REMARKS ON SYMMETRICAL TENSOR FIELDS

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For a spin-2, free quantum field there exists a Lagrange formulation leading to the Klein-Gordon equation, and the subsidiary conditions are added separately. This reproduces the usual interpretation of the field by spin-2 particles. If subsidiary conditions are not introduced, the symmetrical tensor field of rank two is decomposed into its components of spin-2,1,0 and only the spin-2 component defines a physical theory. Various Lagrange formulations become inequivalent when a tensor interaction is introduced because the current can fix the trace and/or the divergence of the field.

1. Introduction

Proca's formulation [1] of a free, massive, spin-1 field builds in the subsidiary condition into the Lagrangian. The same sensible theory results in Bogoliubov's treatement [2] where the divergence condition is introduced separately, after independent field variations. The equivalence remains in the interacting case, too, for a divergenceless current and field. If only the current is conserved, the field equations are still the same, but while in Proca's treatement the divergence of the field is identically vanishing, in Bogoliubov's treatement it is a nontrivial solution of the free Klein-Gordon equation (a spin-0 field is present). Then, the two treatements are formally different, but the physics is still the same [3]. If the current is nonconserved, the two field equations are no longer the same and in Proca's case the divergence of the field is proportional to that of the current. In the latter case an equivalent vector field has been defined [4,5] which made it possible to have good asymptotic physics of spin-one particles.

The aim of the present paper is to investigate similar problems for the case of the more complicated symmetrical tensor field of rank two.

In Section 2 it is shown that a Bogoliubov's type Lagrangian together with subsidiary conditions defines a quantum field theory of massive, spin-2, free particles.

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Furthermore, in the lack of subsidiary conditions, the spin-2 part of the tensor field can be constructed leading to a physical theory, but the spin-1, 0 parts are nonphysical.

Different interacting theories can be introduced (Section 3) depending on the starting free Lagrangian. In general, these are inequivalent since the tensor current influences the trace and/or the divergence of the field differently.

2. Free tensor fields of rank two

The second quantization of an arbitrary, integer spin field has been described by Fierz [6]. He has postulated the field equations and written down the energy-momentum tensor and four-current showing the quantum interpretation of his scheme. Here we will verify for a symmetrical, complex tensor field $\phi_{\mu\nu}$ (+---) that the same interpretation follows from the Lagrangian

$$L = \partial_\mu \phi^*_{\nu\rho} \partial^\mu \phi^{\nu\rho} - m^2 \phi^*_{\nu\rho} \phi^{\nu\rho}, \tag{1}$$

if the subsidiary conditions

$$\partial^\mu \phi_{\mu\nu} = 0, \quad \phi^\mu_\mu = 0 \tag{2}$$

are added after variations.

From (1) $\phi_{\nu\rho}$ satisfies the Klein-Gordon equation,

$$\left(\partial_\mu \partial^\mu + m^2\right)\phi_{\nu\rho} = 0, \tag{3}$$

as well as the conserved energy-momentum tensor and four-current are

$$T^{\mu\nu} = \partial^\mu \phi^*_{\lambda\rho} \partial^\nu \phi^{\lambda\rho} + \partial^\mu \phi^{\lambda\rho} \partial^\nu \phi^*_{\lambda\rho} - g^{\mu\nu} L, \tag{4}$$

$$j^\mu = -i\left(\phi_{\nu\rho} \partial^\mu \phi^{\nu\rho *} - \phi^*_{\nu\rho} \partial^\mu \phi^{\nu\rho}\right). \tag{5}$$

The spin momentum tensor is

$$S^\mu_{\nu\rho} = 2\partial^\mu \phi^{\lambda *}_\nu \cdot \phi_{\rho\lambda} - 2\partial^\mu \phi^{\lambda *}_\rho \cdot \phi_{\nu\lambda} + h.c.. \tag{6}$$

Solving (3) we have

$$\phi_{\mu\nu}(x) = \int \frac{d^3k}{\sqrt{(2\pi)^3 2k_0}} \left(A^+_{\mu\nu}(k) e^{ikx} + A^-_{\mu\nu}(k) e^{-ikx}\right), \quad k_0 = \sqrt{\underline{k}^2 + m^2}, \tag{7}$$

and similarly for $\phi^*_{\mu\nu}(x) \to A^{*\pm}_{\mu\nu}(k)$, where

$$A^{*\pm}_{\mu\nu}(k) = A^{\mp *}_{\mu\nu}(k). \tag{8}$$

This determines the energy P^0, the charge Q and the x component of the spin as follows

$$P^0 = \int d^3k k_0 \left(A^{*+}_{\mu\nu}(k) A^{\mu\nu -}(k) + A^{*-}_{\mu\nu}(k) A^{\mu\nu +}(k)\right), \tag{9}$$

$$Q = \frac{1}{2} \int d^3k \left(A^{*+}_{\mu\nu}(k) A^{\mu\nu -}(k) - A^{*-}_{\mu\nu}(k) A^{\mu\nu +}(k) - A^+_{\mu\nu}(k) A^{\mu\nu -}(k) + A^-_{\mu\nu}(k) A^{\mu\nu +}(k)\right), \tag{10}$$

$$S_x = -2i \int d^3k \left(A^{\lambda *+}_2(k) A^-_{3\lambda}(k) - A^{\lambda *+}_3(k) A^-_{2\lambda}(k) + A^{\lambda *-}_3(k) A^+_{2\lambda}(k) - A^{\lambda *-}_2(k) A^+_{3\lambda}(k)\right). \tag{11}$$

Now, we take into account the subsidiary conditions (2) assuring that $\phi_{\mu\nu}$ transfers only the single spin equal to two. In the rest frame the eigenfunctions of the x component of the spin can be written as

$$A^0 = \frac{1}{\sqrt{6}} \begin{pmatrix} 0 & & & \\ & 2 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}, \quad A^{-1} = \frac{1}{2} \begin{pmatrix} 0 & & & \\ & 0 & 1 & -i \\ & 1 & 0 & 0 \\ & -i & 0 & 0 \end{pmatrix}, \quad A^{+1} = \frac{1}{2} \begin{pmatrix} 0 & & & \\ & 0 & 1 & i \\ & 1 & 0 & 0 \\ & i & 0 & 0 \end{pmatrix},$$

$$A^{+2} = \frac{1}{2} \begin{pmatrix} 0 & & & \\ & 0 & & \\ & & -i & 1 \\ & & 1 & i \end{pmatrix}, \quad A^{-2} = \begin{pmatrix} 0 & & & \\ & 0 & & \\ & & i & 1 \\ & & 1 & -i \end{pmatrix}. \tag{12}$$

These are transformed by a Lorentz transformation into the general polarization tensors $A^i_{\mu\nu}(k)$ satisfying

$$A^i_{\mu\nu}(k) A^{j\mu\nu *}(k) = \delta_{ij},$$

$$\sum_{l=-2}^{2} A^l_{\mu\nu}(k) A^{l*}_{\rho\lambda}(k) = \frac{1}{2}\left(R'_{\mu\rho} R'_{\nu\lambda} + R'_{\mu\lambda} R'_{\nu\rho} - \frac{2}{3} R'_{\mu\nu} R'_{\rho\lambda}\right) \tag{13}$$

$$R'_{\mu\nu} = g_{\mu\nu} - \frac{k_\mu k_\nu}{m^2}.$$

The tensor field $\phi_{\mu\nu}(x)$ of spin-two is given by (7) where $A^{\pm}_{\mu\nu}(k)$ is expanded in the basis $A^{j}_{\mu\nu}$:

$$A^{\pm}_{\mu\nu}(k)=\sum_{j=-2}^{2}c^{j\pm}(k)A^{j}_{\mu\nu}(k). \tag{14}$$

For the field $\phi_{\mu\nu}(x)$ we require [6] the quantization

$$\left[\phi_{\mu\nu}(x),\phi_{\rho\lambda}(y)\right]=0,$$

$$\left[\phi^{*}_{\mu\nu}(x),\phi_{\rho\lambda}(y)\right]=\frac{1}{2i}(R_{\mu\rho}R_{\nu\lambda}+R_{\mu\lambda}R_{\nu\rho}-\frac{2}{3}R_{\mu\nu}R_{\rho\lambda})D(x-y) \tag{15}$$

with

$$R_{\mu\nu}=g_{\mu\nu}+\frac{\partial_{\mu}\partial_{\nu}}{m^{2}} \tag{16}$$

and $D(x)$ is the commutation function of a scalar field. From (15) the only nonvanishing commutators of the operators $c^{(*)\pm}_{j}$ are

$$\left[c^{*\mp}_{j}(p),c^{\pm}_{k}(q)\right]=\pm\delta_{jk}\delta^{(3)}(p-q). \tag{17}$$

P^{0}, Q and S_{z} can be expressed by the operators $c^{(*)\pm}_{j}$ and after normal ordering the result is

$$P^{0}=\int_{-\infty}^{\infty}d^{3}kk_{0}\sum_{j=-2}^{2}\left(c^{*+}_{j}(k)c^{-}_{j}(k)+c^{+}_{j}(k)c^{*-}_{j}(k)\right), \tag{18}$$

$$Q=\int_{-\infty}^{\infty}d^{3}k\sum_{j=-2}^{2}\left(c^{*+}_{j}(k)c^{-}_{j}(k)-c^{+}_{j}(k)c^{*-}_{j}(k)\right), \tag{19}$$

$$S_{z}=\int_{-\infty}^{\infty}d^{3}k\sum_{j=-2}^{2}\left(jc^{*+}_{j}(k)c^{-}_{j}(k)+jc^{+}_{j}(k)c^{*-}_{j}(k)\right). \tag{20}$$

Here $c^{*+}_{j}(k)[c^{+}_{j}(k)]$ creates a positive (negative) charged free particle of three-momentum $\underline{k}$, rest mass m and j is the x componenet of the spin, j=2, ...,-2. c^{-}_{j} and c^{*-}_{j} are the corresponding annihilation operators. This completes the particle interpretation of the field $\phi_{\mu\nu}$.

Next, we consider the Lagrangian (1) under more general conditions, namely without the subsidiary conditons (2). In this case (3)-(11) are still valid but the energy is no

longer positive. This has been remarked in ref. [7] pointing out that $\partial^{\mu}\phi_{\mu\nu}\neq 0$ creates particles of negative energy. In what follows we will show how one can define a physical quantum field theory even in this case.

Decompose the symmetrical tensor $\phi_{\rho\lambda}$ into its definite spin parts by the projections

$$P^{(2)\mu\nu}_{\rho\lambda}=\left(\delta_{\rho\mu}-\frac{k_{\rho}k^{\mu}}{m^{2}}\right)\left(\delta_{\lambda\nu}-\frac{k_{\lambda}k_{\nu}}{m^{2}}\right)-\frac{1}{3}\left(g_{\rho\lambda}-\frac{k_{\rho}k_{\lambda}}{m^{2}}\right)\left(g^{\mu\nu}-\frac{k^{\mu}k^{\nu}}{m^{2}}\right),$$

$$P^{(1)\mu\nu}_{\rho\lambda}=\frac{k_{\rho}k^{\mu}}{m^{2}}\left(\delta_{\lambda\nu}-\frac{k_{\lambda}k^{\nu}}{m^{2}}\right)+\frac{k_{\lambda}k^{\nu}}{m^{2}}\left(\delta_{\rho\mu}-\frac{k_{\rho}k^{\mu}}{m^{2}}\right),$$

$$P^{(d)\mu\nu}_{\rho\lambda}=-\frac{1}{3}\left(g_{\rho\lambda}-4\frac{k_{\rho}k_{\lambda}}{m^{2}}\right)\left(\frac{k^{\mu}k^{\nu}}{m^{2}}\right), \tag{21}$$

$$P^{(sp)\mu\nu}_{\rho\lambda}=\frac{1}{3}\left(g_{\rho\lambda}-\frac{k_{\rho}k_{\lambda}}{m^{2}}\right)g^{\mu\nu},$$

where k^{μ} is on the mass shell. In x-space $P^{(2)\mu\nu}_{\rho\lambda}\phi_{\mu\nu}(x)$ is a spin-2 tensor field with a vanishing trace and divergence. Similarly, $P^{(1)}$ singles out the pure spin-1 component, two spin-0 components are coming from the traceless $P^{(d)}$ incorporating $\partial_{\mu}\partial_{\nu}\phi^{\mu\nu}$ and $P^{(sp)}$ reproducing the trace of $\phi_{\mu\nu}$. The original 10 degrees of freedom are reproduced as 5+3+1+1.

In order to decompose P^{0} and Q into contributions of definite spin we need the commutator of the general field $\phi_{\mu\nu}$. This is generalized from the quantization of the scalar field adding symmetry,

$$\left[\phi^{*}_{\mu\nu}(x),\phi_{\rho\lambda}(y)\right]=\frac{1}{2i}\left(g_{\mu\rho}g_{\nu\lambda}+g_{\mu\lambda}g_{\nu\rho}\right)D(x-y), \tag{22}$$

$$\left[\phi_{\mu\nu}(x),\phi_{\rho\lambda}(y)\right]=0.$$

In momentum-space we get for the nonvanishing commutators

$$\left[A^{*\mp}_{\mu\nu}(p),A^{\pm}_{\rho\lambda}(q)\right]=\pm\frac{1}{2}\left(g_{\mu\rho}g_{\nu\lambda}+g_{\mu\lambda}g_{\nu\rho}\right)\delta^{(3)}(p-q). \tag{23}$$

It is easy to show that (22) leads to the correct quantization (15) for the spin-2 part. This results in that $P^{(2)\mu\nu}_{\rho\sigma}\phi_{\mu\nu}$ defines a positive contribution to the energy P^{0}, and (18), (19), (20) hold for the spin-2 part.

The energy associated with the spin-1 component is negative because in the rest frame only the (0i) and (i0) fields are present and their commutators have an extra minus sign. Indeed, the spin-1 polarization tensors in the rest frame can be written as

$$A^1 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 & & \\ 1 & 0 & & \\ & & 0 & \\ & & & 0 \end{pmatrix}, \quad A^2 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & & \\ 1 & & 0 & \\ 0 & & & 0 \end{pmatrix}, \quad A^3 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & & \\ 0 & & 0 & \\ 1 & & & 0 \end{pmatrix}. \tag{24}$$

The completeness relation is

$$m^2 \sum_{l=1,2,3} A^l_{\mu\nu}(k) A^{l*}_{\rho\lambda}(k) =$$
$$= -k_\rho k_\mu \left(g_{\nu\lambda} - \frac{k_\nu k_\lambda}{m^2} \right) - k_\rho k_\nu \left(g_{\mu\lambda} - \frac{k_\mu k_\lambda}{m^2} \right) - k_\lambda k_\mu \left(g_{\nu\rho} - \frac{k_\nu k_\rho}{m^2} \right) - k_\lambda k_\nu \left(g_{\mu\rho} - \frac{k_\mu k_\rho}{m^2} \right), \tag{25}$$

and the commutator of the spin-1 field turns out to be (-1) x the completeness relation. Therefore, the creation and annihilation operatort have an opposite commutator, and the energy cannot be positive,

$$P^0_{(1)} = \int d^3k k_0 \sum_{l=1,2,3} \left(c_l^{*+}(k) c_l^-(k) + c_l^+(k) c_l^{*-}(k) \right), \tag{26}$$

$$P^0_{(1)} \le 0.$$

Finally, we will argue that the energy $P^0_{(0)}$ of the spin-0 part is positive. $P^0_{(0)}$ has the form

$$P^0_{(0)} = \int d^3k k_0 \frac{4}{3} \left(c_{sp}^{*+}(k) c_{sp}^-(k) + c_{sp}^+(k) c_{sp}^{*-}(k) + c_d^{*+}(k) c_d^-(k) + c_d^+(k) c_d^{*-}(k) \right), \tag{27}$$

where

$$c_{sp}^{(*)\pm}(k) = \frac{1}{2} A^{(*)\pm}_{\mu\nu}(k) g^{\mu\nu},$$

$$c_d^{(*)\pm}(k) = A^{(*)\pm}_{\mu\nu}(k) \frac{k^\mu k^\nu}{m^2}, \tag{28}$$

$$[c_x^{(*)\mp}(p), c_x^{(*)\pm}(q)] = \pm\delta^{(3)}(p-q), x = sp, d.$$

In such a way $P^0_{(0)} \ge 0$, but it contains the factor of 4/3. The same factor emerges also in the charge operator. Since (21) defines an orthogonal decomposition, in the energy P^0 no interference appears and

$$P^0 = \sum_{j=2,1,0} P^0_{(j)}, \tag{29}$$

where j denotes the spin-j projection. P^0 is nonphysical, but the subspace of spin-2 states is physical describing m mass and integer charges. Absorbing the factor 4/3 in (22), the spin-2 sector contains the fractional mass and charges. This phenomemon is present even if $\partial_\mu \phi^{\mu\nu} = 0$, but $\phi^\mu_\mu \ne 0$.

3. Interacting tensor fields

In general, the introduction of the interaction of the symmetrical tensor field $\phi_{\mu\nu}$ results in different theories depending on the starting free Lagrangian. We illustrate this in three examples.

First, we take the Lagrangian (1) adding other fields and interactions. Then, one gets for $\phi_{\nu\rho}(x)$:

$$\left(\partial_\mu \partial^\mu + m^2\right) \phi_{\nu\rho} = j_{\nu\rho}, \tag{30}$$

where the symmetrical tensor source is a functional of the fields. (As a simple example, we add $j^*_{\nu\rho}\phi^{\nu\rho} + h.c.$ to (1) and $j_{\nu\rho}$ does not contain $\phi_{\mu\nu}$.) From (30) we obtain

$$\left(\partial_\mu \partial^\mu + m^2\right) \phi^\nu_\nu = j^\nu_\nu, \tag{31}$$

$$\left(\partial_\mu \partial^\mu + m^2\right) \partial^\nu \phi_{\nu\rho} = \partial^\nu j_{\nu\rho}.$$

If one can choose $\phi^\nu_\nu = 0 \left(\partial^\nu \phi_{\nu\rho} = 0\right)$, then $j^\nu_\nu \ne 0$ $\left(\partial^\nu j_{\nu\rho} \ne 0\right)$ cannot hold, and for $\phi^\nu_\nu \ne 0$, $j^\nu_\nu \ne 0$ etc., but for $j^\nu_\nu = 0$ ϕ^ν_ν may be a free field etc.

As usual, assuming for $t \to \pm\infty$ $j_{\nu\rho} \to 0$, we see that asymptotically $\phi_{\nu\rho}$ is a free field and once again the decomposition (29) emerges, that is, for instance, the spin-2 part of $\phi^{\nu\rho}_{in/out}(x)$ may be physical. The spin decomposition can be carried out in the field equation (30). In x-space the spin projections $P^{(j)\mu\nu}_{\rho\lambda}$ are obtained from (21) by replacing $k_\lambda \to -i\partial_\lambda$, $m^2 \to -\partial_\lambda \partial^\lambda$.
Thus,

$$\left(\partial_\mu\partial^\mu + m^2\right)P_{\rho\lambda}^{(l)\sigma\nu}\phi_{\sigma\nu} = P_{\rho\lambda}^{(l)\sigma\nu} j_{\sigma\nu} \tag{32}$$

$$l = 2, 1, d, sp.$$

The physical, spin-2 part has a complicated source which fulfils the subsidiary conditions, and by the asymptotic condition one can define an S-matrix in this subspace.

Now, let us turn to a more restricted system coming from the field equation

$$\left(\partial^\mu\partial_\mu + m^2\right)\phi_{\nu\rho} - \partial^\mu\partial_\nu\phi_{\mu\rho} = j_{\nu\rho}, \tag{33}$$

$\phi_{\nu\rho}$, $j_{\nu\rho}$ are symmetric tensors. $\phi_{\nu\rho}$ is similar to a Proca's field. Applying ∂^ν we have

$$m^2\partial^\nu\phi_{\nu\rho} = \partial^\nu j_{\nu\rho}, \tag{34}$$

and (33) reads as

$$\left(\partial^\mu\partial_\mu + m^2\right)\phi_{\nu\rho} = j_{\nu\rho} + \frac{1}{m^2}\partial^\mu\partial_\nu j_{\mu\rho}\,. \tag{35}$$

ϕ_ν^ν is determined by the trace of (35).

From tensor symmetry and (35) it follows

$$\partial^\mu\partial_\nu j_{\mu\rho} = \partial^\mu\partial_\rho j_{\mu\nu}, \tag{36}$$

as a consistency condition. Solutions of (36) are the sources

$$j_{\mu\nu}(x) = \partial_\mu\partial_\nu f_1(x) + g_{\mu\nu} f_2(x), \tag{37}$$

(with scalars $f_{1,2}$) and any divergenceless symmetric tensor field.

We learn that in the general case the systems (30)-(31) and (33) describe different $\phi'_{\nu\rho}\,s$. This is valid even if $\partial^\mu j_{\mu\nu} = 0$. However, for $\partial^\mu\phi_{\mu\nu} = 0$ the two formulations agree irrespective of the value of ϕ_ν^ν. Asymptotically $\phi_{\nu\rho}$ is conserved and only spin-0 quanta due to the trace should be removed in the second formulation. Motivated by (34), a divergenceless tensor field can be defined as

$$\phi'_{\mu\nu} = \phi_{\mu\nu} - \frac{1}{m^2} j_{\mu\nu}\,. \tag{38}$$

Asymptotically $\phi'_{\mu\nu}$ is equivalent to the field $\phi_{\mu\nu}$ [4,5], although its quantum field theory is more complicated than that of $\phi_{\mu\nu}$.

An even more restrictive formulation is due to Fierz [8] who has shown a Lagrangian containing also the subsidiary conditions:

$$L = \partial_\rho\phi^*_{\mu\nu}\partial^\rho\phi^{\mu\nu} - 2\partial_\rho\phi^*_{\mu\nu}\partial^\mu\phi^{\rho\nu} + \partial^\mu\phi^*_{\mu\nu}\partial^\nu\phi + \partial^\nu\phi^*\partial^\mu\phi_{\mu\nu} - \partial_\mu\phi^*\partial^\mu\phi - m^2\phi^*_{\mu\nu}\phi^{\mu\nu} + m^2\phi^*\phi, \tag{39}$$

$$\phi \equiv \phi^\rho_\rho.$$

Adding $j^*_{\nu\rho}\phi^{\nu\rho} + h.c.$ to (39) where $j_{\nu\rho}$ is independent of $\phi_{\mu\sigma}$ we get the field equation

$$\left(\partial_\rho\partial^\rho + m^2\right)\phi^{\mu\nu} = j^{\mu\nu} + g^{\mu\nu}\left(m^2\phi + \partial_\rho\partial^\rho\phi - \partial^\rho\partial^\sigma\phi_{\rho\sigma}\right) - \partial^\mu\partial^\nu\phi + \partial_\rho\left(\partial^\mu\phi^{\rho\nu} + \partial^\nu\phi^{\rho\mu}\right). \tag{40}$$

Combining the derivatives $\partial_\mu, \partial_\mu\partial_\nu$ and the trace of (40), ϕ and $\partial_\mu\phi^{\mu\nu}$ can be expressed by the source $j_{\mu\nu}$,

$$3m^2\phi = \frac{2}{m^2}\partial_\rho\partial_\sigma j^{\rho\sigma} - j^\rho_\rho,$$

$$m^2\partial_\mu\phi^{\mu\nu} = \partial_\mu j^{\mu\nu} + m^2\partial^\nu\phi. \tag{41}$$

The right hand side of (40) is determined by $j_{\rho\sigma}$, substituting (41) into (40). If the source is asymptotically vanishing, $\phi^{\mu\nu} \to$ *free field*, $\phi \to 0, \partial_\mu\phi^{\mu\nu} \to 0$, that is pure spin-2 asymptotic states are present with a physical four-momentum operator, and an S-matrix exists. In the general case, however, the interpolating field $\phi^{\mu\nu}$ builds in spin-1,0 components due to (41), except $j^\rho_\rho = \partial_\rho j^{\rho\sigma} = 0$, when $\phi = 0$, $\partial_\mu\phi^{\mu\nu} = 0$. In general, the formulations based on (33) and (40) are different. For a conserved and traceless source in Proca`s formulation ϕ and $\partial_\mu\phi^{\mu\nu}$ are free fields.

In conclusion, by making use of a Bogoliubov's type Lagrangian (1), there exists a quantum field theory of a symmetrical tensor field of rank two and it has a physical interpretation provided subsidiary conditions are separately assumed. In the general case the tensor field is decomposed into its definite spin parts and a field theory can be defined in the spin-2 subspace.

We have discussed the problem of choosing the field equation in the interacting case where nonequivalent formulations are possible. Fierz's Lagrangian turns out to be very useful since the field equation fixes also the trace and divergence of the field in terms of the trace and divergence of the tensor current. Asymptotically only physical spin-2 particles are present.

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