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Cyclic Representations of $U_q(sl_2)$ in Hofstadter Model*

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CYCLIC REPRESENTATIONS OF $U_q(sl_2)$ IN HOFSTADTER MODEL: Preprint ITEP 86-94/

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The $U_q(sl_2)$-symmetry structure of the Hofstadter model established in [1] is reviewed and refined. An explicit realization of the quantum algebra cyclic representations in the space of states of the model is presented, three continuous parameters of the representations being identified with two Bloch momenta and the anysotropy parameter. For some particular points of bands in the anysotropic case the exact Bethe-ansatz-type solution is obtained.

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1 Introduction and Description of the Model

In this note we review and clarify the hidden $U_q(sl_2)$-symmetry structure of the Hofstadter model established in [1], with laying special emphasis on cyclic (unrestricted) representations of the q-deformed algebra.

Hofstadter model is the commonly used name for a peculiar model of Bloch particles in magnetic field [2], [3], [4], [5]. The spectrum of this two-dimensional lattice system has an extremely reach structure of Cantor set and exibits a multifractal behaviour [4], [6], [7] (see also [8] for a review). The model has interesting physical applications [9], [8], [10], [11] and it has been extensively studied for some time. Recently, we revealed [1] the hidden dynamical symmetry of the problem. The quantum algebra $U_q(sl_2)$ acts in the space of states of the model, allowing one to get a partial exact solution of the Bethe-ansatz type.

The hamiltonian of a charged particle on a two-dimensional square lattice with integer coordinates $\vec{n} = (n_x, n_y)$ in homogeneous magnetic field is ($c^\dagger_{\vec{n}}$, $c_{\vec{n}}$ are usual creation-annihilation operators)

$$H = \sum_{<n,m>} \lambda_{n,m} e^{iA_{\vec{n},\vec{m}}} c^\dagger_{\vec{n}} c_{\vec{m}}, \tag{1}$$

the abelian lattice gauge field $A_{\vec{n},\vec{m}}$ being subject to the constraint

$$\prod_{plaquette} e^{iA_{\vec{n},\vec{m}}} = e^{i\Phi}$$

where Φ is the magnetic flux per plaquette. We suppose that the (real) hopping amplitudes $\lambda_{\vec{n},\vec{m}}$ in (1) are non-zero only for nearest neighbour sites: $\lambda_{\vec{n},\vec{n}\pm\vec{1}_x} = \lambda_x$, $\lambda_{\vec{n},\vec{n}\pm\vec{1}_y} = \lambda_y$ ($\vec{1}_x$ and $\vec{1}_y$ are unit vectors along x and y directions). Denoting $A_x(\vec{n}) = A_{\vec{n},\vec{n}+\vec{1}_x}$, $A_y(\vec{n}) = A_{\vec{n},\vec{n}+\vec{1}_y}$, we rewrite (1) as follows:

$$H = \lambda_x \sum_n e^{iA_x(\vec{n})} c^\dagger_{\vec{n}} c_{\vec{n}+\vec{1}_x} + \lambda_y \sum_n e^{iA_y(\vec{n})} c^\dagger_{\vec{n}} c_{\vec{n}+\vec{1}_y} + \text{h.c.} \tag{2}$$

The wave functions of the particle in magnetic field form a representation of the *group of magnetic translations* [2]: let generators of the translations be

$$T_{\vec{\mu}}(\vec{n}) = e^{iA_{\vec{n},\vec{n}+\vec{\mu}}} \mid \vec{n} >< \vec{n} + \vec{\mu} \mid$$

(here $\vec{\mu} = \pm\vec{1}_x$ or $\pm\vec{1}_y$). They form the algebra

$$\begin{aligned} &T_{\vec{\mu}} = T^{-1}_{-\vec{\mu}}, \;\; T_{\vec{n}} T_{\vec{m}} = q^{-\vec{n}\times\vec{m}} T_{\vec{n}+\vec{m}}, \\ &T_y T_x = q^2 T_x T_y, \; T_y T_{-x} = q^{-2} T_{-x} T_y \end{aligned} \tag{3}$$

where we have denoted $T_x = T_{\hat{1}_x}$, $T_y = T_{\hat{1}_y}$ and

$$q = \exp(\frac{1}{2}i\Phi) \tag{4}$$

Below we assume that

$$\Phi = 2\pi\frac{P}{Q} \tag{5}$$

where P and Q are coprime integers, so q is a primitive root of ± 1 of degree Q. This algebra is also known as the finite Heisenberg-Weyl group. The hamiltonian (2) therefore is

$$H = \lambda_x(T_x + T_{-x}) + \lambda_y(T_y + T_{-y}) \tag{6}$$

As it was shown in [1], the $U_q(sl_2)$-symmetry becomes the most transparent in the lattice chiral gauge (this gauge was also considered in [12]):

$$A_x = -\frac{\Phi}{2}(n_x + n_y), \quad A_y = \frac{\Phi}{2}(n_x + n_y + 1). \tag{7}$$

The Schrödinger equation corresponding to the hamiltonian (2) in the first-quantized picture then looks as follows:

$$\lambda_x e^{-\frac{i\Phi}{2}(n_x+n_y)}\Psi(n_x+1, n_y) + \lambda_x e^{\frac{i\Phi}{2}(n_x+n_y-1)}\Psi(n_x-1, n_y) +$$

$$+\lambda_y e^{-\frac{i\Phi}{2}(n_x+n_y)}\Psi(n_x, n_y-1) + \lambda_y e^{\frac{i\Phi}{2}(n_x+n_y+1)}\Psi(n_x, n_y+1) = E\Psi(n_x, n_y) \tag{8}$$

Due to periodicity of the coefficients one can find solutions in the form of the Bloch waves

$$\Psi(n_x, n_y) = e^{ip_+n_+ + ip_-n_-}\chi_{n_+}(p_+, p_-) \tag{9}$$

where

$$n_\pm = n_x \pm n_y, \quad p_\pm = \frac{1}{2}(p_x \pm p_y)$$

are light-cone coordinates and momenta. The equation (8) is then reduced to the following one-dimensional discrete equation for $\chi_n = \chi_n(p_+, p_-)$, $(n = n_+)$:

$$\begin{gathered} e^{ip_+}(\lambda_y e^{\frac{i\Phi}{2}n + \frac{i\Phi}{2} - ip_-} + \lambda_x e^{-\frac{i\Phi}{2}n + ip_-})\chi_{n+1} + \\ + e^{-ip_+}(\lambda_x e^{\frac{i\Phi}{2}n - \frac{i\Phi}{2} - ip_-} + \lambda_y e^{-\frac{i\Phi}{2}n + ip_-})\chi_{n-1} = E\chi_n, \\ \chi_{n+2Q} = \chi_n, \quad n = 0, 1, ..., 2Q-1. \end{gathered} \tag{10}$$

This is a gauge equivalent form of the famous Harper's equation [8].

2 Representations of $U_q(sl_2)$ at Roots of Unity

The algebra $U_q(sl_2)$ (a q-deformation of the universal enveloping of the sl_2 algebra) is generated by the elements A, B, C, D, with the commutation relations [13] [14],[15],[16],[17]

$$\begin{aligned} &AB = qBA,\ BD = qDB, \\ &DC = qCD,\ CA = qAC, \\ &AD = 1,\ [B,C] = \frac{A^2 - D^2}{q - q^{-1}}. \end{aligned} \tag{11}$$

We assume that the deformation parameter q is a root of ± 1 of degree Q. In the classical limit $q \to 1$, $U_q(sl_2)$ turns to $U(sl_2)$ (universal enveloping of the sl_2 algebra): $(A - D)/(q - q^{-1}) \to S_3$, $B \to S_+$, $C \to S_-$.

The central element of this algebra for general q is a $q-$analog of the Casimir operator:

$$c = \left(\frac{q^{-\frac{1}{2}} A - q^{\frac{1}{2}} D}{q - q^{-1}} \right)^2 + BC. \tag{12}$$

When q is a primitive root of unity of degree Q more central elements appear. If Q is odd these operators are

$$x = C^Q, \quad y = B^Q, \quad t = A^Q. \tag{13}$$

They are connected with the $q-$Casimir element (12) by the algebraic equation of degree Q:

$$\prod_{m=0}^{Q-1} (c - c^{(m)}(q)) = xy + (q - q^{-1})^{-2Q}(t^2 + t^{-2} - 2)$$

where

$$c^{(m)}(q) = \left(\frac{q^{(m+1)/2} - q^{-(m-1)/2}}{q - q^{-1}} \right)^2 = [\frac{m+1}{2}]_q$$

and the last notation means the "q-deformed number" $(m+1)/2$.

The representation theory for $U_q(sl_2)$ at roots of 1 was developed in [14], [18]. Here we describe in some detail only the representations which are relevant in the Hofstadter model.

First, there is a "regular" series of finite dimensional irreducible representations characterized by an integer or half-integer number j ("spin"), $j = 0,$

1/2, 1, ..., $(Q-1)/2$. They can be expressed in the weight basis where A and D are diagonal matrices: $A = diag\,(q^j, ..., q^{-j})$. The values of the central elements (12), (13) in spin-j representation are: $x = y = 0$, $t = \pm 1$,

$$c = \left(\frac{q^{j+1/2} - q^{-j-1/2}}{q - q^{-1}}\right)^2 = [j + 1/2]_q^2. \tag{14}$$

These representations can be realized [14] by difference operators acting in the linear space of polynomials $F(z)$ of degree $2j$:

$$\begin{aligned} &AF(z) = q^{-j}F(qz), \quad DF(z) = q^j F(q^{-1}z), \\ &BF(z) = z(q - q^{-1})^{-1}\left(q^{2j}F(q^{-1}z) - q^{-2j}F(qz)\right), \\ &CF(z) = -z^{-1}(q - q^{-1})^{-1}\left(F(q^{-1}z) - F(qz)\right). \end{aligned} \tag{15}$$

Then $F_0(z) = 1$ is the lowest weight vector whereas $F_{2j}(z) = z^{2j}$ is the highest weight vector, i.e. $CF_0(z) = 0$, $BF_{2j}(z) = 0$.

The realization (15) is a smooth q-deformation of the well-known representation of the sl_2 algebra by first order differential operators:

$$S_3 = z\frac{d}{dz} - j, \quad S_+ = z(2j - z\frac{d}{dz}), \quad S_- = \frac{d}{dz}. \tag{16}$$

There are no irreducible representations of the allebra with dimensions greater than Q.

At the "critical" dimension Q, in addition to the "regular" representation with spin $(Q-1)/2$, there exists a *continuous 3-parametric family* of irreducible representations having in general neighter highest nor lowest weight [18]. They are called cyclic (or unrestricted) representations. The continuous parameters are values of the extra central elements (13). In general, the cyclic representations can not be realized in meromorphic functions on the complex plane (at least, such realization is not known). An explicit realization of the cyclic representations by shift operators on a finite ring-like lattice with Q sites was obtained by Sklyanin [14] as early as in 1983.

Recall that $q = \exp(i\pi P/Q)$. Below we restrict ourselves to the case of *even P and odd Q*. In other cases some details of the arguments should be modified. Denote

$$\omega = q - q^{-1} = 2i\sin(\pi P/Q) \tag{17}$$

for brevity. The Sklyanin's realization is the following:

$$
\begin{aligned}
&A = -ie^{i\delta}q^{1/2}T_+, \quad D = ie^{-i\delta}q^{-1/2}T_-, \\
&C = \omega^{-1}e^{-i\gamma}q^{-n}(e^{i\delta+M}T_+ - e^{-i\delta-M}T_-), \\
&B = \omega^{-1}e^{i\gamma}q^{n}(e^{i\delta-M}qT_+ - e^{-i\delta+M}q^{-1}T_-).
\end{aligned} \tag{18}
$$

where δ, γ and M are continuous parameters. We will choose

$$M = -i\pi/2 + M'$$

where M' as well as δ , γ are real. The shift operators $T_\pm$ act as follows: $T_\pm f_n = f_{n\pm1}$ ($f_{n+Q} = f_n$), the commutation relation with q^n being $T_\pm q^n = q^{n\pm1}T_\pm$. Let us denote these representations $R_{\gamma,\delta,M'}$. As a vector space, $R_{\gamma,\delta,M'}$ is Q-dimensional: $\dim R_{\gamma,\delta,M'} = Q$ for any γ, δ, M'. It is easy to find values of the central elements for (18):

$$
\begin{aligned}
&c = \omega^{-2}(e^{M'} - e^{-M'})^2, \\
&x = 2\omega^{-Q}\exp(-i\pi Q/2 - iQ\gamma)\cosh(Q(M' + i\delta)), \\
&y = -2\omega^{-Q}\exp(-i\pi Q/2 + iQ\gamma)\cosh(Q(M' - i\delta)), \\
&t = -i^{Q+P}e^{iQ\delta}.
\end{aligned} \tag{19}
$$

In the case of general position x, y are non-zero and t is not equal to ±1. Such representations are called cyclic because they have no highest and lowest weight vectors. When $x = y = 0$ and t is not equal to ±1 one obtaines the one-parametric series of "nilpotent" representations with both highest and lowest weights. (It can be seen from (19) that they are not realized in the Hofstadter model). When $x = y = 0$ and $t = \pm1$ ($M = 0$, $\delta = \frac{\pi}{2}$) one gets the representation of the "regular" series (15) which can be realized in polynomials.

The realization (18) is not unique. There is another useful realization on a finite Q-periodic lattice:

$$
\begin{aligned}
&A = q^{-n+\mu}, \quad D = q^{n-\mu}, \\
&C = \omega^{-1}q^{n}(q^{n-1/2-\mu} + (q^{\sigma} - q^{-\sigma}) - q^{-n+1/2+\mu})T_-, \\
&B = -\omega^{-1}q^{-n}(q^{n+1/2-\mu} - (q^{\sigma} - q^{-\sigma}) - q^{-n-1/2+\mu})T_+
\end{aligned} \tag{20}
$$

where ω, μ, σ are parameters. This realization is a "symmetrized" (with respect to the action of B and C) version of the more familiar one [18]. We

denote the representation (20) as $\tilde{R}_{\alpha,\mu,\sigma}$. The central elements are:

$$\begin{aligned}
c &= \omega^{-2}(q^{\sigma} - q^{-\sigma})^2, \\
x &= -\omega^{-Q}(-1)^{P/2} q^{\alpha Q}(q^{\mu Q} - q^{-\mu Q} - (-1)^{P/2}(q^{\sigma Q} - q^{-\sigma Q})), \\
y &= \omega^{-Q}(-1)^{P/2} q^{-\alpha Q}(q^{\mu Q} - q^{-\mu Q} + (-1)^{P/2}(q^{\sigma Q} - q^{-\sigma Q})), \\
t &= q^{\mu Q}.
\end{aligned} \tag{21}$$

The expressions like $q^{\mu Q}$ are to be understood as $\exp(i\pi\mu P)$.

Equivalent representations have identical values of x, y, t and c. So, to identify $R_{\gamma,\delta,M'}$ and $\tilde{R}_{\alpha,\mu,\sigma}$ one should compare the values (19) with (21) and find how the new parameters are expressed through old ones.

3 Some Exact Results in the Anysotropic Case

Let us recall the representation of the quantum algebra $U_q(sl_2)$ in terms of magnetic translations [1]:

$$\begin{aligned}
i\omega C &= \lambda^{-1/2} T_x + \lambda^{1/2} T_y \\
-i\omega B &= \lambda^{-1/2} T_{-x} + \lambda^{1/2} T_{-y} \\
A^2 &= -q T_{-y} T_x \\
A^2 &= -q^{-1} T_{-x} T_y
\end{aligned} \tag{22}$$

where

$$\lambda = \lambda_y / \lambda_x. \tag{23}$$

is the anysotropy parameter. The q-Casimir operator (12) for (22) is equal to

$$c = \left(\frac{\lambda^{1/2} - \lambda^{-1/2}}{q - q^{-1}} \right)^2.$$

This representation allows one to realize $U_q(sl_2)$ in the space of states of the model. In particular, the hamiltonian (6) is expressed through $U_q(sl_2)$ generators:

$$H = i\omega\sqrt{\lambda_x \lambda_y}(C - B). \tag{24}$$

Substituting (18) into this hamiltonian we get the Harper's equation (10) in the chiral gauge with

$$p_+ = \delta, \quad p_- = -\gamma, \quad \lambda = exp(-2M'). \tag{25}$$

One concludes that continuous parameters of the cyclic representations are identified in this way with two components of the Bloch momentum of the particle and the lattice anysotropy parameter.

Let us start with the isotropic case: $M' = 0$, $\lambda_x = \lambda_y = 1$. It is possible to show that due to the gauge invariance the spectrum of (10) with $\lambda_x = \lambda_y = 1$ depends on only one parameter $\Lambda = \Lambda(p_+, p_-)$ that for P even and Q odd looks as follows [1]:

$$\Lambda = \cos(Qp_+)\cos(Qp_-). \tag{26}$$

The value $\Lambda = 0$ is known to correspond to the mid band spectrum of the hamiltonian (6) with $\lambda_x = \lambda_y = 1$. In other words, solving (10) for $\Lambda = 0$, one gets Q solutions χ_n which according to (9) yield eigenstates of (6) for the mid points of the bands.

From the point of view of the representation theory the value $\Lambda = 0$ ($\delta = p_+ = \pi/2$, p_- is arbitrary) means that one deals with a representation of the "regular series" (15) (with $j = (Q-1)/2$). To see this, it is sufficient to substitute (25) into (19) at $\Lambda = 0$. Using (15) one rewrites the spectral problem for (24) in the form of the difference equation

$$i(z^{-1} + qz)F(qz) - i(z^{-1} + q^{-1}z)F(q^{-1}z) = EF(z) \tag{27}$$

wich, by constraction, *must* have polynomial solutions. The original equation (10) at $p_+ = \pi/2$ becomes equivalent to (27) after the substitution

$$\chi_n = F(e^{-ip_-}q^n).$$

As it is shown in [1], the ansatz

$$F(z) = \prod_{m=1}^{Q-1}(z - z_m). \tag{28}$$

gives a solution to (27) with the energy

$$E = i(q - q^{-1})\sum_{l=1}^{Q-1} z_l, \tag{29}$$

provided the roots of $F(z)$ satisfy the system of algebraic Bethe equations

$$\frac{z_l^2+q}{qz_l^2+1} = -\prod_{m=1,m\neq l}^{Q-1} \frac{qz_l - z_m}{z_l - qz_m}, \quad l = 1, ..., Q-1. \tag{30}$$

To see this, one should divide both sides of (27) by $F(z)$ and cancel the residues in the left hand side.

A simple generalization of the above argument allows one to obtain some exact results in the anysotropic case as well. Let us recall another realization of cyclic representations given by (20) and plug it into (24):

$$\begin{aligned}\omega(C-B) = q^{-\alpha}(q^{-\mu+1/2+n} - q^{\mu-1/2-n} - (q^{\sigma} - q^{-\sigma}))T_+ + \\ +q^{\alpha}(q^{-\mu-1/2+n} - q^{\mu+1/2-n} + (q^{\sigma} - q^{-\sigma}))T_-.\end{aligned} \tag{31}$$

Consider the spectral problem (10) with $p_+ = \pi/2$, $p_- = -\pi/2$ and arbitrary anysotropy $\lambda = \lambda_y/\lambda_x$. As we have seen, it is associated with the cyclic representation $R_{\pi/2,\pi/2,-(\log\lambda)/2}$ of $U_q(sl_2)$ given by (18). Using formulas (19) and (21) for the central elements one finds that the realization (20) with

$$q^{\sigma} = \lambda^{\pm 1/2}, \quad q^{\mu Q} = e^{i\pi P/2}, \quad q^{\alpha Q} = \pm e^{i\pi Q/2} \tag{32}$$

gives an equivalent representation $\tilde{R}_{\alpha,\mu,\sigma}$. So, the spectra of (10) and (31) are identical.

Now we use the following trick. Consider the auxiliary hamiltonian

$$H' = \omega(q^{1/2}B + q^{-1/2}C) - (-1)^{P/2}(\lambda^{1/2} - \lambda^{-1/2})(A+D). \tag{33}$$

It turns out that the difference spectral problem for (31) at $q^{2\alpha+1} = -1$ *coincides* with the spectral problem for (33) taken in the representation of the "regular series" with spin $j = (Q-1)/2$. At the same time, one of the values $q^{\alpha} = \pm i q^{-1/2}$ is always contained among the solutions of (32). This means that the spectral problem for (33) in the difference realization

$$q^{1/2}(-z + z^{-1} + \kappa)F(qz) + q^{-1/2}(q^{-1}z - qz^{-1} + \kappa)F(q^{-1}z) = EF(z) \tag{34}$$

(here $\kappa = \lambda^{-1/2} - \lambda^{1/2}$) leads, similarly to the case considered above, to the functional Bethe ansatz solution in the anysotropic case. (Note that if $\lambda = 1$ the spectral problem (33) gives the same result as in the isotropic case though the hamiltonian is slightly different. The explanation is very simple:

the coefficients in front of B and C can be "gauged away" by rescaling the functional variable z that does not change the spectrum).

After a suitable rescaling of the variable z in the functional realization of (33) we get the following result in the anysotropic case. The spectrum is given by

$$E = (q^{1/2} - q^{-1/2})(\lambda_x - \lambda_y) + i\lambda_x(q - q^{-1}) \sum_{j=1}^{Q-1} z_j \tag{35}$$

where z_j's satisfy the system of Bethe equations:

$$\frac{(z_k - iq^{1/2}\lambda)(z_k + iq^{1/2})}{(q^{1/2}z_k + i\lambda)(q^{1/2}z_k - i)} = -\prod_{l=1, l\neq k}^{Q-1} \frac{qz_k - z_l}{z_k - qz_l} \tag{36}$$

This result coincides with the equations obtained for the anysotropic case in [19] by means of a completely different method.

4 Conclusion

We have explained how the symmetry algebra $U_q(sl_2)$ acts in the space of states of the Hofstadter model. For the model with anysotropy λ the whole space of states V_λ is a direct integral

$$V_\lambda = \int_B dp_+ dp_- R_{-p_-, p_+, -(\log\lambda)/2} \tag{37}$$

of cyclic representations (18) of the symmetry algebra over an elementary "Brillouin zone" B. The hamiltonian is a simple linear combination (24) of $U_q(sl_2)$ generators. It's action respects the decomposition (37), so the spectral problem can be considered in each $R_{\gamma,\delta,M'}$ separately. Physically, this means the reduction to Bloch waves (9). For some particular parameters the representations $R_{-p_-,p_+,M'}$ can be realized in the entire functions on the complex plane (these values of parameters in isotropic case correspond to the midband spectrum) that allows one to apply the functional Bethe ansatz.

In conclusion, let us mention two results that have not been discussed in this paper. First, the Hofstadter model on the triangular lattice can be treated in the frame of a similar approach. In this case the expression for the hamiltonian is quadratic in $U_q(sl_2)$ generators. Second, it is possible to obtain explicit expressions for zero mode wave functions in terms of q-hypergeometric

series. For example, the midband spectrum always (for odd Q) contains a zero mode, the corresponding wave function being expressed through continuous q-Legendre polynomials.

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