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MULTIKINKS INTERACTIONS IN THE $\lambda\phi^4$ THEORY AND THE ARNOL'D DIFFUSION

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MULTIKINKS INTERACTIONS IN THE $\lambda\phi^4$ THEORY AND THE ARNOL'D DIFFUSION: preprint ITEP 92-65/
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As it is well known \1,2\ there are resonances structures in kink-antikink (KK) scattering in the initial velocity interval $0.18<V_{KK}<0.26$ in the $\lambda\phi^4$ theory. More than that it is found \3,4\ that the behavior of KK-scattering has "quasi-fractal" character in the impact velocity space. The bound state for kink-antikink-kink (KKK)system was found in \5\. Here numerically for antisymmetrical KKK systems it was examed the initial kink velocity interval $0.05< V_{KKK}<0.9$. It was found that the bound state of KKK system has taken place for the initial velocities $V_{KKK}<0.72$ and for $0.72 < V_{KKK}<0.764$ there were found the resonances structures similar to the KK scattering.

For nonsymmetrical case and for more or equal than four kinks scattering there is the Arnol'd diffusion has taken place which stochastizied the process of bound state formation.

In the Appendix it is regarded the example of three-soliton bound state for the $\lambda|\phi|^4$ theory.

Fig. - 10, ref. - 16

1. Introduction

Some numerical and theoretical results of resonance KK interactions in the classical one-dimensional space $\lambda\phi^4$ theory are known \1,2\. More careful investigations show that whether a KK interaction settles to a bound state or a two-soliton solution depends "quasi-fractally" on the impact velocity \3,4\.

Here it is continued the systematic study of multikink resonance interactions in the nonintegrable $\lambda\phi^4$ field theory.

The model is defined by the Lagrangian density:

$$\mathcal{L}(x,t)=1/2\ \partial_\mu\phi\ \partial^\mu\phi-1/4\ (\phi^2-1)^2. \qquad (1)$$

As it is well known (see e.g. reviews \6,7\) among solutions $\phi(x,t)$ of the Euler equation for the Lagrangian density (1):

$$\phi_{tt}-\phi_{xx}-\phi+\phi^3=0 \qquad (2)$$

there exist two vacuum solutions $\phi_\pm=\pm1$, and the static kink (K) and antikink ($\bar{K}$) solutions:

$$\phi_{K(\bar{K})}=\pm\text{th}\ [(x-x_0)/\sqrt{2}] \qquad (3)$$

they are stable topologically. The multikinks interactions are described by following initial conditions:

$$\phi(x,0)=\text{th}[(x-x_0)\beta]-\text{th}(x/\sqrt{2})+\text{th}[(x+x_0)\beta]\ , \qquad (4a)$$

$$\phi_t(x,0)=-V\beta\ [(\text{th}[(x-x_0)\beta])^2-(\text{th}[(x+x_0)\beta])^2]\ , \qquad (4b)$$

here V is the kink velocity and $\beta=[2\ (1-V^2)]^{-1/2}$ (see Fig.1a) and for four kink interactions we have:

$$\phi\ (x,0)=\text{th}[(x-x_1)\beta_1]-\text{th}[(x-x_0)\beta_0]+\text{th}[(x+x_0)\beta_0]-\text{th}[(x+x_1)\beta_1]+1 \qquad (5a)$$

here $\beta_0=[2(1-V_0^2)]^{-1/2}$, $\beta_1=[2(1-V_1^2)]^{-1/2}$, V_0, V_1 are initial velocities of kinks and antikinks (see Fig.1b). In the problem (2)-(5) the energy is conserved:

$$E=\int dx\ W(x,t)=\int dx\ [\ |\partial\phi/\partial t|^2/2+|\partial\phi/\partial x|^2/2+(\phi^2-1)^2/4\ ]. \qquad (5b)$$

2. Numerical results and theoretical discussions

Earlier investigations of the KKK bound state have confirmed \5\ that there is the one in the impact velocity region $V=0.75\pm0.03$ (in light velocity units). Here numerically for KKK system it was examed the initial kink velocity interval $0.05< V <0.9$ with the step $\Delta V=10^{-4}$ and it was found that the KKK bound state has taken place for $V < 0.72$. For the region $0.72< V < 0.764$ we have resonances structures similar to ones which had been found for KK systems. For this purpose it was computed the eq.(2) with the initial conditions (4a),(4b). It was used the method of characteristics \8\ earlier it have been applied in \9\. The computed results are shown at Fig.2 as the dependence on V the energy flux F(V) through the plane $x_f=15$ at the $t_f=150$. The rest kink mass M is equal to:

$$M=\int_{-\infty}^{\infty} dx[1/2(\partial\phi_K/\partial x)^2+1/4(\phi_K^2-1)^2]=2\sqrt{2}/3. \qquad (6)$$

It is seen at Fig.2 that the flux is less than the rest kink mass for the $V <0.72$ so we have KKK bound states with well detected radiation flux. There is no monotonous dependence of the flux on kink velocity at the interval $0.72 < V <0.764$. Here we have the resonance structures similar to ones (see e.g. Fig.3) discovered in \1,2\. The "quasi-fractal" structures are also presented here as much as in the KK-scattering (see Fig.4). It is seen through computations of eq.2 that the KKK-scattering is similar to the

$K(K+\bar{K}+1)$-scattering as it is shown at Figs.5a,b,6 for V=0.7600,so it is possible to discuss "quasi-supersymmetry" here having taken into account that a kink has the "fermion number" which is equal to the value :

$$\xi_K = [\phi_K(+\infty)-\phi_K(-\infty)]/4. \qquad (7)$$

The kink escape to the infinity can be detected from the Fig.7 if we compare the monotony of the energy flux for the three-kink bound state with the jump of the flux when the kink passes through the plane x_f. Here we have for comparison of the resonance structure with n_ν=7 for V=0.7600 (Fig.5a,b) and the bound state which is situated at V=0.7590 (see Fig.7a,b).

Earlier the bound state of $K\bar{K}K$-system was observed in the paper \5\ at V≅0.75±0.03. We can suppose that it was one of the long lived resonance structures such as it is shown at Fig.8 for the initial velocity V=0.7646,here the strongly perturbed kink escapes to infinity.

Here the computed results of typical four-kinks interactions are shown at the Fig.9 for initial conditions $V_{K\bar{K}}$ =0.99 (we used that the $K\bar{K}$-interaction had taken place and the bound system $K\bar{K}$ interacted with such other one).There is no $K\bar{K}K\bar{K}$-bound state because it is not stochastically stable due to additional degree of freedom if comparing with $K\bar{K}$- or $K\bar{K}K$-systems bound states (the same explanation is correct for multi-kinks bound states).For estimating the time living of the bound states we use the time living formula of the Arnol'd diffusion \10\ :

$$\tau_D \sim \omega_0^{-1}\varepsilon^{-1}\exp(\varepsilon^{-a}), \qquad (8)$$

here

a=2/(12ζ+3N+14) , ζ ≥N(N-1)/2

and: a is the function of degrees-of-freedom number N ;

ω_0 is the frequency of unperturbed oscillations (here we suppose that it is equal to the one of $K\bar{K}$- oscillations or $K\bar{K}K$-oscillations or multi-$K\bar{K}$ ones);

ε is the parameter of perturbation for the energy,(here we can use the ratio of the internal K-oscillations energy to the energy of interacted multi-kinks).

The bound state is stochastically stable if the degrees-of-freedom number is less or equal to two and the resonances are not crossing over\10\. It is fulfilled for the $K\bar{K}$-system far from the critical velocity $V_{cr}\approx$ 0.259 (see Fig.4) and for anti-symmetrical $K\bar{K}K$-system it is far from the $V_{cr}\approx$ 0.764. But if we exam the $K\bar{K}K\bar{K}$-system then there are three or more degrees of freedom always and so such system is stochastically unstable. (Here we must notice that the crystal-like structures are stochastically stable because they can be described by one degree of freedom-lattice parameter).

3. Acknowledgments

The author thanks to B.S.Getmanov,A.E.Kudryavtsev and A.S.Schwarz for useful discussions on problems of this paper.

4. Appendix

If we regard other nonlinear equations, for example the $\lambda|\phi|^n$ theory (n=4,6), which have soliton solution then it is possible to have stochastically stable bound states taking into account the Arnol'd diffusion.

Let us regard the Klein-Gordon nonlinear equation of type \11-15\ :

$$\partial^2\phi/\partial t^2-\partial^2\phi/\partial x^2+m^2\phi-\mu^2\phi\,|\phi|^2=0\,. \qquad (A1)$$

There is well known soliton solution for this equation of scalar charged field:

$$\phi_s=\sqrt{2}/\mu(m^2-\omega^2)^{1/2}\,e^{-i\omega t}ch^{-1}[x(m^2-\omega^2)^{1/2}]\,, \qquad (A2)$$

here ω is the frequency of complex field and the stable solutions are for $1/\sqrt{2} \le \omega \le m$. The soliton charge for (A2) is:

$$Q_s=8\omega(m^2-\omega^2)^{1/2}\mu^{-2}\,. \qquad (A3)$$

If we change the sign of the frequency $\omega \to -\omega$ then the anti-soliton charge will be :

$$Q_A=-Q_s\,.$$

As it is shown in paper \16\ there is exist the soliton-soliton bound state (SS) for $\omega_1=\omega_2=0.95$ and their interaction velocity $V_1=-V_2=0.25$. Here we demonstrate that it is possible to have the three-soliton bound state (SSS) for $\omega_1=\omega_2=0.972$ and $V_{s1}=-V_{s3}=0.70$, $V_{s2}=0$ (see Fig.10b). For comparing it is shown nearby the evolution for $V_{s1}=-V_{s3}=0.78$, $V_{s2}=0$ (Fig.10a). The possibility of existence of multi-solitons bound states with four or more solitons will be regarded next paper.

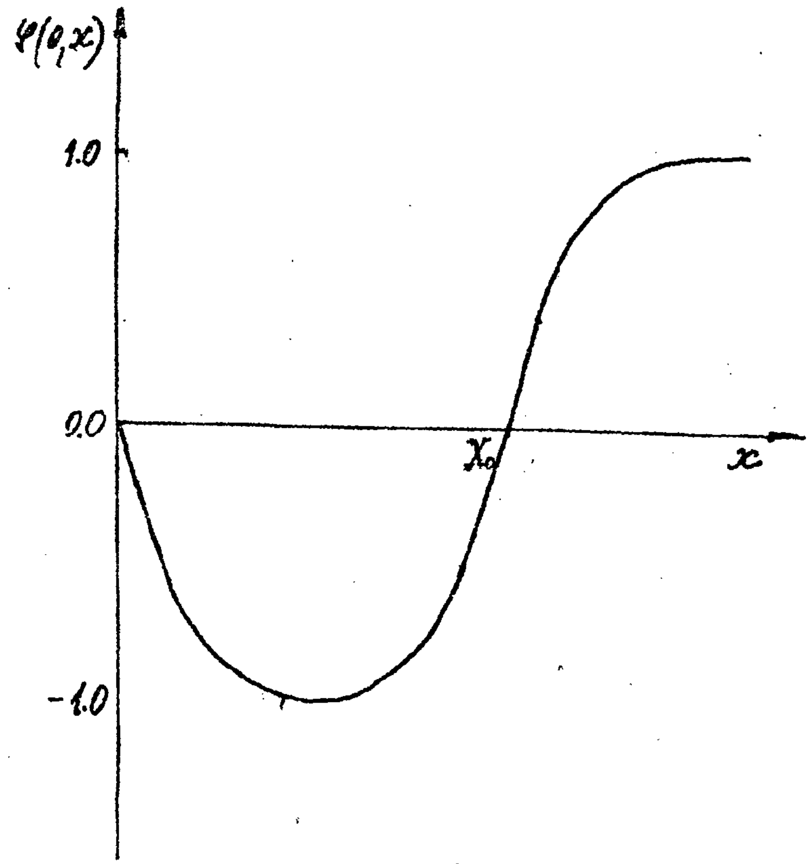


Fig.1a The initial condition for KKK-system. Here it is shown right part of the system, the left one is the anti-symmetrical continuation to axis $x < 0$.

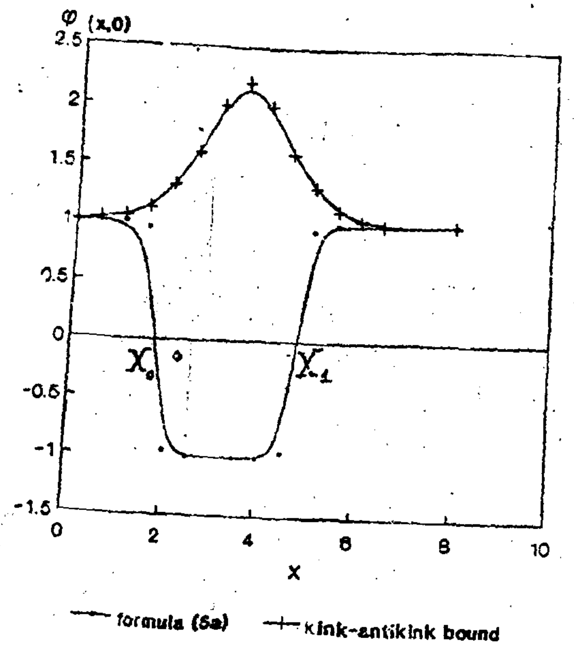


Fig. 1b The initial condition for KKKK-system. Here the left part of the system is symmetrical continuation to axis $x < 0$.

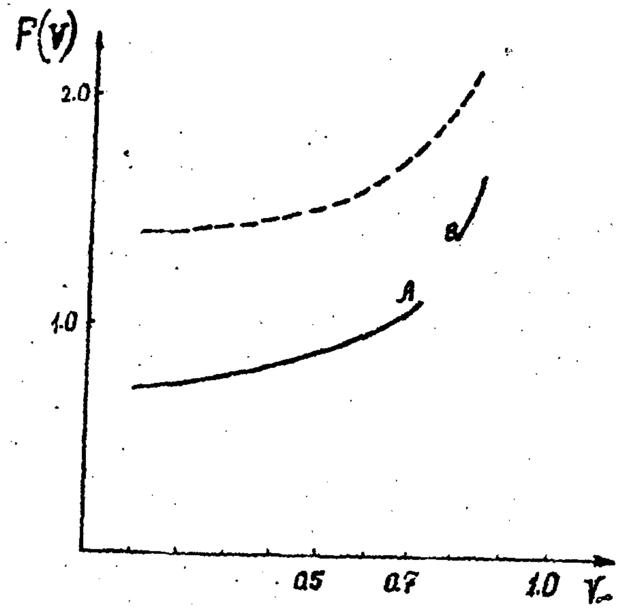


Fig. 2 The flux of energy through the plane x_f=15 at the time Δt=150 for KKK-system as function of kink initial velocity V. Between points A and B the region of resonances is situated The dashed line is the full energy of KKK-system at half-axis $x > 0$.

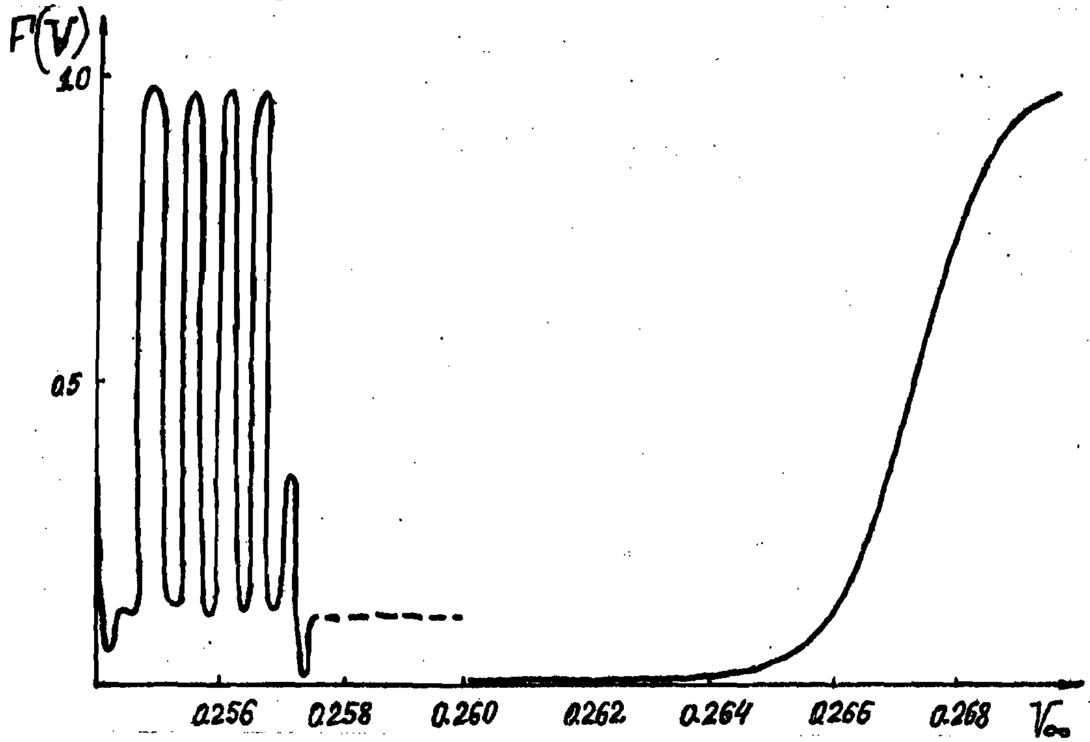


Fig. 4 The flux of energy through the plane x=10 and Δt=200 for the KK-system as function of kink initial velocity. There is shown the region of resonances accumulation point near V_{cr}=0.2598... (as the resonances "windows" are too small they are replaced by the dashed line).

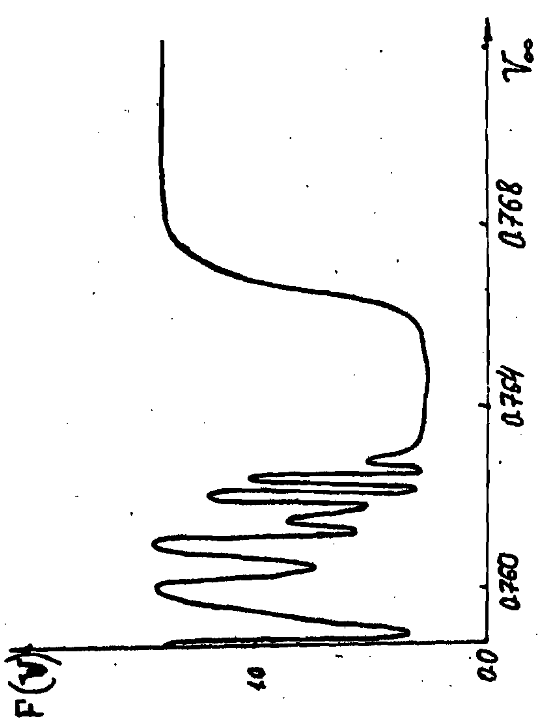


Fig.3 The flux of energy as at Fig.2a for the region of resonances accumulation point.

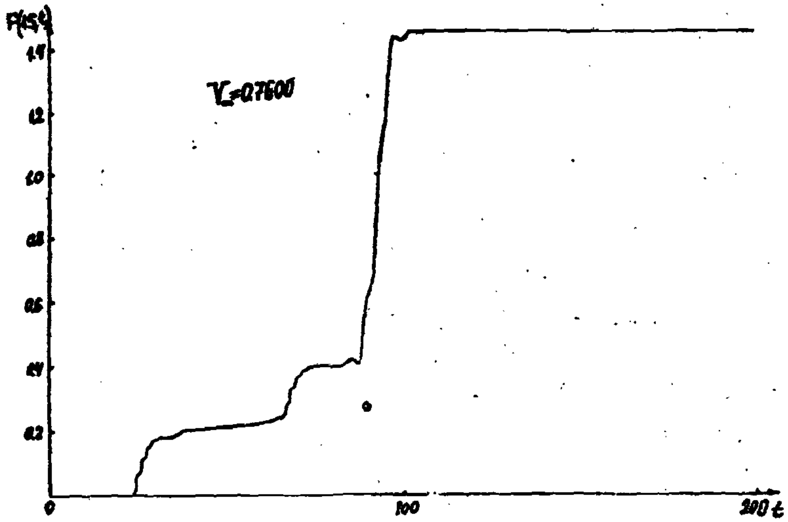


Fig. 5a The flux of energy as a function of time for KKK-interaction for V_K=0.7600. There is well shown resonance structure with n_ν=7 and the jump of radiation equal to rest kink mass M .

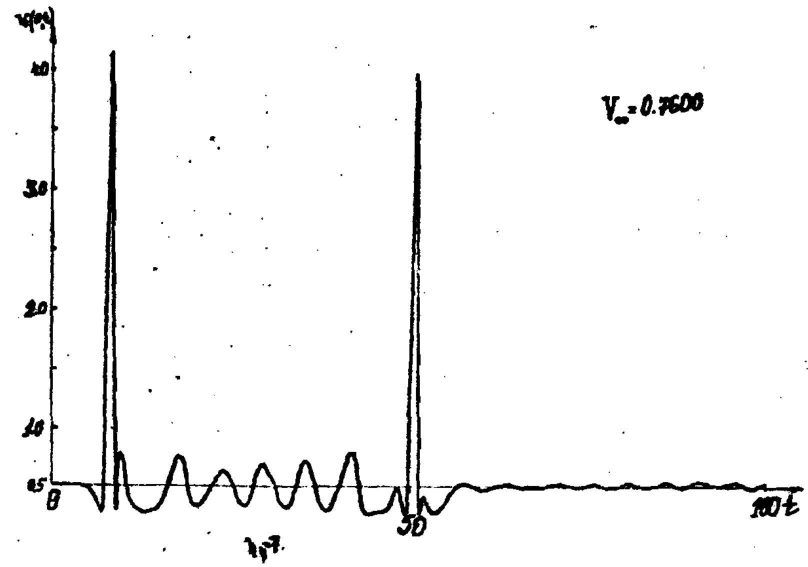


Fig. 5b The energy density of KKK-system as a function of time for V_K=0.7600 at the point x=0.

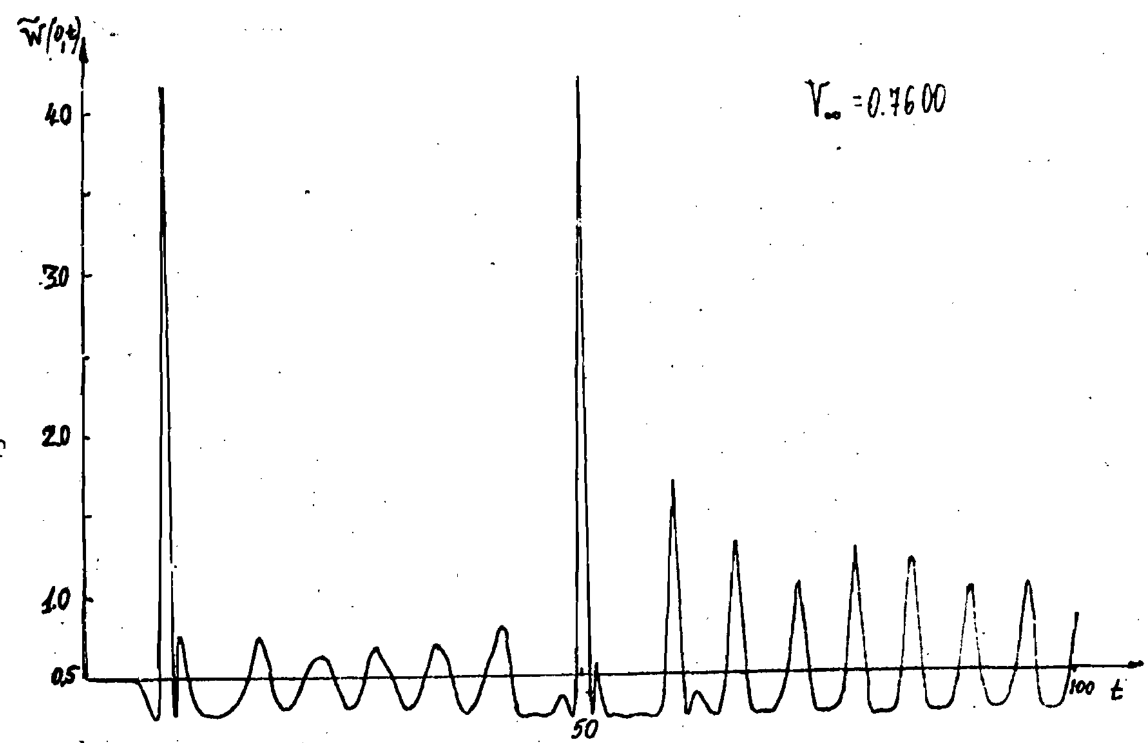


Fig.6 The energy density of K(K+K̄+1)-system as a function of time for $V_K=-V_{\bar{K}}=0.7600$. The resonances interaction of such system is similar to the KKK-one (see Fig.5b)

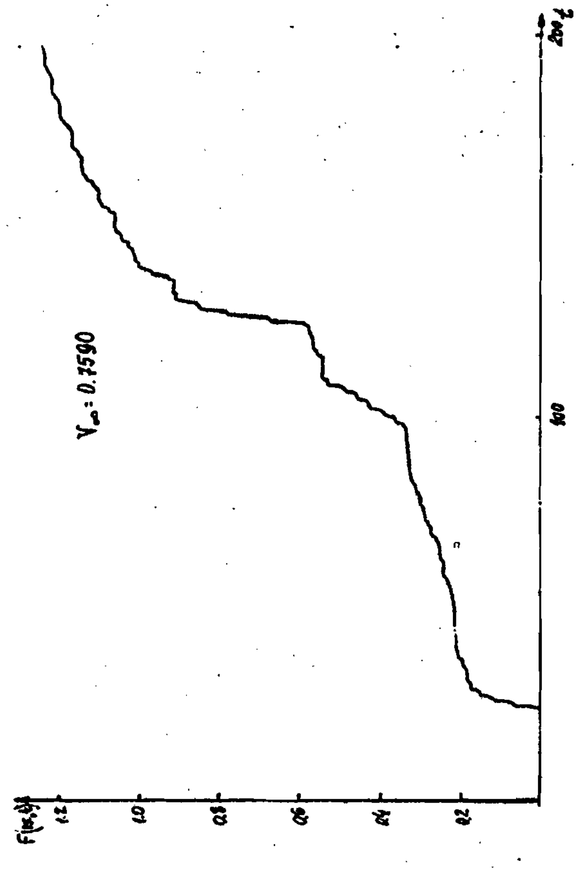


Fig.7a The flux of energy as a function of time for KKK-interaction for V_K=0.7590. Here it is shown the KKK-bound state after the first KKK-collision.

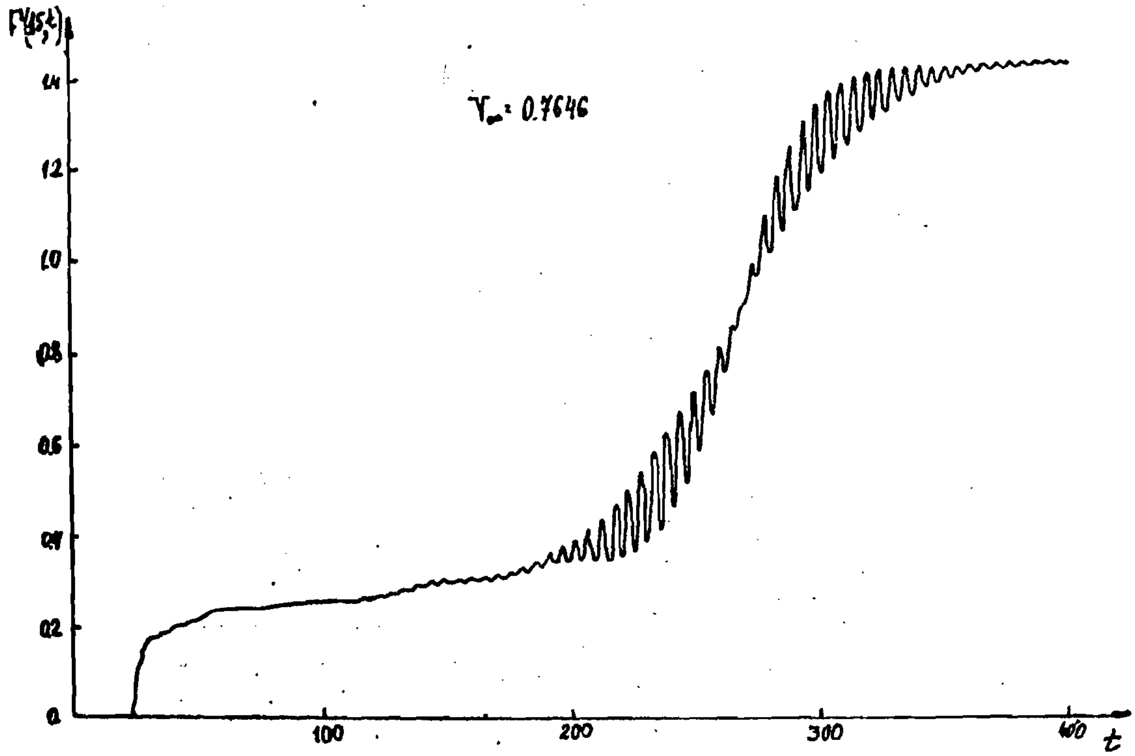


Fig. 8 The flux of energy as a function of time through x_f=15 for long living resonance structure KKK. The escaping kink is perturbed very much (see oscillations of flux). The initial kink velocity V_K=0.7646.

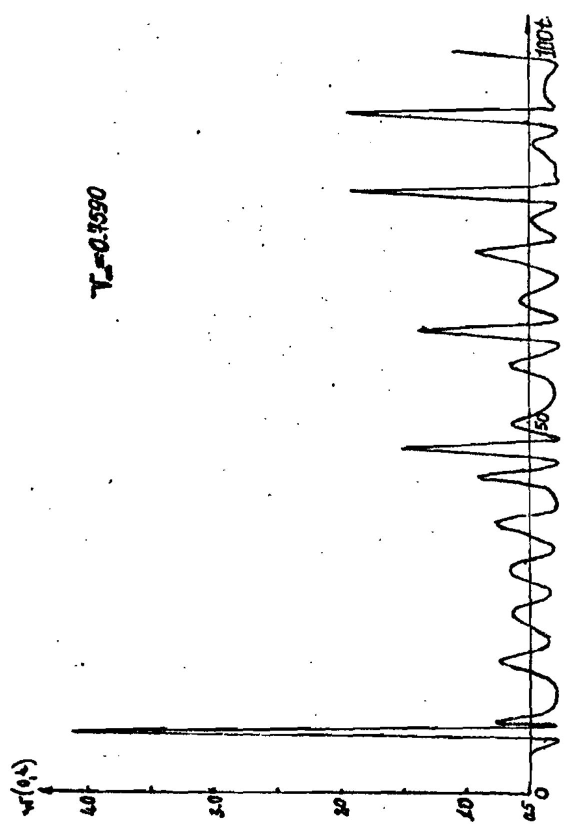


Fig. 7b The energy density of KKK-system as a function of time for V_K=0.7590 at the point x=0 (center-of-mass point).

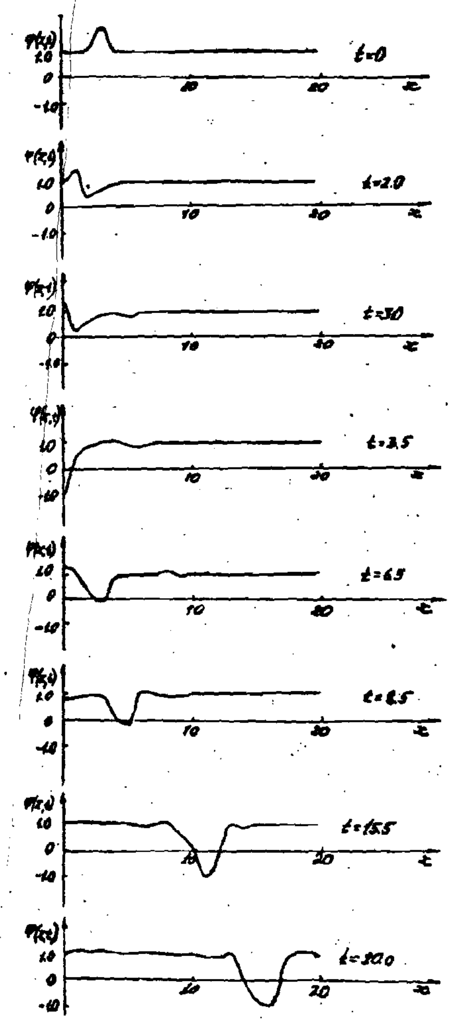


Fig.9 The four-kinks interaction evolution in time . As initial condition was taken the KK-bound state instead of (5a). The initial velocity of the KK-system is 0.9.

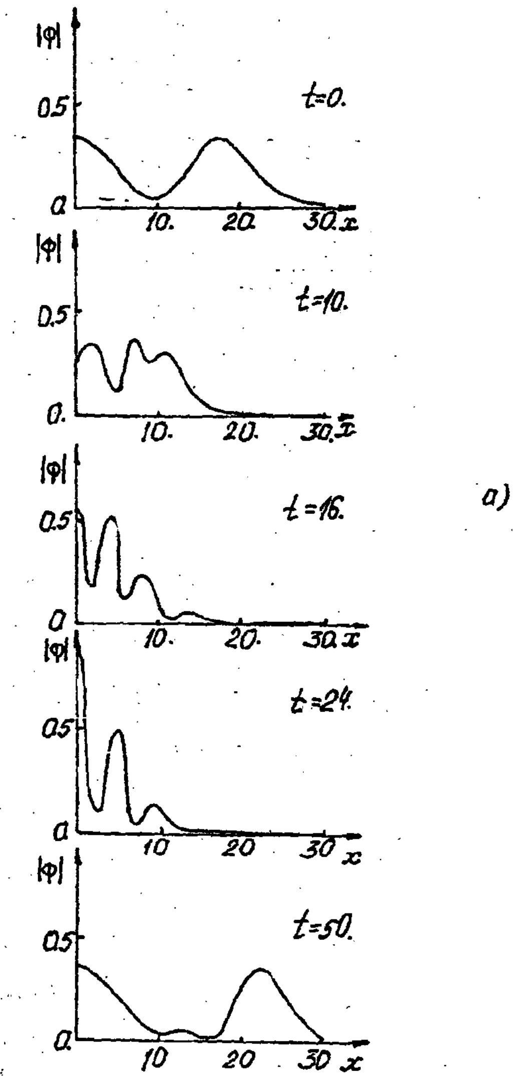
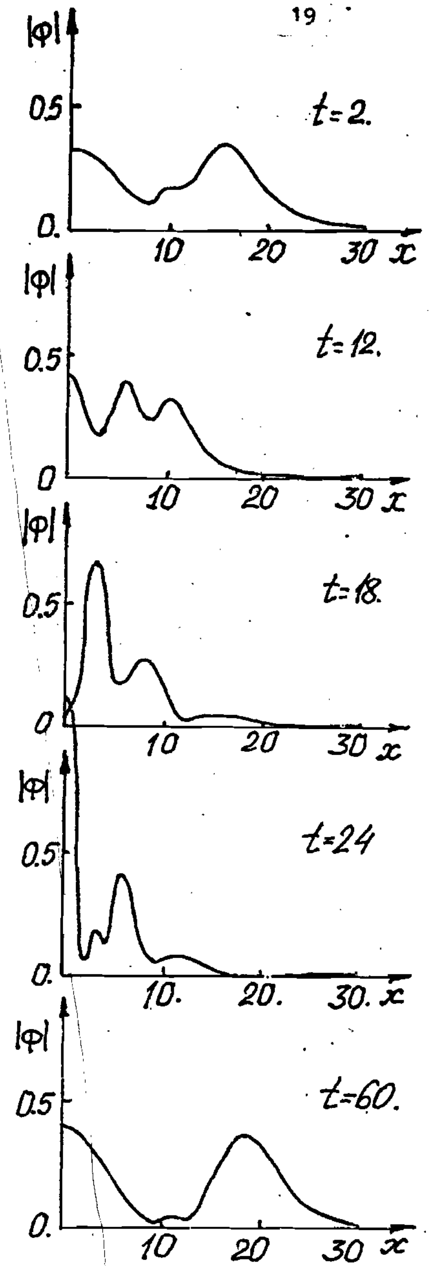


Fig.10a,b The formation of three-solitons bound state for the case of V_{in}=0.70 and $\omega_1=\omega_2=\omega_3$=0.972 (b) and V_{in}=0.78 (a) as compairing for the $\lambda|\phi|^4$ theory.



b)

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Т.И.Белова

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