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ON METASTABILITY OF GAUGED NON – TOPOLOGICAL SOLITONS

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ON METASTABILITY OF GAUGED NON-TOPOLOGICAL SOLITONS: Preprint ITEP 92-68/

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We note that, although the gauged non-topological soliton (GNTS) is unstable against fragmentation, it may live quite long due to the Coulomb potential barrier. We estimate the probability of the fragmentation for separation of a small fragment from the fermion-field GNTS in theory with degenerate vacuums, and briefly discuss the fragmentation of the gauged Q-ball.

Мы замечаем, что, хотя калибровочные нетопологические солитоны (КНТС) нестабильны относительно фрагментации, они могут жить очень долго вследствии кулоновского потенциального барьера. Мы оцениваем вероятность фрагментации в случае выделения малого фрагмента из фермионного КНТС в теории с вырожденными вакуумами и кратко обсуждаем фрагментацию калибровочных Q-шаров.

Fig. - , ref. - 7.

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Non-topological soliton is the minimum energy solution of classical equations of motion for some theories with broken symmetry (see for references /1,2/ and review /3/). The NTS field configuration is a spherically symmetric one and carries a conserved Noether charge Q. In case of global symmetry the configuration is stable against a decay into Q free particles of the field if the sum of masses of the particles Qm exceeds the NTS energy (mass) E(Q). The NTS with the conserved charge associated with the local (gauged) symmetry is especially interesting since the standard model fields do possess the gauge charge of unbroken U(1) symmetry and, although the charged matter cannot exist in nature, the gauged NTS may be created (if it is theoretically admissible) artificially, in a laboratory.

We demonstrate herein that the gauged NTS may live quite long (to be metastable) even though the condition of stability is not satisfied, i.e. E(Q)>Qm. The phenomenon is nothing new but Coulomb's barrier well known for the α -decay of nuclei. The reason of the phenomenon lies simply in the Coulomb repulsion: the classical decay cannot happen if the NTS energy is smaller than the energy of the repulsion of the same spatial configuration of free particles plus the sum of their masses. What can happen in this case, that is the quantum tunnelling of

the particles far enough from the NTS, at the distance where the repulsion becomes weak. The same is true for fragmentation (fission) of the NTS.

Consider first the gauged NTS with fermions in the theory with the degenerate potential of the scalar field /1/:

$$\mathcal{L} = i\bar{\psi}(\partial_\mu + ieA_\mu)\gamma^\mu\psi + \frac{1}{2}(\partial_\mu\sigma)^2 - m\left(1-\frac{\sigma}{\sigma_0}\right)\bar{\psi}\psi - \frac{1}{2}\mu^2\sigma^2\left(1-\frac{\sigma}{\sigma_0}\right)^2 \quad (1)$$

Inside the NTS σ lies in the false vacuum state $\sigma = \sigma_0$ so that the fermions are massless. The estimates for the NTS radius and energy are

$$R = \left[\left(\alpha Q^{4/3} + \beta e^2 Q^2\right) \Big/ \frac{4\pi\mu\sigma_0^2}{3}\right]^{1/3} \quad (2)$$

$$E = \left(\frac{9\pi\mu\sigma_0^2}{2}\right)^{1/3}\left(\alpha Q^{4/3} + \beta e^2 Q^2\right)^{2/3} \quad (3)$$

Here $\alpha = (4\pi)^{1/3}\ (3/4)^{5/3} \simeq 1.4$, the number coefficient takes into account the distribution of the charge density inside the NTS. To make the formulae transparent, hereafter we assume such parameters of the theory that NTS is still classically stable for Q so large that

$$\alpha \ll \beta e^2 Q^{2/3}, \quad (4)$$

i.e. as the Coulomb energy dominates over the kinetic one (gathering the particles near the NTS surface: $\beta \simeq (6\pi)^{-1}$).

Thus $E \simeq \frac{2}{3}\pi\mu\sigma_0^2 R^2 + \frac{e^2Q^2}{8\pi R}$

has to be compared with the energy of the same spatial particle

configuration but with the true vacuum everywhere,

$$E_t = \frac{e^2 Q^2}{4\pi R} + Q m \tag{5}$$

(once again neglecting the kinetic energy). The comparison gives for the maximum NTS charge stable against classical decay into Q particles:

$$\beta e^2 Q_{met}^{1/2} < m^{3/2}\left(\frac{6}{\pi\mu\sigma_0^2}\right)^{1/2} . \tag{6}$$

Q_{met} is 9 times larger (independently of ~~e and g~~ the parameters) than Q_{stab} given by the condition

$$E(Q > Q_{stab}) > m \tag{7}$$

Between Q_{stab} and Q_{met}, the NTS is metastable.

The dependence E(Q) (3) has a positive second derivative for any Q, so that the NTS is unstable against fragmentation. For large Q, as $\frac{\partial E}{\partial Q}(Q) > m$, it is possible separation of a particle. Taking into account the Coulomb repulsion of the fragments again changes the situation. Immediately after a classical fission, the fragments are still close (approximately at the distance R(Q)) and, if there are only two of them, with the charges q and Q-q>q, we estimate the full energy gain as

$$\Delta E = E(Q) - E(Q-q) - E(q) - \frac{\beta e^2 q(Q-q)}{R(Q)} \tag{8}$$

or, using (2) and (3) under the condition

$$\alpha \ll \beta e^2 q^{2/3} \tag{9}$$

$$\Delta E = \left(\frac{9\pi\mu\sigma_0^2}{2}\right)^{1/3}(\beta e^2)^{2/3} Q^{4/3}\left[1 - \left(1 - \frac{q}{Q}\right)^{4/3} - \left(\frac{q}{Q}\right)^{4/3} - \frac{2}{3}\frac{\xi}{\beta}\frac{q}{Q}\left(1 - \frac{q}{Q}\right)\right] . \tag{10}$$

The number coefficient ξ depends on the charge distribution in the fragments and upon the way the fission occurs (we do not carry out herein the detailed analysis of the fission process). If ΔE is negative (that is certainly the case for any q if $\xi / \beta \gg 1$), then the fission is energetically unprofitable. The more so if the lightest of the fragments is unbound (e.g. because q is too small for the Fermi gas approximation to be valid) and E(q) in (8) should be replaced by $qm > E(q)$.

Even if the GNTS does not undergo the classical fragmentation, the quantum tunneling is still possible. To estimate its probability we consider the separation of one particle from the GNTS with $Q \gg 1$. The particle passes through the Coulomb potential barrier created by the rest of the GNTS, carring away the energy

$$\mathcal{E} \simeq E(Q) - E(Q-1) - m \simeq \frac{4}{3}\left(\frac{9}{2}\pi\mu\sigma_0^2\right)^{1/3}(\beta e^2)^{2/3} Q^{1/3} \gg m . \tag{11}$$

Since we are not interested in the spin of the particle, we use the squared Dirac equation for the particle wave function (see /4/ for the correctness of the procedure):

$$\left((\mathcal{E} - e\varphi)^2 + \Delta - m^2\right)\psi = 0 \tag{12}$$

with

$$\varphi = \frac{e(Q-1)}{4\pi r} , \quad r > R(Q) \simeq \frac{\beta^{1/3}(eQ)^{2/3}}{\left(\frac{4}{3}\pi\mu\sigma_0^2\right)^{1/3}} \tag{13}$$

Although inside the NTS the particle is massless and acquires the mass only approaching to the NTS surface, for estimate we assume the mass of the particle coming up to the barrier to be constant and equal to that for the particle in the region after the barrier. Such an approximation is justified in case if the thickness of the region where the mass changes (i.e. the thickness of the NTS supface $\sim \mu^{-1}$) is small as compared with the thickness of the barrier, so that the particle passes almost all its way under the barrier with the constant nonzero mass m.

Since $dp/dr \simeq m \dfrac{e(Q-1)}{R(Q)} \ll p^2$,

one can use the semiclassical approximation and obtain the passing coefficient

$$D = \exp\left(-2\int_{R(Q)} \sqrt{m^2 - (\varepsilon - e\varphi)^2}\, dr\right). \qquad (14)$$

$e\varphi = \varepsilon - m$

Denote $R=R(Q)$, $e\varphi = \Phi R/r$ and dimensionless $l = r/R - 1$. The thickness of the barrier $\Delta l \sim Q^{-1/3} \ll 1$, its altitude $\Phi - \varepsilon = m - Q\left(e^{1/2}\mu\sigma_0^2/Q^2\right)^{1/3}$.

Then the integral is approximately equal to

$$R\int_0^{\frac{2m}{\Phi}} \left(m^2 - (m - \Phi e)^2\right)^{1/2} dl = \frac{\pi}{2}\frac{m^2 R}{\Phi} = 2m^2\left(\frac{3\pi^2\beta}{4e\mu\sigma_0^2}\right)^{2/3} Q^{1/3} \qquad (15)$$

The probability of the particle separation ($\sim D$) is exponentially small for large Q (but, of course, $Q<Q_{mel}$; otherwise the rest of the NTS, which creates the "external" field for the tunneling particle, is unstable itself). The calculation

above certainly is still valid for a fragment with small $q \neq 1$ ($q \ll Q-q$) if one changes $m \to E(q)$, $(Q-1) \longrightarrow q(Q-q)$.

Now let us discuss gauged Q-balls. If the complex scalar field $\Phi = (f/\sqrt{2})\exp(i\vartheta)$ possesses a gauged charge,

$$\mathcal{L} = \frac{1}{2}(\partial_\mu f)^2 + \frac{1}{2} f^2 (\partial_\mu \vartheta - eA_\mu)^2 - U(f) - \frac{1}{4}(F_{\mu\nu})^2, \qquad (16)$$

gauged Q-ball arises in a theory with suitable potential $U(|\Phi|)$ /2/. For the potential

$$V = \frac{\mu^2}{2} f^2 - \frac{f^2}{4} + \frac{\lambda^2 f^6}{6\mu^2}, \qquad (17)$$

the Q-ball energy

$$E = \mu\alpha Q\left[1 + \frac{3}{20}\left(\frac{e^3 Q}{\pi\lambda\alpha}\right)^{2/3}\right] \qquad (18)$$

where Q is the charge of the ball and $\alpha = \left(1 - 3/16\lambda^2\right)^{1/2}$.

Since $d^2E/dQ^2 > 0$, fragmentation (fission) is profitable for any Q.

The gauged Q-ball does not hold the particle gas and is described in terms of classical fields (three of them). Therefore one can try to calculate the possibility of its fragmentation looking for the transition amplitude

$$\langle f | e^{i\hat{H}t} | i \rangle = \int [d\vartheta][dA_\mu][df]\, e^{iS} \qquad (19)$$

by the steepest descent method, similar to the problem of nucleation of new vacuum bubbles for a single scalar field /5/.

We do not accomplish herein the calculation and only make some first steps.

The initial state $|i\rangle$ is the field configuration satisfying the classical equation of motion with $\vartheta = \omega t$ in the thin wall approximation:

$$f(\vec{x}) = f_c\,\theta(R-r)$$
$$A_0(\vec{x}) = \begin{cases} \dfrac{eQ}{4\pi R}, & r \le R \\ \dfrac{eQ}{4\pi r}, & r > R \end{cases} \qquad (20)$$

where the constants ω, f_0 and the radius of the ball R are determined by the constants of the theory and by the charge $Q \gg 1$.

The final state $\langle f|$ for two of the fragments with the charges Q_1 and Q_2 $(Q_1+Q_2=Q)$ is just two (20) with Q_i, R_i, $i=1,2$ at the distance where the Coulomb repulsion between the fragments is equal to the $E(Q)-E(Q_2)-E(Q_1)$.

It is possible to demonstrate that, if we deal with the transition between the states with fixed values of the charge, only those pathes give contribution in the (19), which preserve the charge in every their point. Then, before we look for the saddle path of (19) (i.e. for the classical solution of Euclidean equation of motion), we have to impose the constraint of charge conservation, integrating over the "position" b(t), which changes under the global U(1)-rotation, i.e. is canonically conjugated to "momentum" $Q = \dfrac{\partial \int d^3x\, \mathcal{L}}{\partial \dot{\vartheta}}$ /6/. After the fixing of gauge and the Euclidean continuation $t=-i\tau$, $\dot{\vartheta}=i\dot{\vartheta}_E$, $A_0 = iA_{0E}$, one gets the effective action

$$S_{E,eff} = S_E(f, \vartheta_E, A_{\mu E}) + \frac{1}{2}\,\frac{\left[\int d\tau\left[Q - i\int d^3x\, f^2\left(\dot{\tilde{\vartheta}}_E - eA_{0E}\right)\right]\right]^2}{\int d^3x\, f^2} \qquad (21)$$

where $\tilde{\vartheta} = \vartheta - b(t)$.

The similar problem (but non-gauged) has been solved in /7/ for wormholes.

The equations of motion are integro-differential and nonlinear, and the trajectory A_{0E} is a complex one. However, the problem may be simplyfied if to linearize the equations and to find the action at the small initial pieces of the trajectories (as a function of Q), that gives the upper estimate for the probability $\exp(-S_{E,eff})$.

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