (π^+, η) inclusive cross sections on ^{12}C target at $p_\pi^{lab} = 680$ MeV/c. Inclusive cross sections, in μb/MeV-sr, vs. η kinetic energy in MeV. The data are from Ref. [7]. Solid curve: full results of Ref. [9]. Dashed curve: calculations using free N*(1535) width.

3. (π^+, η) inclusive cross sections on ^{12}C target. Notation is that of Fig. 2. Histogram: INC calculations of Ref. [10]; solid curve: DWIA calculation of Ref. [6]. Data are from Ref. [7].

4. (π^+, η) inclusive cross sections on ^{12}C target at $p_\pi^{lab} = 620$ MeV/c. Inclusive cross sections, in μb/MeV-sr, vs. η kinetic energy in MeV. This calculation includes only on–shell pion rescattering effects.

5. $^{13}C(\pi^+, \eta)^{13}N$ cross sections, in μb/sr, vs. η CM angle in degrees. Dashed curve: at $p_\pi^{lab} = 680$ MeV/c; solid curve: $p_\pi^{lab} = 650$ MeV/c; long–dashed curve: $p_\pi^{lab} = 620$ MeV/c.

6. $^{13}C(\pi^+, \eta)^{13}N$ cross sections, in μb/sr, vs. η CM angle in degrees. Dashed curve: full calculation including pion single charge exchange; solid curve: calculation without pion charge exchange; long–dashed curve: includes both elastic and charge exchange pion scattering but without $N(1535)$ self–energy corrections.

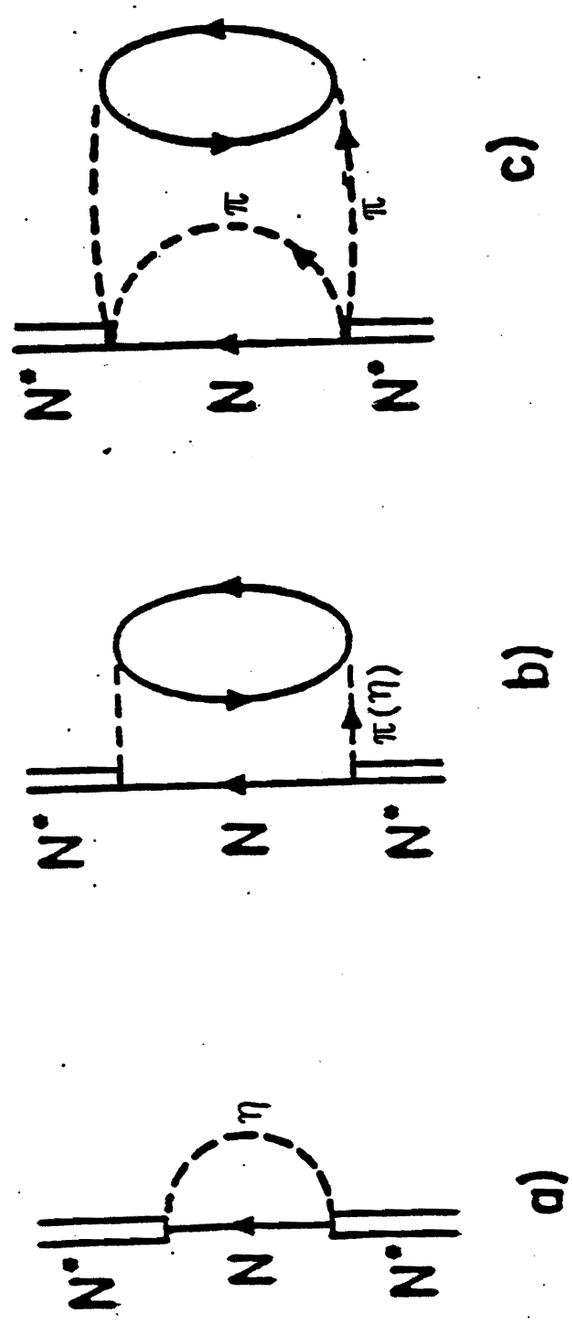

Fig. 1

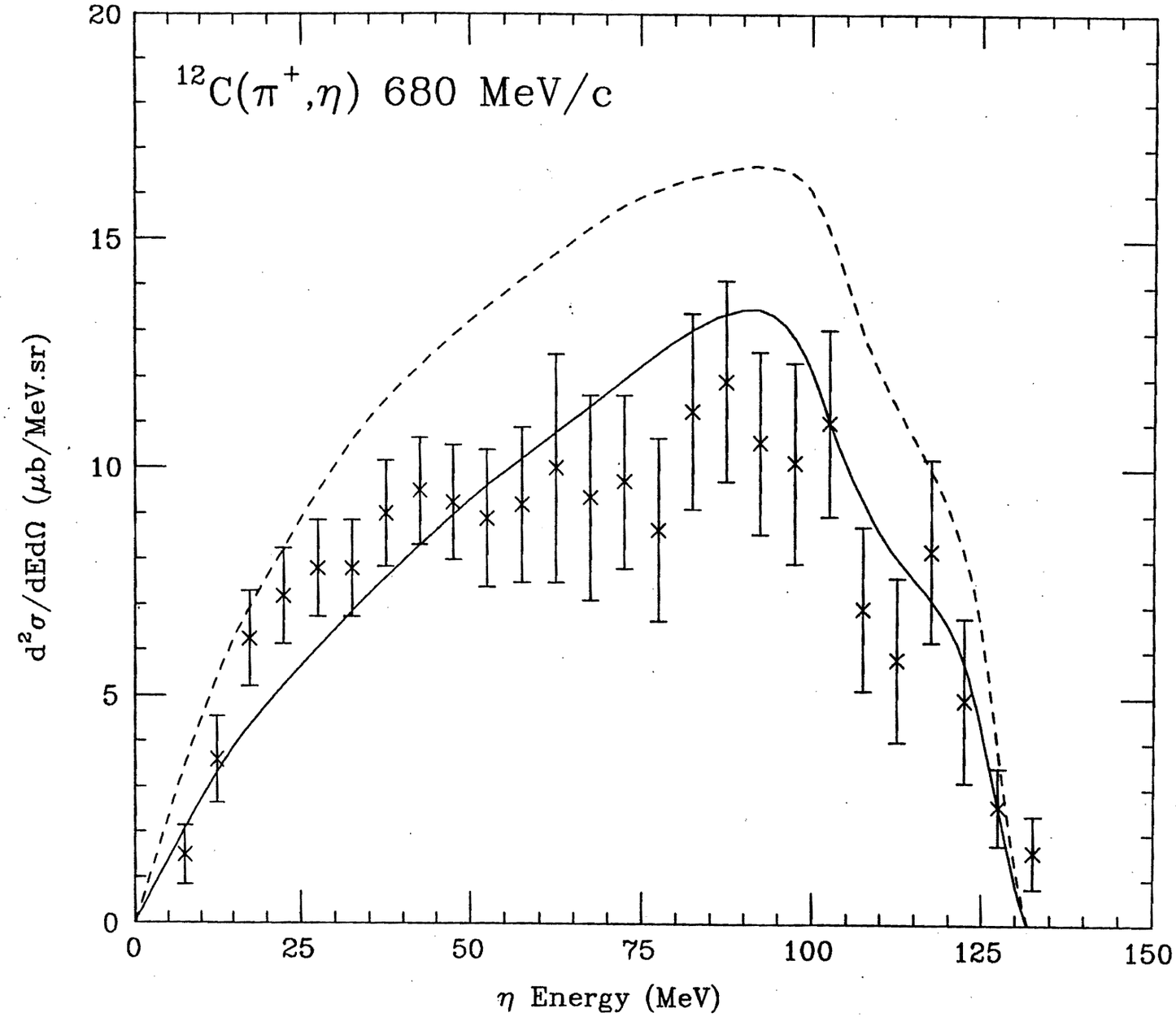
$^{12}C(\pi^+,\eta)$ 680 MeV/c
$d^2\sigma/dEd\Omega$ (μb/MeV.sr)
η Energy (MeV)
0
5
10
15
20
0
25
50
75
100
125
150

Fig. 2

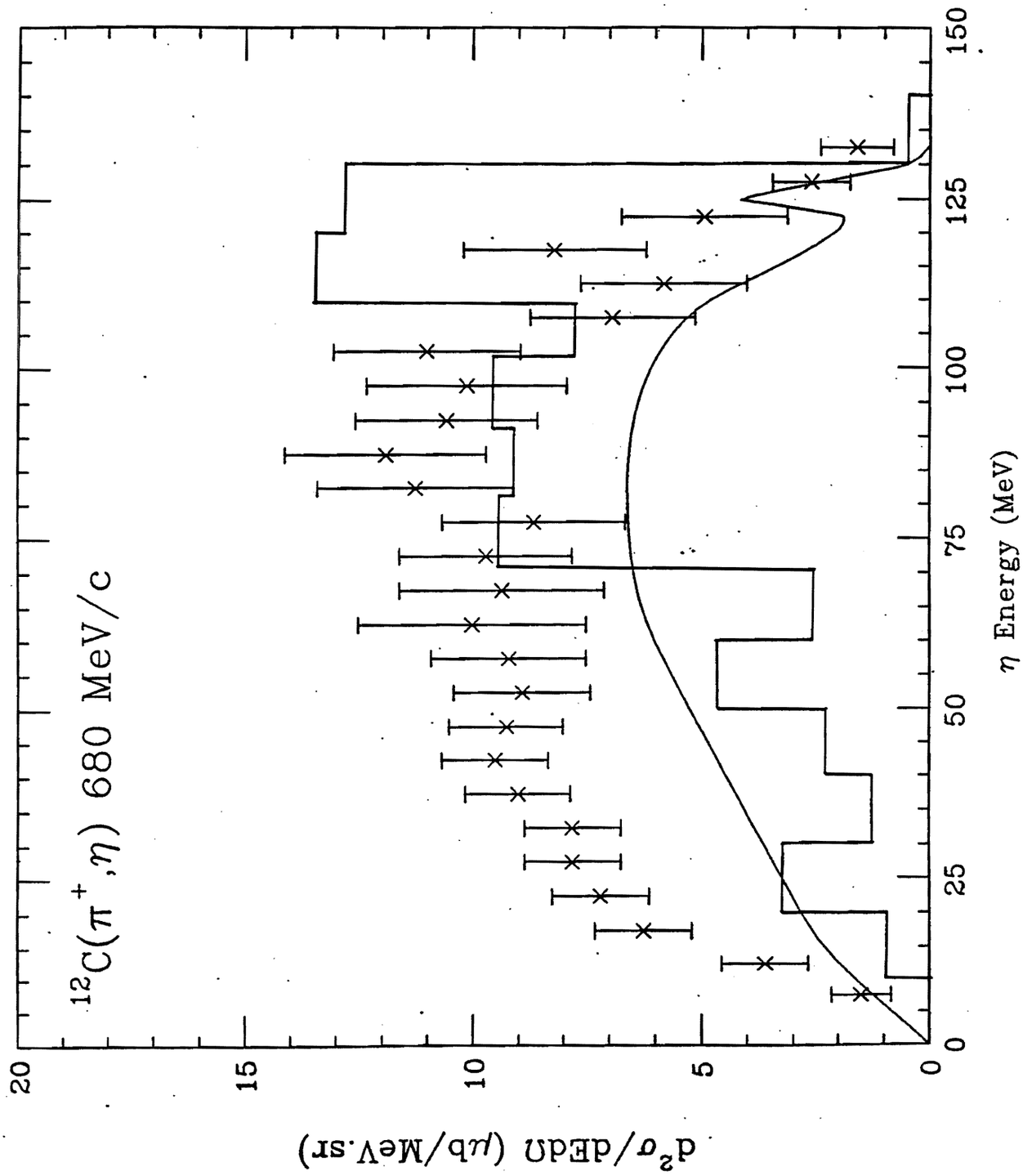
$^{12}C(\pi^+,\eta)$ 680 MeV/c
$d^2\sigma/dEd\Omega$ (μb/MeV.sr)
η Energy (MeV)
0
5
10
15
20
0
25
50
75
100
125
150

Fig. 3

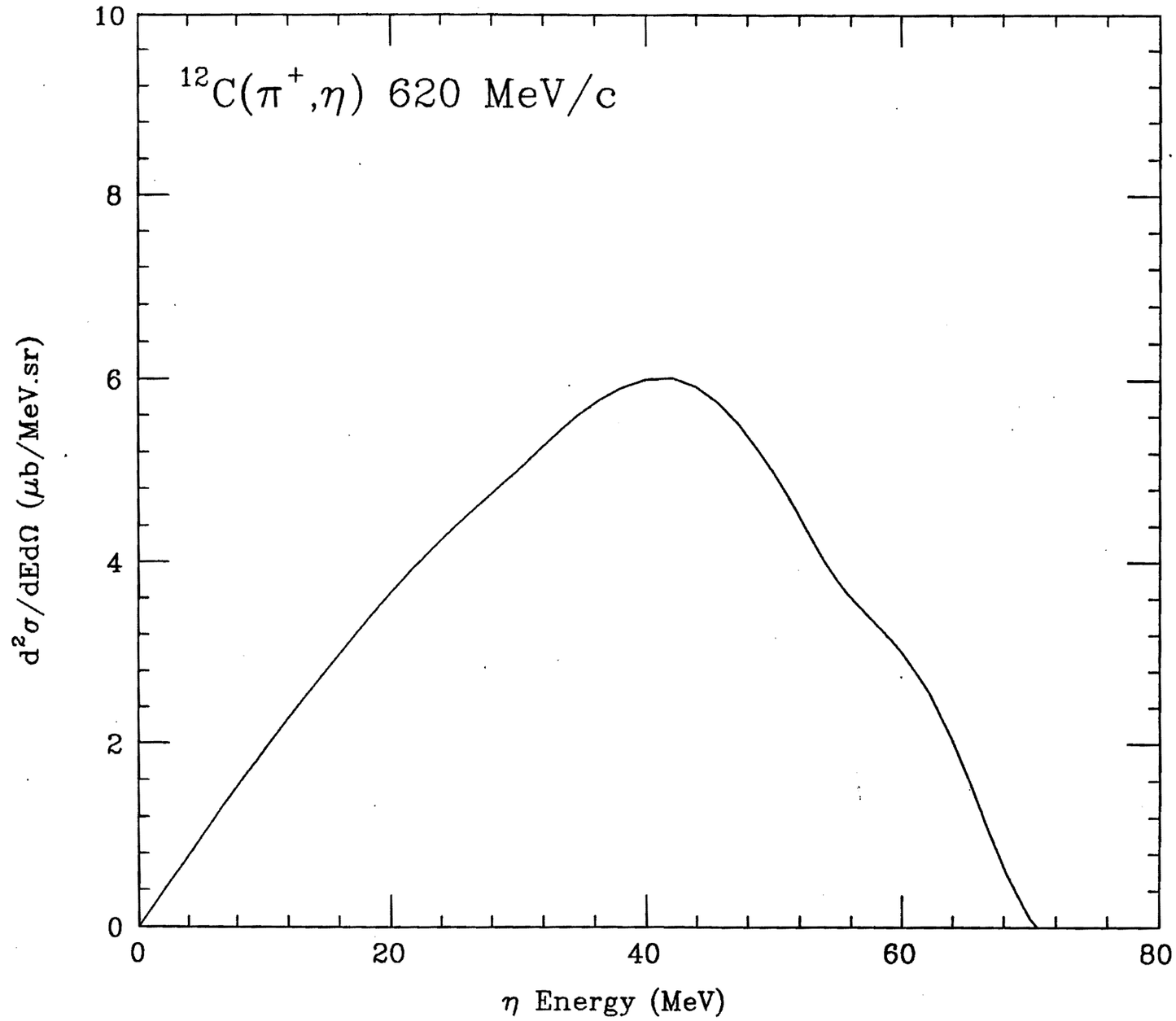
$^{12}C(\pi^+,\eta)$ 620 MeV/c
$d^2\sigma/dEd\Omega$ (μb/MeV.sr)
η Energy (MeV)
0
2
4
6
8
10
0
20
40
60
80

Fig. 4

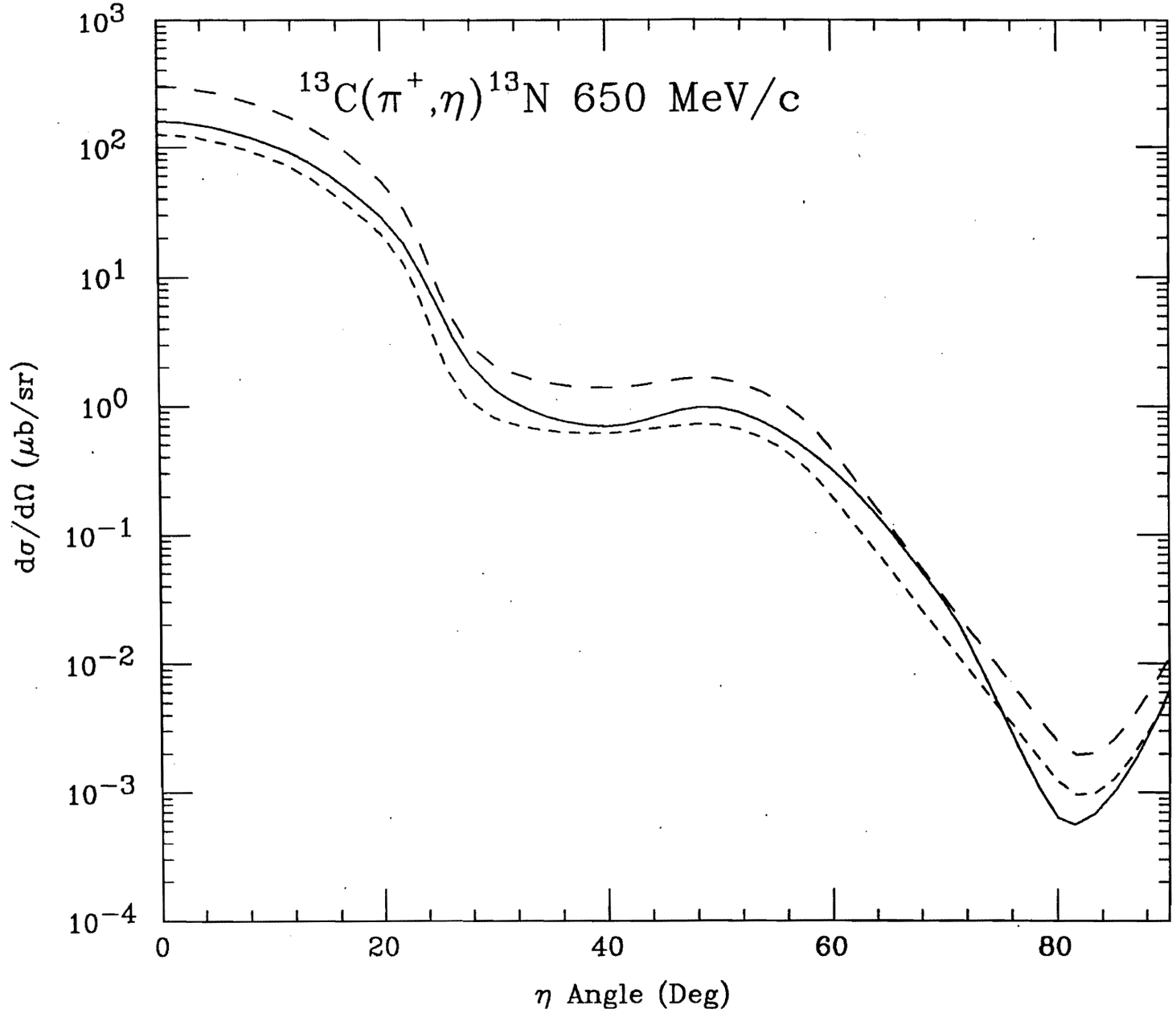
$^{13}C(\pi^+,\eta)^{13}N$ 650 MeV/c
$d\sigma/d\Omega$ (μb/sr)
10^3
10^2
10^1
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
0
20
40
60
80
η Angle (Deg)

Fig. 5

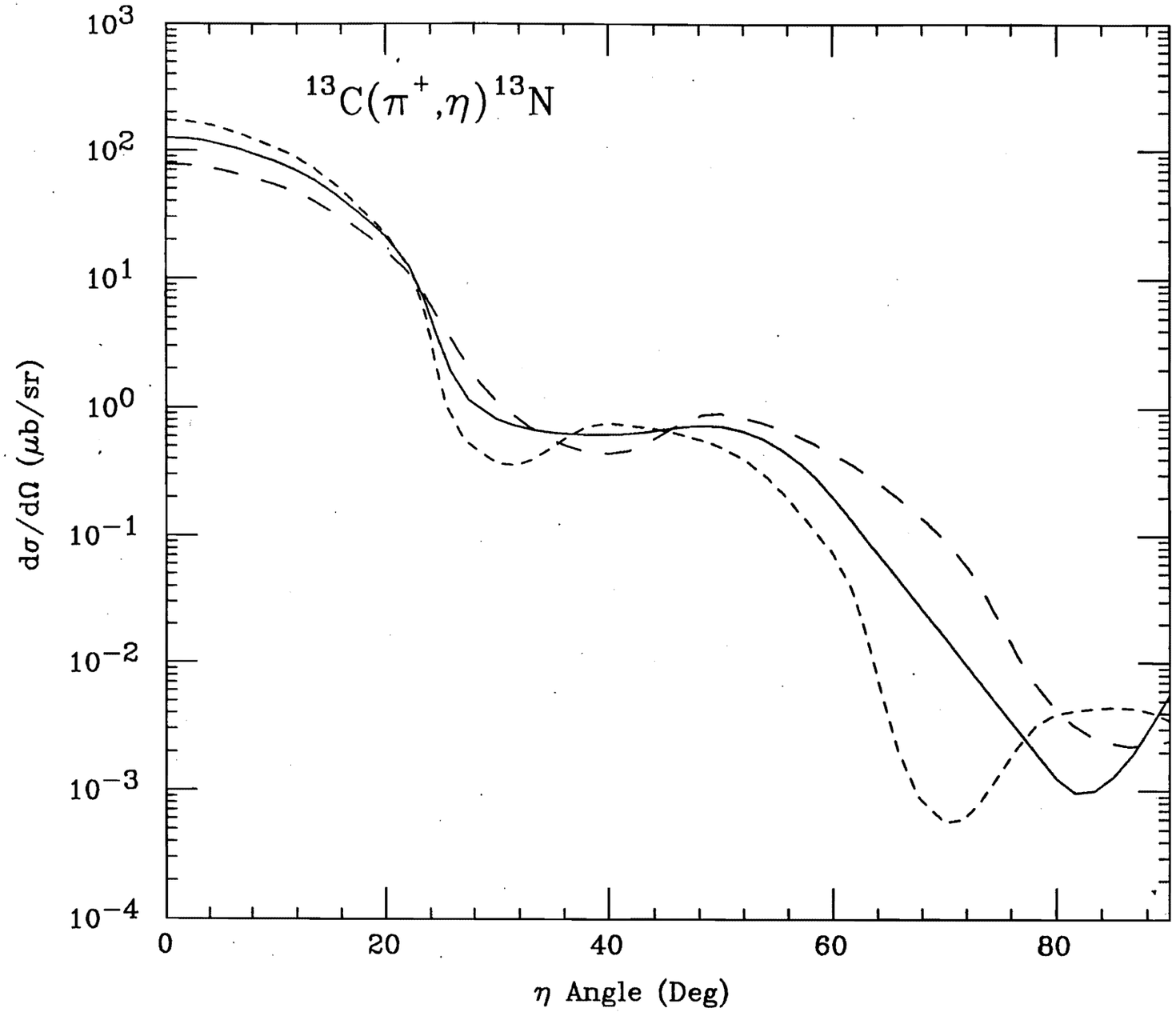
$^{13}C(\pi^+,\eta)^{13}N$
10^3
10^2
10^1
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
$d\sigma/d\Omega$ (μb/sr)
0
20
40
60
80
η Angle (Deg)

Fig. 6