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91–92

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MOSCOW 1992

УДК 530.145 М-16

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B.O.Kerbikov, A.E.Kudryavtsev - M., 1992 - 8p.

From a combined analysis of the data of Cyclotron Trap ($p\bar{p}$-atom) and OBELIX scattering experiments at LEAR we deduce the characteristics of $p\bar{p}$-interaction in the singlet spin state. The ratio of the triplet and singlet widths $x = \Gamma(^3S_1)/\Gamma(^1S_0) \simeq 0.42$. We discuss also on how the inequality $x < 1$ influences on the value of electromagnetic proton formfactor in the time-like region at threshold. The so deduced quantity is $|G(k \to 0)| = 0.39$ instead of the value 0.53 presented in the paper [8]. Thus the slope at threshold of the formfactor as function of k disappears.

Fig. - , ref. - 11

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Nowdays the $p\bar{p}$-interaction in the low-energy region is known mainly for laboratory momentum $p_{lab} \geq 190 MeV/c$, see, e.g., the papers [1-3]. The direct data just near the threshold were presented recently by OBELIX group [4]. In this paper the annihilation $p\bar{p}$-cross section was presented at the three momenta near $p_{lab} = 70 MeV/c$. The value $p = 70 MeV/c$ is low enough to use the S-wave scattering length approximation. One may compare scattering length deduced from this analysis with scattering length from atomic data on shift and width of the $p\bar{p}$-atom, see e.g. the review paper [5]. Unfortunately all the abovementioned results were peformed with nonpolarized tergets and beams. So no direct data are known on triplet and singlet scattering parameters separately [1].

The separate value of the triplet width was first reported by the Cyclotron Trap experiment at LEAR [7]. The extracted value of $\Gamma(^3S_1)$ is [7]

$$\Gamma(^3S_1) = (708 \pm 160 \pm 10) eV \tag{1}$$

For the spin averaged width the same experiment yielded a result [7]

$$\Gamma(S) = (1180 \pm 80 \pm 10) eV \tag{2}$$

If one would naively assume that $\Gamma(S)$ is averaged over singlet and triplet states with $1:3$ statistical distribution, one would get from (1) and (2) a result $\Gamma(^1S_0) \simeq 2.6 KeV$. So we get for the ratio $x = \Gamma(^3S_1)/\Gamma(^1S_0)$

$$x \simeq 0.27 \ . \tag{3}$$

However such an assumption is probably incorrect since the relative weights of the singlet and triplet components depend upon the width of the $2P$ hyperfine levels [6]. In particular the result (1) for $\Gamma(^3S_1)$ was obtained using the intensity ratio $1:2.2$. As argued by Prof. L.M.Simons (private communication) the extracted value of $\Gamma(^3S_1)$ is rather stable with respect to the intensity ratio, whereas $\Gamma(^1S_0)$ is much more sensitive to it. The first purpose of the present note is to provide a reliable estimate for the singlet width.

Use will be made of the two independent experimental results: (i) the value of $\Gamma(^3S_1)$ given by Eq.(1), and, (ii) the value of anihilation cross section at very low energy reported by OBELIX collaboration [4]

$$v\sigma_{ann} = 33.2 mb \ \text{ at } \ p_{lab} = 70 MeV/c. \tag{4}$$

In the Eq.(4) and in equations to follow the error bars are omitted since the experimental results are continuously refined. The decomposition of $v\sigma_{ann}$ into triplet and singlet contributions reads

$$v\sigma_{ann} = \frac{3}{4} v\sigma^t_{ann} + \frac{1}{4} v\sigma^s_{ann} \tag{5}$$

The value of the momentum $p_{lab} = 70 MeV/c$ is low enough in order to use the S-wave scattering length approximation. On the other hand at such low energies the Coulomb effects become important. Keeping this in mind we write the following equations for the annihilation cross sections

$$\sigma^j_{ann} = \sigma^j_{tot} - \sigma^j_{el} \ \ j = t, s,$$

$$v\sigma^j_{ann} = \frac{8\pi}{m} C^2(k) \{ Jm f^j - k C^2(k) \mid f^j \mid^2 \} \ , \tag{6}$$

$$C^2(k) = (2\pi/ka_B)/(1 - exp(-2\pi/ka_B)) \ , \tag{7}$$

$$f^j = a^j/(1 - ika^j) \ , \tag{8}$$

where m is a nucleon mass, $k = p_{lab}/2$ is the center of mass momentum, $C^2(k)$ is the Coulomb Gamow factor, a^j — the complex scattering length, $a^j \equiv a^j_1 + ia^j_2$. Keeping leading terms with respect to (ka^j) we get

$$v\sigma^j_{ann} = \frac{8\pi}{m} C^2(k) \frac{a^j_2 + k \mid a^j \mid^2 (C^2(k) - 1)}{(1 + ka^j_2)^2 + k^2 (a^j_1)^2} \simeq \frac{8\pi}{m} C^2(k) \frac{a^j_2}{1 + 2ka^j_2} \ , \tag{9}$$

where we have also used the fact that $C^2(k) \simeq 1.4$ at $p_{lab} = 70 MeV/c$. From eqs. (5) and (9) we obtain

$$\frac{m}{2\pi C^2(k)} (v\sigma_{ann}) = \frac{3a^t_2}{1 + p_{lab} a^t_2} + \frac{a^s_2}{1 + p_{lab} a^s_2} \ . \tag{10}$$

The imaginary part of the triplet scattering length a^t_2 can be expressed through the triplet width $\Gamma(^3S_1)$ (1) according to the well known equation [2]

$$\Gamma(^3S_1) = \frac{8}{m a_B^3} a^t_2 = \alpha^3 m^2 a^t_2 \ , \tag{11}$$

which yields

$$a^t_2 = 0.408 fm \tag{12}$$

Substituting this result into Eq. (10) we obtain

$$a_2^s = 0.96 fm \quad ^{3)} \tag{13}$$

Then applying relation (11) to the singlet state we get

$$\Gamma(^1S_0) = 1.70 keV \tag{14}$$

Thus the singlet state is broader than the triplet one and the ratio of their width is

$$x = \Gamma(^3S_1)/\Gamma(^1S_0) = 0.416 \simeq 0.42 \tag{15}$$

It is necessary to take in view also the following: the OBELIX result on $v\sigma_{ann}$ at $p_{lab} = 70 MeV/c$ is the only one available at such low energy. If one estimates the value of $v\sigma_{ann}$ at $p_{lab} = 70 MeV/c$ according to the fit of Heidelberg group [3], the result would be $v\sigma_{ann} = 47.7 mb$. Using this quantity instead of that given by the Eq.(4), we get $\Gamma(^1S_0) = 5.6 KeV$ and $x = 0.126$. Of course one should keep in mind that the Heidelberg fit describes the annihilation cross section in the momentum interval $180 MeV/c \leq p_{lab} \leq 600 MeV/c$ and does not pretend to the extrapolation toward threshold. However the above example clearly demonstrates a very tight correlation between the experimental quantities $(v\sigma_{ann})$ and $\Gamma(^1S_0)$. So the tendency $x < 1$ is confirmed in all our estimates. Notice that the result (15) was obtained using only the direct experimental information with no additional theoretical asumption upon the width of the 2p-levels or some extrapolation of the experimental data. So the result (15) is indeed more preferable.

The second part of our paper is devoted mainly to the consequences from inequality $x < 1$. Consider first of all the problem of electromagnetic formfactor in the time-like region at threshold point $(k \to 0)$. Reaction $p\bar{p} \to e^+e^-$ at rest proceeds from $^3S_1$ states of antiprotonic atoms. In the paper [9] the value of the formfactor $\mid G(k \to 0) \mid$ was extracted from this reaction,

$$\mid G(k \to 0) \mid = 0.53 \tag{16}$$

Notice that this value in the ref.[9] was obtained under assumption $\Gamma(^3S_1) = \Gamma(^1S_0)$ or $x = 1$ [4]. Let us get formula for probability of the reaction $p\bar{p} \to e^+e^-$ at rest. We use the following notations: $N_{\bar{p}}(0)$ - number of stopped antiprotons forming the $p\bar{p}$-atom. $N_{\bar{p}}^{(s)}(0)$-number of stopped antiprotons, captured into S-states. According to the ref. [10]

$$N_{\bar{p}}^{(s)}(0) = (1 - \alpha_p) N_{\bar{p}}(0) \tag{17}$$

where $\alpha_p = 0.065 \pm 0.015$ in liquid hydrogen case. The observed number of e^+e^--pairs in the experiment [9] is connected with $N_{\bar{p}}^{(s)}(0)$:

$$N_{e^+e^-} = \frac{3}{4} N_{\bar{p}}^{(s)} \cdot Br_{e^+e^-}(^3S_1), \tag{18}$$

where $Br_{e^+e^-}(^3S_1)$ is branching ratio into e^+e^--pairs from the triplet state of the $p\bar{p}$-atom. Evidently, that this branching is connected with the $p\bar{p}$ cross section at flight via the following equation

$$Br_{e^+e^-}(^3S_1) = \lim_{v \to 0}(v\sigma_t^{e^+e^-}) / \lim_{v \to 0}(v\sigma_t^{ann}). \tag{19}$$

Index t here denotes triplet state. On other hand the triplet $p\bar{p}$ annihilation cross section is connected with the total annihilation cross section

$$\sigma^{ann} = \frac{3}{4}\sigma_t^{ann}\left(1 + \frac{1}{3}\frac{\Gamma(^1S_0)}{\Gamma(^3S_1)}\right). \tag{20}$$

Correspondingly the cross section of the reaction in the triplet state is connected with total cross section for this reaction

$$\sigma_t^{e^+e^-} = \frac{4}{3}\sigma^{e^+e^-}. \tag{21}$$

Taking in view (20) and (21), we get

$$N_{e^+e^-} = N_{\bar{p}}^{(s)}(0)\frac{3}{4}\left(1 + \frac{1}{3x}\right)\frac{\lim_{v \to 0}(v\sigma^{e^+e^-})}{\lim_{v \to 0}(v\sigma^{ann})} \tag{22}$$

Number of the $\pi^+\pi^-$ pairs is given by the following expression

$$N^{\pi^+\pi^-} = N_{\bar{p}}(0) Br_{\pi^+\pi^-} = N_{\bar{p}}(0) \cdot f_{LH}(\pi^+\pi^-). \tag{23}$$

Here we use notations of the paper [10],

$$f_{LH}(\pi^+\pi^-) = (1 - \alpha_p)B_0 + \alpha_p B_1 \tag{24}$$

where $B_0 = 0.00319$ and $B_1 = 0.00481$. Combining Eqs. (22) and (23), we finally have

$$\lim_{v \to 0}(v\sigma^{e^+e^-}) = \frac{N_{e^+e^-}}{N_{\pi^+\pi^-}} \cdot \frac{f_{LH}(\pi^+\pi^-)}{1 - \alpha_p} \cdot \frac{1}{\frac{3}{4}(1 + \frac{1}{3x})} \cdot \lim_{v \to 0}(v\sigma^{ann}) \tag{25}$$

This formula looks like that one which was used in the paper [9] to extract factor $| G(k \to 0) |$, but still there are two different points. The right part of the Eq. (25) contains the factor $\lim_{v\to 0}(v\sigma^{ann})$ instead of $\lim_{v\to 0}(v\sigma^{tot})$ in the analogous formula of the paper [9]. Formally these two limits are to be equal. But practically we get different numbers. So using the parametrization for σ^{tot} given in the Buggs paper [1], we get

$$\lim_{v\to 0}(v\sigma^{tot}) = 57.4mb. \tag{26}$$

Using Heidelberg parametrization [3] we get

$$\lim_{v\to 0}(v\sigma^{ann}) = 49.7mb. \tag{27}$$

If we use the parametrization of the paper [11], which fits the OBELIX data [4], we get

$$\lim_{v\to 0}(v\sigma^{ann}) \simeq 41mb. \tag{28}$$

The second reason for deducing of the formfactor in the Eq.(25) is the spin factor ζ, depending on ratio x,

$$\zeta(x) = \frac{3}{4}(1 + \frac{1}{3x}) \tag{29}$$

In the widths of triplet and singlet states are the same, or $x = 1, \zeta = 1$. As it follows from our previous consideration, $x < 1$. It increases $\zeta(x)$ and leads to smaller quantities of formfactor $| G(k \to 0) |$.

Consider both corrections numerically. Let us begin from OBELIX data [4]. Substituting (26) in the Eq. (25) by (28) we get $| G |= 0.45$ instead of 0.53. Taking in view Eq. (15), we finally get [5]

$$| G(k \to 0) |= 0.39 \tag{30}$$

So we get 0.39 instead of 0.53 from the paper [9]. This quantity is worth comparing with analogous quantities for formfactor at very low energies at flight, which were extracted in the [9] using another procedure: $| G |= 0.39 \pm 0.05$ at $75 < k < 100MeV/c$; $| G |= 0.34 \pm 0.04$ at $100 < k < 125MeV/c$ and $| G |= 0.31 \pm 0.03$ at $125 < k < 150MeV/c$. These three points are measured ones nearest to the atomic point $k \to 0$. We conclude that the slope at threshold of the formfactor as function of k disappears or at least is getting less drastic than discussed in [9].

Notice in addition that using Heidelberg parametrization [3] as input, we get after substitution (27) instead of (26) $| G |= 0.493$. Taking in view that $x = 0.126$ in this case, we get

$$| G(k \to 0) |= 0.30. \tag{31}$$

So in this case we obtain even some fall off the formfactor in the region near threshold.

In conclusion it worth mentioning that there are other consequences from inequality $x < 1$. Near the threshold annihilation reactions which go through different intermediate S-states (triplet and singlet) may have different energy dependence. As consequences one may get different branchings for two such channels at rest and at flight. We are going to discuss this possibility in more details elsewhere.

The authors are thankful for useful discussion to P.Dalpias and P.F. Dalpiaz as well as M.G. Sapozhnikov.

1 Notes

1) The polarization data known for today refer to more higher momenta $p_{lab} \geq 600MeV/c$, see e.g. the paper [6].

2) Use of the Deser et al. formula (11) in this case is quite valid because the scattering length a_2 is not large, of order of range of nuclear force.

3) In our previous publication [8] the result $a_2^2 = 1.02fm$ was presented. It was obtained with the preliminary OBELIX result $v\sigma_{ann} = 33.7mb$ at $p_{lab} = 70MeV/c$.

4) It is argued in the ref. [9], that possible errors due to the $x \neq 1$ were taken into account with changing x in range $0.5 < x < 2$.

5) In the paper [9] all the events with $p_{lab} \leq 150MeV/c$ were reffered to be the events at rest. Evidently that this condition doesn't exclude for part of these events to take place in flight. So the numbers $N_{\pi^+\pi^-}$ and $N_{\pi^+\pi^-}$ used in the ref. [9] to extract $| G(k \to 0) |$ are to be corrected.

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Доля синглетного спина в $p\bar{p}$ и протонный формфактор на пороге.

Подписано к печати 21.10.92 Формат 60x90 1/8 Офсетн.печ.
Усл.-печ.л.0,5. Уч.-изд.л.0,3. Тираж 150 экз. Заказ 91
Индекс 3649

Отпечатано в ИТЭФ, 117259, Москва, Б.Черемушкинская,25