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IDEAS IN NONPERTURBATIVBE QCD

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The structure of the hadron spectrum is discussed in connection with the main phenomena of nonperturbative QCD: confinement and chiral symmetry breaking (CSB). For the higher part of the spectrum ($M \geq 2GeV$) spin and chiral effects are unimportant; spectrum of $q\bar{q}$ system is described by an effective Hamiltonian deduced from QCD. The Hamiltonian reduces to relativistic quark potential model or to the open string model in two opposite limits. Hybrids are shown to appear naturally in theory and enter the multiplets which are compared to experiment and bosonic string theory. The phenomenon of conspiracy of the spectrum of radial excited states producing operator product expansion is discussed.

The lower part of the spectrum ($M < 2GeV$) is influenced by spin and chiral effects. CSB and the chiral quark mass are deduced for the vacuum containing instantons and confining background.

Main points are summarized in conclusion.

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1 Introduction

The main nonperturbative properties of the QCD vacuum – confinement and CSB – influence hadron spectrum in a different way for higher ($M > 2Gev$) and lower ($M < 2Gev$) part.

The former is shaped by confinement and mesons are mostly $q\bar{q}$ connected by a string. The lower part feels strongly CSB especially in the PS channel. After a short discussion of the QCD vacuum in section 2 we devote next 4 sections to derivation of $q\bar{q}$-string Hamiltonian from first principles and compare resulting multiplets to experiment. The last sections are devoted to chiral effects interconnected with confinement. A summary of results is given in conclusion.

2 Nonperturbative properties of the QCD

The QCD vacuum is known to be occupied by the nonperturbative configurations, which lead to the scale anomaly and produce the nonperturbative shift of the vacuum energy density ε [1]

$$\varepsilon = \frac{\beta(\alpha_s)}{16\alpha_s} < F^a_{\mu\nu}F^a_{\mu\nu} > \cong -\frac{11}{3}N_c\frac{\alpha_s}{32\pi} < F^2 > \tag{1}$$

Note that asymptotic freedom which ensures the negative sign of $\beta(\alpha_s)$, makes the nonperturbative QCD vacuum advantageous as compared to the empty (perturbative) one.

The nonperturbative QCD vacuum can be characterized by vacuum field correlators
$< F(1)\Phi(1,2)F(2)...F(n)\Phi(n,1) >$ where $\Phi(x,y)$ are parallel transporters

$$\Phi(x,y) = P\ exp\ ig\int_y^x A_\mu(z)dz_\mu \tag{2}$$

The dynamics of confinement enters through the area law of the Wilson loop

$$< W(c) > \equiv \frac{1}{N_c} < tr\Phi(x,x) > = exp(-\sigma S_{min}) \tag{3}$$

where the string tension σ is computed through the vacuum correlators [2]

$$\sigma = \frac{1}{24}\int d^2x g^2 < F_{\mu\nu}(x)\Phi(x,0)F_{\mu\nu}(0)\Phi(0,x) > + ... \tag{4}$$

and dots refer to higher order correlators.

Another important characteristics of the nonperturbative vacuum is the chiral condensate [3]

$$< \bar{q}q > \cong -(250MeV)^3 \tag{5}$$

and the topological susceptibility [4]

$$\int d^4x < Q(x)Q(0) >= (180MeV)^4 \tag{6}$$

which suggests that the topological charge density is of the order of one unit per $1fm^4$. We shall see that the latter quantity is connected to chiral condensate (5).

3 The $q\bar{q}$ Green's function

The Green's function of the $q\bar{q}$ system can be written using the Feynman-Schwinger representation [5] as a double path integral over paths of a quark Dz and antiquark $D\bar{z}$ with the proper-time integrations $ds\ d\bar{s}$

$$G(x\bar{x}, y\bar{y}) = \int ds \int d\bar{s} Dz D\bar{z} e^{-K-\bar{K}} < W(C) > \tag{7}$$

where we have omitted spin degrees of freedom having in mind to concentrate on higher levels, where spin interactions are unimportant. We also neglected the quark determinant (sea quark loops) in the large N_c limit (quenched approximation).

Here kinetic energy terms are defined as

$$K = \frac{1}{4}\int_0^s \dot{z}_\mu^2(\lambda)d\lambda, \bar{K} = \frac{1}{4}\int_0^{\bar{s}} \dot{\bar{z}}_\mu^2(\lambda)d\lambda. \tag{8}$$

All interaction between q and $\bar{q}$ is contained in the Wilson loop in (7). To understand better the origin of confinement - the area law (3) - one may apply to $W(C)$ the nonabelian Stokes theorem and use the cluster expansion, which yields [2]

$$< W(C) >= exp\{-\frac{g^2}{2!}\int d\sigma_{\mu\nu}(u)d\sigma_{\rho\lambda}(u') < F_{\mu\nu}(u,z_0)F_{\rho\lambda}(u'z_0) > + \tag{9}$$
$$+\frac{g^4}{4!}\int d\sigma(1)d\sigma(2)d\sigma(3)d\sigma(4) \ll F(1,z_0)F(2,z_0)F(3,z_0)F(4,z_0) \gg +...\}$$

where $F_{\mu\nu}(u,z_0) = \Phi(z_0,u)F_{\mu\nu}(u)\Phi(u,z_0)$ and z_0 is an orbitrary point, on which the whole sum (9) is independent. It is convenient to choose it in the plane of the contour C. It was shown in [2] that each term of the cluster

expansion in (9) provides the area law (4) when the area S is much larger than the correlation length T_g of field correlators $< FF >, < FFFF >$ etc. Recently the lowest order correlator $< FF >$ was measured on the lattice [6]. It consists of two independent Lorentz structure functions D and D_1,

$$g^2 < F_{\mu\nu}(z,0)F_{\rho\lambda}(0,0) >= (\delta_{\mu\rho}\delta_{\nu\lambda} - \delta_{\mu\lambda}\delta_{\nu\rho})D(z) + \\ +\frac{1}{2}[\partial_\mu z_\rho \delta_{\nu\lambda} - \partial_\mu z_\lambda \delta_{\nu\rho} + \mu\nu \leftrightarrow \rho\lambda]D_1(z), \tag{10}$$

which both decrease exponentially [6]

$$D_1(z) \sim \frac{1}{3}D(z) \approx const\ exp(-|z|/T_g), \quad T_g \approx 0.2fm \tag{11}$$

Therefore one may use the area law (3) to calculate spectrum of $q\bar{q}$, when the size of the $q\bar{q}$ system R is much larger than T_g,

$$R \gg T_g \tag{12}$$

Condition (12) is fulfilled for most existing hadronic systems, except for the ground state of bottomonium, where one must exploit for $< W(C) >$ the more detailed form (9). On the other hand, the condition (12) puts some limits on the $q\bar{q}$ system considered as a string, as we shall see below.

4 The $q\bar{q}$ -string Hamiltonian

Our aim now is to calculate the spectrum of the $q\bar{q}$ system [7], described by (7). To this end we rewrite identically the kinetic terms (8)

$$K + \bar{K} = \int_0^T \frac{d\tau}{2}[\frac{m_1^2}{\mu_1(\tau)} + \mu_1(\tau)(1 + \dot{z}_i^2(\tau)) + \frac{m_2^2}{\mu_2(\tau)} + \mu_2(\tau)(1 + \dot{\bar{z}}_i^2)] \tag{13}$$

introducing an important new quantity – to be the dynamical q and $\bar{q}$ masses:

$$\mu_1(\tau) = \frac{dz_4(\lambda)}{d\lambda}; \quad \mu_2(\tau) = \frac{d\bar{z}_4(\lambda)}{d\lambda}, \quad \tau \equiv z_4 = \bar{z}_4 \tag{14}$$

We choose $z_4 = \bar{z}_4 \equiv \tau$ as an integration variable in (13), and neglect the backtracking in time i.e. take $\mu_i(\tau) > 0$. Justification for it may be found on dynamical grounds - when coming back and forth in time, the quark is dragging with itself the heavy string; the action sharply increases due to that, so such motion is dynamically suppressed.

An effective action for our system can be read off from Eq.(7) and (3).

$$A = K + \bar{K} + \sigma S_{min} \tag{15}$$

where S_{min} – the minimal area inside the contour made of q and $\bar{q}$ trajectories $z(\lambda)$ and $\bar{z}(\lambda)$ – can be constructed by connecting $z(\lambda)$ and $\bar{z}(\lambda)$ by straight lines:

$$S_{min} = \sqrt{\dot{w}^2 w'^2 - (\dot{w}w')^2}\ , \quad w_\mu = z_\mu \beta + \bar{z}_\mu (1-\beta), \tag{16}$$
$$\dot{w}_\mu = \frac{\partial w_\mu}{\partial \tau}\ , \quad w'_\mu = \frac{\partial w_\mu}{\partial \beta} = r_\mu = z_\mu - \bar{z}_\mu.$$

The square-root form of S_{min} can be eliminated in the standard way [7] introducing the auxiliary functions $\nu(\tau,\beta)$ and $\eta(\tau,\beta)$. After integrating out the latter and the center-of-mass coordinate R_μ, one is left with the following effective action (we consider equal mass case, $m_1 = m_2 = m$ and consequently, $\mu_1 = \mu_2 = \mu$)[7].

$$A = \int_0^T d\tau \{ \frac{m^2}{\mu(\tau)} + \mu(\tau) + \frac{1}{2}[\frac{\mu(\tau)}{2}\dot{\vec{r}}^2 + \tag{17}$$
$$+ \int_0^1 d\beta (\beta - \frac{1}{2})^2 \nu(\tau,\beta) \frac{(\dot{\vec{r}} \times \vec{r})^2}{\vec{r}^2} + \sigma^2 \vec{r}^2 \int_0^1 \frac{d\beta}{\nu} + \int_0^1 \nu d\beta]\}$$

The function ν introduced as an auxiliary function, actually has important physical meaning - it describes the energy density of the string. We note first of all that $\nu(\tau,\beta)$ and $\mu(\tau)$ have no canonical momenta and should be found from the minimum of the effective action (17).

Below we consider several limiting cases of (17) following discussion given in [7].

i) nonrelativistic case, $m \gg \sqrt{\sigma}$. One finds $\mu \sim m \gg \nu$, in the leading order $\mu(\tau) = m, \quad \nu = \sigma|\vec{r}|$,

$$A \cong \int_0^T d\tau [2m_q + \frac{m_q}{4}\dot{\vec{r}}^2 + \sigma|\vec{r}|] \tag{18}$$

ii)relativistic case, $L = 0$. The term $(\dot{\vec{r}} \times \vec{r})^2$ disappears in (17) and minimization of ν, μ yields the Hamiltonian

$$H = 2\sqrt{\vec{p}^2 + m^2} + \sigma|\vec{r}| \tag{19}$$

This is exactly the Hamiltonian of the relativistic potential model, assumed in many papers [8-10] and studied both numerically and and quasiclassically

in [9,10]. The spectrum is given by a simple formula

$$M^2(n_r, L) = 4\pi\sigma(n_r + \frac{L}{2}) + \Delta \tag{20}$$

where Δ has been computed numerically and quasiclassically in [9,10]

$$\Delta(n_r, L) = 2\sigma(4 - \pi - \gamma)L + 2m_q^2 + 4m_q^2 ln\frac{M}{m_q} + m_0^2 \tag{21}$$

$\gamma = 0.12$ for n_r large.

It is interesting that asymptotics at large n_r,$M^2 \approx 4\pi\sigma n_r$, is twice that of bosonic string for large L, $M_L^2 \approx 2\pi\sigma L$. On the other hand at large L the asymptotics of (20),

$$M^2(L \gg 1) \approx 8\sigma L \tag{22}$$

differs from that of bosonic string.

Actually this happens because we have used in (22) the Hamiltonian (19) in the region $L \gg 1$, where it is not applicable. To find out what regime takes place at large L, one must consider

iii) the limit of pure string dynamics, $L \gg 1$, $L \gg n_r$. In this case the minimization in (17) yields $\nu \gg \mu$ and [7]

$$M^2(L) = 2\pi\sigma L(1 + (\frac{1.46(n_r + 1)}{L})^{4/5} + ... \tag{23}$$

The extremal value of ν is the energy density of rotating string

$$\nu \rightarrow \nu_L(\beta) = \frac{\rho_L}{\sqrt{1 - v^2(\beta)}}, \tag{24}$$

where the mass density of the string ρ_L and the velocity $v(\beta)$ are given by

$$\rho_L = (\frac{8\sigma\sqrt{L(L+1)}}{\pi})^{1/2}\ , \quad v(\beta) = 2(\beta - 1/2)\ , \tag{25}$$

Thus one can see that $\mu(\tau)$ and $\nu(\tau, \beta)$ refer to the energy density of quarks and string respectively. At small L, we have $< \mu >=< \nu >$ and ν describes the potential energy, $\nu = \sigma|\vec{r}|$; μ is dynamical mass of quark. At large L we have $\nu \gg \mu$ and most energy is carried by string (ν) and not by quarks (μ). This yields correct mass relation (23).

5 Structure of the $q\bar{q}$ spectrum at $M \geq 2GeV$

One can write the general form of the spectrum, corresponding to the action (17) in the form (20) where Δ is given by (21) at moderate $L, L \leq 2$ while at large L, the value of Δ is frozen, effectively one can put $4-\pi-\gamma \to 0, L \to \infty$ in (21). This form of answer is also suggested by numerical quantization of the $q\bar{q}$ system in [11].

Since Δ is thus limited for large L, the gross features of the high excited spectrum are given by the first term on the r.h.s. of (20), suggesting degeneration of states with the same $N \equiv n_r + L/2$.

Taking $\Delta(n_r, L)$ into account splits the masses within the multiplet with a given value of N.

Qualitatively Δ grows with L for L not large; $L \geq 2$ and this agrees with experiment. E.g. for the doublet $N = 1$ $\rho(1450)(n_r = 1, L = 0)$ and $\rho(1700)(n_r = 0, L = 2)$ we find from (21), $\delta \equiv \Delta(1,0) - \Delta(0,2) \cong 0,58GeV^2$ (we choose $m_q = 0.2GeV$ and $\sigma = 0.17GeV^2$, for discussion of m_q see last section).

Experimentally $\delta_{exp} \equiv \Delta(1,0) - \Delta(0,2) \cong 0.79GeV^2$.

Now for the triplet $N = 2$

$$\rho_5(2350)(n_r = 0, L = 4), \rho_3(2250)(n_r = 1, L = 2), \rho_1(2150)(n_r = 2, L = 0)$$

the theoretical difference δ between ρ_5 and ρ_3, or ρ_3 and ρ_1 is again $0.58GeV^2$ (however it is actually smaller for the first pair because for $L > 2$ the value of Δ start to saturate), while experimentally $\delta_{53} = \delta_{31} = 0.46GeV^2$. We list in Table 1 the theoretically computed masses using eqs. (20-21), and compare these with experimental values. One can notice that agreement is good, except for the lowest state – ρ and π mesons which should not be described by our formulas (20-21).

We conclude this section by several remarks:

1) lowest states need corrections from spin-spin, spin-orbit interactions and gluon exchanges, which are not taken into account in (20-21)

2) at large L, fixed $n_r, L \gg n_r$, the splitting Δ does not depend on L - this is the string regime, [7] described by (23). E.g. from Table 1 one obtains that already ρ_5 state with $l = 4$ is close to the string regime but correction in (23) is already large,

$$\Delta M^2 \approx 2\pi\sigma(\frac{1.46}{L})^{4/5} \cdot \sqrt{L(L+1)} \approx 0.4M^2, (L = 4) \qquad (26)$$

3) at large n_r, fixed L, one has relativistic potential regime, $M^2 = 4\pi\sigma n_r$. It is remarkable that the "radial trajectory" $\rho(2.11), \rho(1.45), \rho(0.77)$ has a slope $4\pi\sigma$ different from lowest part of orbital Regge trajectory $\rho(0.77), \rho_3(1.69), \rho_5(2.35)$ with the slope which is close to 8σ. This is in nice agreement with theoretical prediction (20).

4) There are many states especially in the $I = 0$ channel, which do not fit into the spectrum of multiplets $M^2 = 4\pi\sigma(n_r + L/2) + \Delta$. Some of them –hybrids– will be discussed in section 7. They fit into generalized multiplets of the QCD string type. Others do not and they are probably not of the $q\bar{q}$ structure - they might be gluebals or multiquark states. We obtain a fit of mesons in Table 1 taking $m_q = 0.2GeV$. In section 9 we justify this choice showing that m_q is actually not the current mass (as $\bar{m}_q$ in (39)) but the chiral mass $M(0)$ (71).

Table 1

$L\backslash n_r$	0	1	2
0	0.767 $\rho(0.77)$	1.47 $\rho(1.45); \pi(1300)$	2.07 $\rho(2.11); \pi(1.80?)$
1	1.328 $a_2(1.32)$	1.90	2.34 $f_2(2.34)$
2	1.715 $\rho_3(1.69), \pi_2(1670)\rho(1700)$	2.19 $\rho_3(2.25)$	2.58
3	2.029 $a_4(2.04)a_3(2.05)$	2.44	
4	2.3 2.31 $\rho_5(2.35)$		
5	2.544 2.52 $a_5(2.45)$		

Masses of mesons computed from (20) (upper entry) *vs* experimental values from Particle Data Booklet, June 1992 (lower entry). For $L = 4.5$ in the right upper corner are listed values computed in the corrected string regime, Eq. (23).

Parameters used in Eq.(20-21) are:$m_q = 0.2GeV, (8\sigma)^{-1} = 0.85GeV^{-2}$, m_0^2 was fitted to $\rho(0.77)$.

6 OPE: condensates from hadronic spectra

Consider as a first example the process $e^+e^- \to$ everything, its crossection is given by $Im\Pi_{\mu\nu}(q^2)$, where $\Pi_{\mu\nu}$ is

$$\Pi^i_{\mu\nu}(x) = \frac{1}{2} < 0|Tj^i_\mu(x)j^i_\nu(0)|0 > \tag{27}$$

and i denotes the sort of quark. One can write

$$\Pi^i_{\mu\nu}(Q^2) = (Q_\mu Q_\nu - Q^2\delta_{\mu\nu})\Pi^i(Q^2), \tag{28}$$

$$\Pi^i(Q^2) = \frac{1}{\pi}\int_{s_0}^{\infty}\frac{ds Jm\Pi^i(s)}{s+Q^2}, \tag{29}$$

Introducing the standard hadronic ratio $R_i(s)$, one has

$$Im\Pi^i(s) = \frac{1}{12\pi\ e_i^2}R_i(s) \tag{30}$$

Consider now large $N_c, N_c \to \infty$. This is a realistic limit, since it provides a linearity of Regge trajectories which is observed with a few percent accuracy. Also, all hadronic masses are constant in this limit, while decay widths are $0(1/N_c)$ and are indeed smaller than masses (at large masses $\Gamma/M \sim 10\%$).

It is important that Π^i contains only poles when $N_c \to \infty$

$$\Pi_i(Q^2) = \frac{1}{12\pi^2}\sum_{n=0}^{\infty}\frac{c_n^i}{M_n^2+Q^2} \tag{31}$$

while

$$R_i(s) = \sum_{n=0}^{\infty} c_n^i\delta(s-M_n^2) \tag{32}$$

As symptotically at large s the quark-hadron duality tells us that the averaged $R_i(s)$ is constant

$$\int_{\Delta s} R_i(s)ds = e_i^2 N_c\Delta s = \sum_{n<\Delta s} c_n^i = c_\infty^i\Delta n \tag{33}$$

Hence the quark-hadron duality (QHD) means that [9,12]

$$c_\infty^i = e_i^2 N_c\frac{dM_n^2}{dn} \tag{34}$$

Let us check it now with M_n^2 and c_n for our Hamiltonian (19). The c_n^i have been computed quasiclassically in [9]:

$$\text{For} L = 0, c_\infty^i = N_c e_i^2\frac{2}{3}m^2, m^2 = 4\pi\sigma \tag{35}$$

$$\text{For} L = 2, \bar{c}^i_\infty = N_c e_i^2 \frac{1}{3} m^2 \tag{36}$$

Since $M^2(n_r, L = 0) = M^2(n_r - 1, L = 2)$, both states asymptotically degenerate and one has

$$c^i_\infty = N_c e_i^2 (\frac{2}{3} m^2 + \frac{1}{3} m^2) = N_c e_i^2 m^2, \tag{37}$$

while from (20) for large $n \equiv n_r$ one has

$$\frac{dM_n^2}{dn} = m^2 \equiv 4\pi\sigma \tag{38}$$

Hence the QHD (34) is asymptotically satisfied, as was recognized in [9].

Let us now make a step forward: we can expand (31) at large Q^2 in powers of $1/Q^2$ using for the l.h.s. the operator product expansion (OPE) (this idea without reference to large N_c and in a bit different setting has been first used in [12].). Using OPE from [1] one gets

$$\begin{aligned} \Pi^i_{\mu\mu}(Q^2) = \frac{Q^2}{4\pi}\{\frac{c_0^i}{M_0^2 + Q^2} + \frac{c_1^i}{M_1^2 + Q^2} + \sum_{n=2}^{\infty} \frac{c_n^i}{M^2 + Q^2}\} = \\ \frac{e_i^2 N_c}{4\pi^2}\{-(1 + \frac{\alpha_s}{\pi}) ln \frac{Q^2}{\mu^2} + \frac{24\pi^2 \bar{m}_q^2}{Q^2} + \frac{8\pi^2 \bar{m}_q < \bar{q}q >}{Q^4} + \\ + \frac{\pi^2}{3Q^4} \frac{\alpha_s}{\pi} < G^a G^a > - \frac{8\pi^3 \alpha_s}{Q^6} [< j^a_{\mu 5} j^a_{\mu 5} > + \frac{2}{9} < j^a_\mu j^a_\mu >] \end{aligned} \tag{39}$$

We have separated out the first two poles to approximate $c^i_n = c^i_\infty$ for $n \geq 2$. The sum is then

$$\sum_{n=n_0}^{\infty} \frac{1}{M_n^2 + Q^2} = -\frac{1}{m^2} \Psi(\frac{Q^2 + \Delta + n_0 m^2}{m^2}) + divergent\ const. \tag{40}$$

where $\Psi(z) = \frac{\Gamma'(z)}{\Gamma(z)}$ and has an asymptotic expansion

$$\Psi(z)_{z \to \infty} = lnz - \frac{1}{2z} - \sum_{k=1}^{\infty} \frac{B_{2k}}{2kz^{2k}} \tag{41}$$

with B_n Bernulli numbers, $B_2 = \frac{1}{6}$.

Using (41) in (39) one obtains equations, of which we quote only those resulting from terms $lnQ^2, 1/Q^2$ and $1/Q^4$

$$c^i_\infty = N_c e_i^2 m^2 (1 + \frac{\alpha_s}{\pi}) \tag{42}$$

$$c_0^i + c_1^i = c_\infty^i(\frac{\Delta + 2m^2}{m^2} - \frac{1}{2}) \tag{43}$$

$$c_0^i M_0^2 + c_1^i M_1^2 = -e_i^2 \alpha_s \pi < G^a G^a > + \frac{c_\infty^i}{2m^2}[(\Delta + 2m^2)(\Delta + M^2) + B_2 m^4] \tag{44}$$

One deduce from (39) and (42-44) that

1) logarithmic term in OPE is naturally emerging from the sum of high excited states, also with correct coefficient if (42) is satisfied (this is QHD).
2) In the limit $\bar{m}_q \to 0$ ($\bar{m}_q$ -current mass) the OPE has no $1/Q^2$ term, while the sum (31) generally contains it. However, if $\Delta = \frac{1}{2}m^2$, then $\Psi(\frac{Q^2+m^2+\Delta}{m^2})$ contains no terms of $1/Q^2$ in agreement with OPE. This case we shall call "ideal spectrum".
3) Assuming the states with $n = 0, 1$ nonasymptotic as in (39), one gets from (44) that the lowest value of gluonic condensate is $\frac{\alpha_s}{\pi} < G^a G^a > \approx 0.1 GeV^4$ i.e. around 8 times the standard value of [1]. One should have in mind that the limit $N_c \to \infty$ cuts off the quark loops and hence changes gluonic condensate, which can be several times larger than the standard value.

The same type of estimates have been obtained in [12] both from heavy quarkonia and light quark channels.
4) Expansion (41) is at best asymtotic, since B_{2k} grows as $k!$ at large k. Hence the OPE is at best asymptotic expansion with factorially growing coefficients of $(1/Q^2)^n$.
5) In the leading order of $N_c \to \infty$ the relation (39) is exact. It allows to relate the spectrum in each channel J^{PC} to the microscopic characteristics of the vacuum – condensates. Condensates are the same in each channel; therefore one has very rigid conditions on masses and coefficients c_n in each channel. It may lead to an apparent paradox. E.g. in the 1^{++} shannel the OPE for $\bar{m}_q = 0$ looks the same as in the 1^{--} channel (up to terms $1/Q^4$). However the spectrum at least for $M < 2GeV$ looks very different. What is the resolution of this paradox is not yet clear.

7 Hybrids

To define hybrids one has to separate quantum gluon field a_μ from the non-perturbative background B_μ

$$A_\mu = B_\mu + a_\mu, \tag{45}$$

with the background gauge condition $D_\mu(B)a_\mu = 0$. Then the hybrid state w.f. can be formed as

$$\Psi(x,\bar{x},u) = \bar{\Psi}(\bar{x})\Phi(\bar{x},u)\Gamma a_\mu(u)\Phi(u,x)\Psi(x) \tag{46}$$

Thus hybrid is obtaind as a product of the quark bilinear $\bar{\Psi}\Gamma\Psi$ with quantum numbers $0^{++}(\Gamma = 1), 1^{--}(\Gamma = \gamma_\nu), 0^{-+}(\Gamma = \gamma_5)$ and gluon w.f. with $J^{PC} = 1^{--}$. As a result one gets for the hybrid

$$J^{PC}(\Gamma a_\mu) = 1^{--}(\Gamma a_\mu = a_\mu), 0^{++}(\Gamma a_\mu = \hat{a}), 1^{+-}(\Gamma a_\mu = \gamma_5 a_\mu)$$
$$1^{++}(\Gamma a_\mu = \sigma_{\mu\nu}a_\nu), 2^{++}(\Gamma a_\mu = \gamma_\mu a_\nu + \gamma_\nu a_\mu)$$

Those are the lowest states not containing orbital gluon excitations. Introducing into Γ the operator $D_\mu(B)$ one gets all possible gluon excitations with additional quantum numbers e.g. $1^{-+}(\Gamma a_\mu = \gamma_\nu D_\nu a_\mu)$. Lowest states $1^{+-}, 0^{++}, 1^{++}, 2^{++}$ are degenerate modulo spin-spin interactions of quarks and gluons.

The hybrid Green's function can be written in the same way as for $q\bar{q}$ system (cf.(7))

$$G(1;2) = \int \prod_{i=1}^{3}(ds_i Dz_i e^{-K_i}) < W(C)\Phi_{adj}(u,v) > \tag{47}$$

where $W(C)$ is the product of quark parallel transporters Φ, while Φ_{adj} arises from the gluon propagator.

In the large N_c limit the gluon line Φ_{adj} becomes a $q\bar{q}$ line, and we have

$$< W(c)\Phi_{adj} >_{N_c\to\infty} \to < W(C_1) >< W(C_2) > \tag{48}$$

Thus the gluon line becomes a border of two surfaces S_1, S_2 and lies entirely inside the film covering the total contour C. This means that gluon in the hybrid describes (at least at $N_c \to \infty$) vibration of the surface and in this way vibrational degrees of freedom of the string appear, which have been absent in the ground state (where the minimal surface enters). We shall come back to this point in the next section.

It is easy to obtain from (47) the Hamiltonian in the same way as it was done for the $q\bar{q}$ system. For small orbital momenta one gets

$$H = \sqrt{\bar{p}_1^2 + m_1^2} + \sqrt{\bar{p}_2^2 + m_2^2} + |\bar{p}_3| + \sigma|\bar{r}_1 - \bar{r}_3| + +\sigma|\bar{r}_2 - \bar{r}_3| - C_0 \tag{49}$$

Calculation witout one-gluon exchange (OGE) and $\sigma = 0.17GeV^2$ yields lowest mass

$$M = 2.5GeV - C_0 = 0 \tag{50}$$

and including OGE and $C_0 = 0.4 \div 0.2GeV$ [5] one gets

$$M = 1.3 - 1.5GeV \tag{51}$$

This agrees qualitatively with other calculations [13]. Orbital gluon excitations cost $\Delta M \approx 1GeV$ for $L = 1$.

8 QCD string, bosonic string and Veneziano spectrum.

The spectrum of the open bosonic string is [14]

$$M_n^2 = 2\pi\sigma[-\alpha_0 + \sum_{k=1}^{\infty} kN_k] \tag{52}$$

where k denotes the mode and N_k - excitation number. Theory is consistent when $\alpha_0 = 1, d = 26$.

The lowest mode is $k = 1$, which is rotation of a rigid stick with $L = 1$. Next is $k = 2$ which may be rotation with $L = 2$ and vibration–center of the string moves with respect to ends. The lowest vibration mode has $M_2^2 = 4\pi\sigma \equiv m^2$. The open bosonic string has no longitudinal (radial) excitations.

An important characteristics of the spectrum is the multiplicity of the state with given N, which is equal to $exp(a\sqrt{N}), N \to \infty, a = \frac{4\pi}{\sqrt{6}}$ [14]. This exponential growth is needed to get the Veneziano formula for the amplitude. In other words the property of duality of amplitudes $A(s,t,u)$ which is contained in the Veneziano formula, needs the exponential growth of multiplicity.

It is clear that the spectrum (20) cannot ensure this exponential growth – the number of states with given n_r, L grows only like a power, because number of degrees of freedom is fixed. We shall see now that the problem is solved by hybrids.

One can recognize in the hybrid Hamiltonian (49) two pieces of string connected at the gluon position. If one does the same type of treatment as for the $q\bar{q}$ state leading in that case to (17) and considers a generic string excitation with spectrum (23) for each peace of string, one has for the asymptotic hybrid spectrum

$$M^2(l_1, l_2) = 2\pi\sigma(|l_1| + |l_2|), \vec{L} = \vec{l_1} + \vec{l_2} \tag{53}$$

In case of pure vibration $\vec{l_1} + \vec{l_2} = 0, |\vec{l_1}| = |\vec{l_2}| = \nu$ and one has

$$M(\nu) = 4\pi\sigma\nu, \quad \nu = 0, 1, 2... \tag{54}$$

where ν refers to the vibration mode.

For a multihybrid with n gluons sitting on the $q\bar{q}$ string dividing it into $n+1$ cuts, the vibration obtains when internal cuts have angular momentum $2l$ while first and last have l. The total mass again in the regime when $\nu_i \gg \mu_i$ (string regime [7]) is

$$M^2(n,l) = 4\pi\sigma nl \tag{55}$$

Thus every quantum of vibration yields $4\pi\sigma = m^2$ to the squared mass, while every quantum of rotation is $2\pi\sigma$.

The hybrids contribute all necessary vibration modes and correspond to the spectrum (52). Moreover, the multiplicity is now growing exponentially, since the number of degrees of freedom contains an infinite number of gluons on the string.

The hybrids enter the same QCD string multiplets, which we can now write as

$$M^2(n_r, L\nu) = 4\pi\sigma(n_r + \frac{L}{2} + \nu) + \Delta \tag{56}$$

where $\nu \geq$ number of gluons in the multihybrid.
For $\nu = 1$, $M_1 = \sqrt{(1.46)^2 + \Delta} \approx 1.5GeV$
For $\nu = 2$, $M_2 = \sqrt{(2.06)^2 + \Delta} \approx 2.2GeV$

Conclusions on high spectrum:
Spectrum consists of QCD-string multiplets (56), containing radial, orbital and vibrational exitation. This spectrum contains that of the bosonic string plus radial excitations specific for QCD_2.

However, there is a limitation – the finite correlation length T_g (see eg. (11)) makes a natural cut-off at small distances – there is no string at distances $\Delta x < T_g$. Therefore effective number of gluons is less than length of the string divided by T_g. Hence the effective number of degrees of freedom is finite and the string theory is nonlocal. This fact may cure difficulties with string quantization for the real QCD string for $d = 4$.

9 Lowest states – chiral effects.

In the formation of lowest states the broken chiral symmetry plays an important role. In this Section we discuss chiral quark mass and chiral symmetry breaking (CSB) in connection with confinement.

Several statements known in literature are in order.

1) CSB is due to quasizero modes of quarks in the vacuum gluonic field

[15]. If u_n, Λ_n are to be found from

$$i\hat{D}(A)u_n(x) = \Lambda_n u_n(x) \tag{57}$$

then the quark condensate is connected to the density $\nu(\Lambda)$ of quasizero modes [15]

$$< \bar{\Psi}\Psi >= -m_q \int \frac{\nu(\Lambda)d\Lambda}{\Lambda^2 + m_q^2} \to -\nu(0)\pi \tag{58}$$

2) zero modes $u_0(x)$ are provided by instantons [16]. Therefore instantons may be responsible for CSB [17]. This is the simplest possibility. Another is vacuum with magnetic monopoles which also provide zero models [18]. The quark zero mode on (anti) instanton is normalizable [16].

$$u_0^{(\pm)} = \frac{\rho}{\pi(\rho^2 + x^2)^{3/2}} \frac{\hat{x}}{\sqrt{x^2}} \binom{1}{-1} \varphi, \quad \varphi_{am} = \frac{1}{\sqrt{2}} \varepsilon_{am} \tag{59}$$

where ρ is the size of instanton; a, m-color and spin indices.

3) To provide $\nu(0) \neq 0$ and consequently CSB it is necessary for instantons and antiinstantons in the vacuum to overlap their zero modes [19]. This mechanism was realized in [19] neglecting gauge invariance and confinement properties. Here I wil quote results [20] taking these properties into account.

Consider instantons in the confining background

$$A_\mu = B_\mu + \sum_{i=1}^{N} A_\mu^i \,, \quad N = N_+ + N_- \tag{60}$$

where B_μ ensures confinement (i.e. correlators $F_{\mu\nu}(B)$ in (4) yield nonzero string tension), while A_μ is the field of i-th (anti) instanton (instantons do not confine). Now let light quark move in the field (60) where instantons are at $x = R_i, i = 1, ...N$. The scattering amplitude of a quark on center i (instanton or antiinstanton) is given by

$$t_i = \frac{u_0(x - R_i)u_0^+(y - R_i)}{i\bar{m}_q} \tag{61}$$

where u_0 is given in (59), $\bar{m}_q$- the current quark mass. The total quark Green's function is given by the multiple scattering theory as [19]

$$S(x, y) = S_0(x, y) + \sum_{i,j} u^i(x)(\frac{1}{\hat{T} - i\bar{m}_q})_{ij} u^{j+}(y) \tag{62}$$

$$u_i \equiv u_0(x - R_i)$$

where T_{ij} is the overlap integral of zero modes

$$T_{ij} = \int u_i^+(z) i\hat{D}(B) u_j(z) d^4 z \tag{63}$$

Eq.(62) shows that multiple scattering can provide effective quark mass – chiral mass. In solids this effective mass is naturally produced by subsequent collisions. For CSB this is not enough – the quark should return to a given center any number of times. Only in this case occurs a gap equation yielding the chiral mass (similar conclusions are drawn in [19] without confining field B_μ). In case of one flavour, $N_f = 1$, the effective action for the quark can be written in the gauge-invariant way [20]

$$Z_{QCD} = const \int_\mu D_\mu(B) D\Psi D\Psi^+ exp \int dx dy \Psi^+(x)[i\hat{D}\delta(x,y) + iM(x,y)]\Psi(y) \tag{64}$$

where the nonlocal mass operator is

$$M(x,y) = \frac{\varepsilon N}{2N_c V} \int dR i\hat{D} u_+(x-R)\Phi(x,R,y) u_+^+(y-R) i\hat{D} + \\ +(+ \to -) \equiv M_+ + M_- \tag{65}$$

and $\Phi(x,R,y) \equiv \Phi(x,R)\Phi(R,y)$ is product of parallel transporters (3).

Parameter ε is to be defined from the "gap equation", which is gauge invariant and contains confinement.

$$\frac{N}{2} = Tr\left(\frac{M_+ M_+}{-\hat{D}^2 + M_+ M_-}\right) \tag{66}$$

When no confinement is taken into account, $B_\mu = 0$, one can introduce $M(p)$ instead of $M(x,y)$ and (66) becomes [19]

$$\int \frac{d^4 p}{(2\pi)^4} \frac{M^2(p)}{p^2 + M^2(p)} \frac{4VN_c}{N} = 1 \tag{67}$$

with

$$M(p) = \frac{\varepsilon N}{2VN_c} p^2 \varphi^2(p) \tag{68}$$

and $\varphi(p)$ – Fourier transform of the spacial part of $u_0(x)$ (59). One can find $\frac{N}{V} = R^{-4}$ from gluonic condensate [1]

$$\frac{\rho}{R} = \frac{1}{3}, \ \frac{N}{V} = \frac{< G_a G_a >}{32\pi^2} = 1 fm^{-4}, \ \text{and} \ M(p=0) = 345 MeV \tag{69}$$

$M(p)$ is fast decreasing for large p; $< \bar{\Psi}\Psi >= -(255 MeV)^3$ [19].

How confinement modifies this picture? First, the density of instantons $d(\rho)$ is suppressed at large ρ due to the freezing of the coupling constant $\alpha_s(\rho)$ at large distances in the confining background [21]. Rough estimates yield the average instanton size $\rho \approx 0.2fm$ [22].

Second, the density of instantons N/V decreases since now only a part of gluonic condensate is due to instantons.

Keeping $< \bar{\Psi}\Psi >$ at experimental value, $-(250MeV)^3$, one gets roughly [22]

$$R \approx 1.2fm \ , \ \rho = 0.22fm \ \text{and} \ \ M(0) = 0.2GeV. \tag{70}$$

It is interesting that for such small instantons there appears a situation with two scales [23]
chiral scale $R_{ch} \approx \rho \approx 0.2fm$ and confinement scale $R \geq \sigma^{-1/2} \sim 0.5fm - 1fm$.

In the limit $R_c \gg R_{ch}$ one obtains that the mass operator becomes local, and the role of the chiral mass is played by $M(0)$, where $M(p)$ is given in (68). Thus, confinement and chiral effects are separated:
1) chiral mass is created at small distances, $x \sim \rho \sim 0.2fm$ due to quark returns to instantons while passing the vacuum.
2) at large distances, $x > T_g, q$ and $\bar{q}$ form a string, which is described by the Hamiltonian (19) (for $L \leq n_r$) or effective action (17), where now the role of mass in m is played by $M(0) = 0.2GeV$. The situation with chiral mass is the same for more flavours. E.g. for two flavours one makes bosonization (introduces auxiliary scalar and pseudoscalar fiels $\sigma, \eta, \sigma_i, \pi_i$ to disentangle 4 fermion vertices [19]). Integrating out quark and boson fields one gets the gauge invariant effective Lagrangian for pions in the background field B_μ

$$W(\pi) = -Trln(i\hat{D}(B) + i\hat{M}V_5) \tag{71}$$

where $\hat{V}_5 = expi\pi_i\tau_i\gamma_5$, and $\hat{M}$ is given in (66). In the limit $B_\mu \to 0$ one obtains the action studied in [19].

10 Conclusions

Nonperturbative QCD naturally explains confinement and CSB, and through this, the structure of the meson spectrum both in high and low mass region. It is remarkable that QCD forms multiplets, called the QCD- srting multiplets which contain those of bosonic string plus radial excitations – those of QCD_2.

To make this contact with bosonic string one needs string vibrations – and remarkably its QCD counterpart are hybrids.

Their appears a remarkable conspiracy in structure of spectrum which yields through OPE microscopic characteristics of the QCD vacuum – gluonic and quark condensates.

Finally, CSB and confinement work together to provide two distinct scales, and the chiral mass of quarks appears naturally, which enters into Hamiltonian (19). Thus chiral mass is created "before the string appears between quarks".

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References

[1] M.A.Shifman, A.I.Vainshtein and V.I.Zakharov, Nucl. Phys. **B147** (1979) 385,448, Uspekhi Fiz. Nauk **136** (1982) 553.

[2] H.G.Dosch and Yu.A.Simonov, Phys. Lett. B205 (1988) 339.

[3] S.Weinberg, Physica **A96** (1979) 327;
J.Gasser and H.Leutwyler, Ann. Phys. **158** (1984) 142.

[4] A.Di Giacomo, Talk at the Conference Hadron'89, Smolenica, September 24-29, 1989.

[5] Yu.A.Simonov, Nucl. Phys. B307 (1988) 512; B324 (1989) 67. see review in Yad. Fiz. **54** (1991) 192.

[6] A.Di Giacomo and H.Panagopoulos, Phys.Lett. B285 (1992) 133

[7] A.Yu.Dubin, A.B.Kaidalov and Yu.A.Simonov, preprint ITEP 17-93, Yad.Fiz. (in press).

[8] D.R.Stanley and D.Robson, Phys. Lett. **B45**(1980) 235;
P.Cea et al. hys.Rev. **D26** (1982) 1157;
J.Carlson et al. Phys. Rev. **D27** (1983) 233;
N.Isgur and S.Godfrey, Phys. Rev. **D 32** (1985) 189.

[9] P.Cea , G.Nardulli and G.Preparata, Z. Phys. **C16** (1982)135, Phys.Lett. **B115** (1982) 310.

[10] J.L.Basdevant and S.Boukraa, Z. Phys. **C28** (1985)413.

[11] M.G.Olsson, talk at the Como Hadron-93 meeting; C.Olsson, M.G.Olsson and D.La Course, MAD/PH1761.

[12] B.V.Geshkenbein, Yad. Fiz. **49** (1989)1138, **51** (1990)1121;
B.V.Geshkenbein and V.L.Morgunov, prepr. ITEP May 1993.

[13] T.Barnes et al. Nucl. Phys. **B224** (1983) 241;
I.I.Balitsky et al. Phys.Lett. **B112** (1982) 71;
J.Govaerts et al. Nucl. Phys. **B262** (1985) 575;
J.I.Latorre et al. Phys. Lett. **B147** (1984) 169.

[14] M.B.Green, .H.Schwartz and E.Witten, Superstring theory, Cambridge Univ. Press 1987.

[15] T.Banks and A.Casher, Nucl. Phys. **B169** (1980) 102.

[16] G.'tHooft, Phys. Rev. **14** (1976) 3432.

[17] C.G.Callan, R.Dashen and D.J.Gross, Phys. Rev. **D 17** (1978) 717,**D19** (1978) 1826;
D.G.Caldi, Phys. Rev. Lett. **39** (1977) 121;
R.D.Carlitz, Phys. Rev.**D 17** (1978)3225;
E.V.Shuryak, Nucl. Phys. **B203** (1982) 93, 116, 140.

[18] R.Jackiw and C.Rebbi , Phys. Rev. **D 13** (1976) 3398;
R.Callias, Phys.Rev.**D16** (1977) 3068.

[19] D.I.Dyakonov and V.Yu.Petrov, Nucl. Phys. **B272** (1986) 457, preprint LINP 1153 (1986).

[20] Yu.A.Simonov, in preparation.

[21] Yu.A.Simonov, preprint HD-THEP-93-16.

[22] N.O.Agasyan and Yu.A.Simonov, in preparation.

[23] A.Manohar and H.Georgi, Nucl. Phys. **B234** (1984) 189.