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CHIRAL EFFECTS IN THE CONFINING QCD VACUUM

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CHIRAL EFFECTS IN THE CONFINING QCD VACUUM:
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Confining configurations are introduced into the standard instanton vacuum model. This drastically improves theoretical properties of the vacuum: instanton size density $d(\rho)$ stabilizes at $\rho \sim 0.2fm$, all chiral effects are formulated in a gauge-invariant way and quarks are confined. An interesting interplay of chiral and confining dynamics is observed; for the realistic values of parameters the Georgi-Manohar picture emerges with chiral radius $R_{ch} \sim \rho \sim 0.2fm$ much less than confining radius $R_c \sim$ hadron radius $\sim 1fm$. In the limit $R_{ch} \ll R_c$ the chiral mass $M_{ch}(p)$ is unaffected by confinement and can be taken in the local limit $M_{ch}(p=0)$.

Different types of effective chiral Lagrangians (ECL) are obtained, containing all or a part of gluon, quark and Nambu–Goldstone–meson fields. The ECL are manifestly gauge–invariant and in the limit of no gluon fields coincide with those found previously.

The problem of the double role of the pion – as a Goldstone meson or as a $q\bar{q}$ system is briefly disscussed using confining ECL with quarks, mesons and gluons.

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1 Introduction

Effects of chiral symmetry breaking (CSB) and in particular dynamics of Nambu-Goldstone bosons have been studied for a long time even before the discovery of QCD [1].

The effective chiral Lagrangian (ECL) has been introduced [2] before the notion of quarks appeared in physics. Nowadays ECL and chiral perturbation theory is a powerful method [3] and only the loop cut-off problem and numerous phenomenological coefficients of ECL remind that this is an effective theory, not yet deducible from the first principles of QCD.

What is the relation of confinement to chiral physics? At first sight two phenomena are not directly connected. This is explicitly so in the instanton model of chiral vacuum [4,5], where instantons do not bear confinement [6], but create zero modes [7] which are necessary for the quark condensate [8] and a fortiori for CSB [9,10]. At the same time there is a coincidence of chiral and deconfining phase transition temperatures in lattice calculations [11], and some attempts exist [12] to explain the fact in the framework of the instanton model.

At present there are two disconnected approaches in quark physics, which treat confinement or chiral dynamics. The last approach uses ECL and chiral perturbation theory, where the notion of confinement is absent [2,3]. More microscopic among models of this type is the instanton gas or liquid model [4,5]. This model yields microscopic chiral dynamics of quarks and produces ECL with coefficients which are calculable within the model. However quarks are deconfined and gauge invariance in the resulting ECL is violated. This of course is the price for keeping only instantons and disregarding confining degrees of freedom in the vacuum. The first approach is exemplified by the so-called relativistic potential model (RPM) [13], where chiral effects are disregarded fully and all mesons including Nambu-Goldstone bosons are treated as bound state of $q\bar{q}$ interacting via linear potential.

Recently a more general approach has been used for the $q\bar{q}$ system (for a review see [14]), starting from QCD Lagrangian, where due to confinement a string was shown to appear for nonzero orbital momentum L, while at small L one has RPM. In this way in [15] RPM has been derived from QCD.

But chiral as well as spin effects have been disregarded in [15].

In this paper we start to consider systematically chiral and confinement effects on the same footing. To this end we consider QCD vacuum as consisting

of instantons $A_\mu^{(i)}$ plus confining configurations B_μ. Instantons create zero modes, which after being mixed and shifted due interaction give rise to CSB, chiral quark mass and ECL, as it was shown in the instanton liquid model in [10,16].

Confining background, superimposed on instantons, plays double role. First, and very important, it strongly modifies instanton density $d(\rho)$ at large instanton size ρ. This stems from the fact that charge renormalization at large distances in presence of B_μ strongly modifies [17]: logarithmic infrared divergence disappears, and the charge freezes at large distances [17]

$$\alpha_s(R) \approx \frac{4\pi}{b_0 ln \frac{R^{-2}+m^2}{(\Lambda_{QCD})^2}} \tag{1.1}$$

where m is the lowest excitation mass of the order of 1 GeV. As a consequence $g^2(\rho)$ in the renormalized instanton action $S_{inst} = \frac{8\pi^2}{g^2(\rho)}$ does not grow at large ρ and the instanton density converges [18]

$$\frac{d\rho}{\rho^5} d(\rho) = \frac{d\rho}{\rho^5} const [\frac{8\pi^2}{g^2(\rho)}]^{2N_c} e^{-\frac{8\pi^2}{g^2(\rho)}} \tag{1.2}$$

Thus confinement stabilizes the instanton size ρ at some relatively small value $\rho \sim \rho_0$,

$$\rho_0 \sim \frac{1}{m} \sim 1 GeV^{-1} \sim 0.2 fm \tag{1.3}$$

This value is slightly less than obtained in the instanton liquid model [5,10], where the size of instanton is fixed by instanton interaction at $\rho \approx 0.3 fm$. The important point here is that now ρ_0 depends on fundamental properties of the QCD vacuum rather than on model- dependent properties of instanton-instanton interaction. Second, confinement introduces important dynamical effects in calculation of all chiral effects, e.g. in calculation of the coefficients of ECL. In what follows we write a new ECL modified by confinement which is manifestly gauge invariant. Here one can see explicitly how two dynamics, chiral and confining, interplay and in particular one can check some provisions of Georgi and Manohar [19]. It appears that indeed the chiral scale $\Lambda_{CSB} \sim \rho^{-1}$ and the scale of confinement, coinciding with average size of a hadron Λ_{QCD}, are different and one may disentangle in some cases two dynamics. In particular the chiral mass in the limit $\Lambda_{CSB} \ll \Lambda_{QCD}$ becomes a local operator dependent only on chiral dynamics. On larger scale the chiral massive quark interacts with antiquark via a string and this creates a new quark mass – constituent quark mass.

Thus the latter contains three ingredients: (i)current (or Lagrangian) mass, (ii) chiral mass, (iii) string mass. In magnetic moments or spin-dependent forces exactly this constituent mass enters.

Another interesting aspect of the interplay between chiral (instanton) and confining dynamics is the problem of double role of pion – as a Nambu-Goldstone particle or as a bound state of quark and antiquark. We obtain an expression for the $q\bar{q}$ Green's function which contains both chiral and confining effects (i.e. the string due to nonperturbative gluons and pion exchanges) and is an extension of our earlier result [14,20] where only confinement was taken into account, and of earlier result of [16, 21] where only chiral effects have been considered. One can visualize there two possible type of poles, as suggested in [19] when one expands in pion field, and their position is separated (parametrically a Nambu-Goldstone pole at $m = 0$ and $q\bar{q}$ pole at $m_a \sim \sqrt{\sigma}$). We show explicitly that the second type of pole at m_a (a "quark model pole") exactly cancels in the expression for the total Green's function, and only Nambu-Goldstone pole survives, while new poles of unified dynamics appear heavier than m_a. This is in agreement with the scenario suggested by Georgi and Manohar [19].

The paper is organized as follows.

In section 2 we study the quark propagation in the instantonic vacuum in presence of confining background B_μ. Methods of multiple scattering theory in $4d$ are widely used and the framework of the Dyakonov and Petrov approach [10] is modified to include B_μ. In this modification all expressions are manifestly gauge covariant in contrast to the original ones of [10].

Meanwhile we have justified in Appendix A the accuracy of the usually done approximation – keeping only zero mode in the quark Green's function in the field of a given instanton: we show that omitted terms are of the order $m\rho \ll 1$ as compared to the contribution of the zero mode. Another important estimate in Appendix B concerns the shift of the zero mode eigenvalue due to the presence of confining field B_μ. It is shown that the shift $\delta\lambda$ is $\leq 40MeV$ and satisfies $\delta\lambda \cdot \rho \ll 1$, thus making all the method of [10] applicable also for the realistic inclusion of confining field.

In section 3 we are applying another approach using a specific effective action suggested in [21-24] to calculate chiral effects for any number of flavours. To this end a new simple derivation of the effective action is given in Appendix C, which takes into account B_μ. In the same section 3 we consider interplay of chiral and confining effects in the example of the quark chiral mass and

explicitly show that these effects can be disentangled when their ranges are much different.

The case of two flavours is considered in Section 4. Bosonization method of [21] is slightly modified by confinement. One obtains here the same "gap equation" for the chiral mass as for $N_f = 1$.

In section 5 based on bosonization results of the previous section we deduce ECL for quarks, pions and B_μ and also for pions and B_μ only (after integration over quarks).

We also obtain here and in the Appendix D an expression for F_π, which is a modification of that of [10,21] due to confinement. In section 6 we consider the $q\bar{q}$ Green's function and discuss contribution to it from confinement and Nambu-Goldstone modes. Here the double face of pion is discussed and some remarks on the OZI rule and OZI-violating mechanisms are made.

The last section 7 is devoted to summary and an outlook.

2 Quark propagation in the instantonic vacuum with the confining background.

In this Section we shall extend the method of [10] to the case when the QCD vacuum contains in addition to the gas of instantons and antiinstantons also some confining configurations, using mostly the method and notations of ref. [10]. Our goal is to write the quark propagator and effective action in the one-flavour case in the form where the gauge invariance and confinement are present explicitly.

We start with the ansatz for the vacuum

$$A_\mu(x) = \sum_{i=1}^{N} A_\mu^{(i)}(x) + B_\mu(x) \tag{2.1}$$

where $N = N_+ + N_-$ is the total number of instantons and antiinstantons in the 4-volume $V, A_\mu^{(i)}$ is the field of (anti)instanton in the singular gauge and $B_\mu(x)$ is the background confining field which ensures the observed string tension σ through its field correlators $< F_{\mu\nu}(x)F_{\lambda\sigma}(y) >$ etc. [25]. Note that the instanton gas does not provide nonzero string tension,therefore the known value of $\sigma \cong 0.2GeV^2$ fixes the normalization of the background field B_μ.

We define as in [12,26] the total quark Green's function S, "free" Green's function S_0 and the quark propagator $S^{(i)}$ in the i-th (anti)instanton field:

$$S = (-i\hat{D}(A) - im)^{-1}, \quad S^{(i)} = (-i\hat{D}(B + A^{(i)}) - im)^{-1}, \tag{2.2}$$

$$S_0 = (-i\hat{D}(B) - im)^{-1},$$

We also introduce a complete set of real eigenvalues $\lambda_n^{(i)}$ and eigenfunctions $u_n^{(i)}$ on a given center i:

$$-i\hat{D}(B + A^{(i)})u_n^{(i)} = \lambda_n^{(i)} u_n^{(i)}, \quad 1 \le i \le N; 0 \le n < \infty \tag{2.3}$$

So that $S^{(i)}$ can be written as

$$S^{(i)} = \sum_n \frac{u_n^{(i)}(x) u_n^{(i)+}(y)}{\lambda_n^{(i)} - im} \tag{2.4}$$

with the definition

$$t_i \equiv S_0 - S^{(i)} \tag{2.5}$$

one can write the exact equations for the total quark propagator S [26]

$$S = S_0 - \sum_{i,k} Q_{ik} \ , \tag{2.6}$$

$$Q_{ik} = t_i \delta_{ik} - t_i S_0^{-1} \sum_{j \neq i} Q_{jk} \tag{2.7}$$

Eqs.(2.6)-2.7) are exact for the the given decomposition of the gauge field in (2.1). One can now make an approximation for t_i using the fact that the sum over large values of n in (2.4) is close to the "free" Green's function $S_0(x,y)$, since $\lambda_n \sim \sqrt{p^2}$, $n \sim p_\mu$ at large n and $p^2 \gg < B_{\mu\nu}^2 >$. Therefore one can approximate t_i by a finite sum

$$t_i(x,y) = -\sum_{n=0}^{K} \frac{u_n^{(i)}(x) u_n^{(i)+}(y)}{\lambda_n^{(i)} - im} \tag{2.8}$$

In what follows we shall often keep only the lowest term $n = 0$, $\lambda_0^{(i)} \approx 0$, this approximation was exploited in a series of papers [10,23,24]:

$$S^{(i)}(x,y) = S_0(x,y) + \frac{u_0^{(i)}(x) u_0^{(i)+}(y)}{\lambda_0^{(i)} - im} \tag{2.9}$$

We study the accuracy of the approximation (2.9) in Appendix A. We show there that the omitted terms are of the order of $0(m\rho)$ as compared to lowest mode contribution. Another important topic concerns the shift of eigenvalues $\lambda_n^{(i)}$, defined in (2.3-2.4), due to the background field B_μ. We argue in Appendix B, that this shift is insignicant for the realistic values of gluonic

condensate and instanton radius ρ. In particular, $(\lambda_0^{(i)})^2$ is of the order of $(30-40MeV)^2$ and therefore does not spoil the approximation (2.9).

Insertion of the separable form (2.8) into (2.7) yields the following solution for S:

$$S(x,y) = S_0(x,y) - \sum_{i,k;n,m} u_n^{(i)}(x)(im - \tilde{\lambda} + \tilde{V})^{-1}_{nm,ik} u_m^{(k)+}(y) \tag{2.10}$$

where we have defined the matrices $\lambda_{nm,ik}, V_{nm,ik}$

$$\lambda_{nm,ik} = \delta_{ik}\delta_{nm}\lambda_n^{(i)} \tag{2.11}$$

$$\tilde{V}_{nm,ik} = \int u_n^{(i)+}(z)(-i\hat{D}(B) - im)u_m^{(k)}(z)d^4z \tag{2.12}$$

by definition $\tilde{V}_{nm}, (i=k) \equiv 0$.

The form (2.10) coincides with that used in [10], when one keeps in the sum only $n = m = 0$, $\lambda_0 = 0$.

In the rest of this section we shall calculate the quark Green's function S averaging (2.10) over vacuum configurations B_μ and (anti) instanton positions and orientations. We shall follow here the direct approach of [10], which makes explicit the appearing of the chiral mass of the quark, while in the next sections we follow another approach [21-24], where the notion of the effective action is introduced from the beginning. The new element which we shall obtain in this section is the gauge covariant form of the quark propagator with confinement taken into account.

For the latter and for the averaging over B_μ and $A_\mu^{(i)}$ one must define a physical gauge-invariant quantity associated with $S(x,y)$ (which itself is not gauge invariant).

The simplest quantity is the Green's function of one light and one heavy quark in the limit when the heavy mass is infinitely large.

$$G_{HL}(x,y) = < tr\Gamma S(x,y)\Gamma\Phi(y,x) >_{A_\mu^{(i)},B_\mu} \tag{2.13}$$

Here tr is a trace over color and Lorentz indices, $\Gamma = \gamma_5, \gamma_5\gamma_\mu, 1, \gamma_\mu$ etc., and

$$\Phi(y,x) = Pexp\ \ ig\int_x^y B_\mu(z)dz_\mu \tag{2.14}$$

The integral in (2.14) is taken along the straight line connecting x and y- this is the remnant of the heavy quark Green's function.

The angular brackets in (2.13) denote averaging over fields $B_\mu, A_\mu^{(i)}$ defined as follows.

$$< 0(A_\mu^{(i)}, B_\mu) >_{A_\mu^{(i)}, B_\mu} \equiv \int \prod_{i=1}^{N} \frac{d^4 R^{(i)}}{V} d\Omega_i d\mu(B) 0(A_\mu^{(i)}, B_\mu) \tag{2.15}$$

where R_i is the position of (anti)instanton, Ω_i is its color orientation, and $d\mu(B)$ is the standard integration measure for the field B_μ, the specific form of it is not needed for our purposes.

The total gauge transformation for the field A_μ

$$A_\mu(x) \to U(x)(A_\mu(x) + \frac{i}{g}\partial_\mu)U^+(x) \tag{2.16}$$

can be conveniently split into a homogeneous one for $A_\mu^{(i)}$ and inhomogeneous for B_μ

$$A_\mu^{(i)} \to U(x) A_\mu^{(i)} U^+(x) \tag{2.17}$$

$$B_\mu(x) \to U(x)(B_\mu(x) + \frac{i}{g}\partial_\mu)U^+(x) \tag{2.18}$$

From(2.1) and (2.17-2.18) one returns back to (2.16). The color orientation for $A_\mu^{(i)}$ can be made explicit using

$$A_\mu^{(i)}(x) = \Omega_i \bar{A}_\mu^{(i)} \Omega_i^+ \tag{2.19}$$

where $\bar{A}_\mu^{(i)}$ is the standard singular gauge form [7,27]

$$\bar{A}_\mu^{(i)}(x) = \frac{2}{g}\bar{\eta}_{a\mu\nu}(x - R^{(i)})_\nu \frac{\rho^2}{(x - R^{(i)})^2[(x - R^{(i)})^2 + \rho^2]} \tag{2.20}$$

It is clear from (2.17) that under global gauge transformation $\bar{U}$ the constant matrix Ω_i is simply "rotated"

$$\Omega_i \to \bar{U}\Omega_i \tag{2.21}$$

We now turn to eigenfunction $u_n^{(i)}$. Under gauge transformation it transforms as

$$u_n^{(i)}(x) \to U(x) u_n^{(i)}(x) \tag{2.22}$$

To make gauge dependence in $u_n^{(i)}$ explicit, we write it in the form

$$u_n^{(i)}(x) = \Phi(x, R^{(i)}) \Omega_i \varphi_n(x - R^{(i)}) \tag{2.23}$$

where

$$\Phi(x, R^{(i)}) = Pexp(ig \int_{R^{(i)}}^{x} B_\mu dz_\mu), \tag{2.24}$$

and φ_n is the form of solution in the singular gauge, e.g. for the (anti)instanton zero-mode solution one has [7,28]

$$\varphi_0(x) = \bar{\varphi}(x) v^{\pm}_{\alpha m},$$
$$\bar{\varphi}(x) = \frac{1}{\pi}\frac{\rho}{(x^2+\rho^2)^{3/2}} \quad \frac{x_\mu \gamma_\mu}{\sqrt{x^2}} \tag{2.25}$$

with $v^{\pm}_{\alpha m} = \frac{1}{\sqrt{2}}\varepsilon^{\alpha m}\binom{1}{\pm 1}$ and α, m spin and color $SU(2)$ indices, and + and − referring to the instanton and antiinstanton zero mode respectively.

Since $\Phi(x, R^{(i)})$ transforms as

$$\Phi(x, R^{(i)}) \to U(x)\Phi(x, R^{(i)})U^+(R^{(i)}) \tag{2.26}$$

one can satisfy (2.22) imposing on Ω_i the transformation law consistent with (2.21)

$$\Omega_i \to U(R^{(i)})\Omega_i \tag{2.27}$$

It is clear now that $V_{nm,ik}$ (2.12) is gauge invariant, while $S(x,y)$ in (2.10) transforms in standard way:

$$S(x,y) \to U(x)S(x,y)U^+(y) \tag{2.28}$$

and G_{HL} (2.13) is gauge invariant. We do now a drastic approximation as in [10] to keep in (2.10) only terms with $n = m = 0$, and the lowest eigenvalue λ_0. In accordance with [10] we define

$$\left(\frac{1}{i\bar{m} + \hat{V}}\right)_{ik} = \frac{\delta_{ik}}{i\bar{m}} + \begin{cases} D_{ik}(R^{(i)}, R^{(k)}, \Omega_i, \Omega_k), i, k \text{ . of one tipe} \\ P_{ik}(R^{(i)}, R^{(k)}, \Omega_i, \Omega_k), i, k \text{ of diff. types} \end{cases} \tag{2.29}$$

where i, k of one type means both i and k are instantons or both antiinstantons and we have used notation $i\bar{m} = im - \lambda_0$. Following the same line of reasoning as in [10] we obtain the following equations for D, P where we suppress arguments for simplicity

$$P_{ik} = -\frac{1}{i\bar{m}}V_{ik}\frac{1}{i\bar{m}} - \frac{N}{2V}\int d^4R^{(i)}d\Omega_j \frac{1}{i\bar{m}}V_{ij} \times \frac{1}{1 - i\bar{m}\delta}D_{jk} \tag{2.30}$$

$$D_{ik} = -\frac{N}{2V}\int d^4R^{(j)}d\Omega_j \frac{1}{i\bar{m}}V_{ij}\frac{1}{1 - i\bar{m}\delta}P_{jk} \tag{2.31}$$

Several comments are in order. First, P_{ij} and D_{ij} can be considered as probability amplitudes for a quark to travel from a center i to a center j with all possible centers being on its way. Both P_{ij} and D_{ij} are defined not to contain

(infinitely many, in principle) returns to the centers i, j, and those should be accounted for separately (note that (2.29) accordingly is a precise definition only for paths without returns).

This is done introducing the factor

$$(1 - i\bar{m}\delta)^{-1} = \sum_{n=0}^{\infty}(i\bar{m}\delta)^n \tag{2.32}$$

where $i\bar{m}\delta$ is the amplitude of returning to the center only once,

$$\delta \equiv D_{ii}(R^{(i)}, R^{(i)}, \Omega_i, \Omega_i) \tag{2.33}$$

Second, averaging over all $R^{(j)}, \Omega_j$ with $j \neq i, k$ is assumed in D_{ik}, P_{ik}. This can be factorized out only in the limit $N_c \to \infty$ and this limit is assumed everywhere below.

It is convenient to factorize explicitly the dependence on Ω in D_{ik}, P_{ik}

$$D_{ik} = v^{+}_{\alpha,m}(\Omega_i^{+})_{m\nu} d^{ik}_{\nu\nu'}(\Omega_k)_{\nu'\rho} v_{\rho\beta_j} \tag{2.34}$$

$$P_{ik} = v^{+}_{\alpha,m}(\Omega_i^{+})_{m\nu} f^{ik}_{\nu\nu'}\Omega v \tag{2.35}$$

$$V_{ik} = v^{+}_{\alpha,m}(\Omega_i^{+})_{m\nu} V^{ik}_{\nu\nu'}\Omega v \tag{2.36}$$

From (2.34) one concludes that the δ does not depend on i in (2.33) and is a gauge-invariant quantity ($N_c \to \infty$), if one exploits the fact that it is diagonal in spin indices $\alpha, \beta (cf(2.25))$

$$\begin{aligned} \delta &= \frac{1}{2} Tr_\alpha (D_{ii}(R^{(i)}, R^{(i)}, \Omega_i, \Omega_i)) = \\ &= \frac{1}{2} Tr_\alpha v^{+}_{\alpha m}(\Omega_i)_{m\nu} d^{ii}_{\nu\nu'}(\Omega_i)_{\nu'\rho} v_{\rho\alpha} = \\ &= \frac{1}{4} Tr_\nu d^{ii}_{\nu\nu} \end{aligned} \tag{2.37}$$

The color trace appearing in the last equality in (2.37) signals the gauge invariance of δ. We are interested in the limit $m \to 0$, therefore we introduce the reduced quantities $\bar{d}, \bar{f}, \bar{v}$ where dependence on $\bar{m}$ and $\frac{N}{V}$ is made explicit,

$$\begin{aligned} d^{ik} &= \frac{2VN_c}{N\varepsilon}\frac{i}{\bar{m}^2}\bar{d}^{ik}, \quad f^{ik} = \frac{2VN_c}{N\varepsilon}\frac{i}{\bar{m}^2}\bar{f}^{ik} \\ V^{ik} &= \frac{2VN_c}{N\varepsilon i}\bar{V}^{ik}; \quad \varepsilon \equiv \frac{1}{\bar{m}(1 - i\bar{m}\delta)} \end{aligned} \tag{2.38}$$

and obtain equations not containing m any more

$$\bar{f}^{ik} = -\bar{V}^{ik} + \int \bar{V}^{ij} dR^{(j)} \bar{d}^{jk} \qquad (2.39)$$
$$\bar{d}^{ik} = \int \bar{V}^{ij} dR^{(j)} \bar{f}^{jk}$$

The fundamental role in the chiral mass generation is played by the so-called "consistency equation" [10] which in our notations is

$$Sp_c \bar{d}^{ii} = \frac{2N}{VN_c} \qquad (2.40)$$

Using (2.39) it can also be written as

$$\frac{1}{\varepsilon} = \frac{1}{4}\{\frac{N}{2VN_c}\varepsilon Sp_c(V_{ik}V_{ki}) + \qquad (2.41)$$
$$+ \; 0(\frac{N}{2VN_c})^2 \varepsilon^2 tr(VVV)\}$$

The first term in (2.41) can be written explicitly:

$$Sp_c(V_{ik}V_{ki}) \equiv Sp_c \int d^4z d^4z' d^4R^{(k)} \bar{\varphi}^+(z - R^{(i)}) \Phi(R^{(i)}Z) i\hat{D}\Phi(z, R^{(k)} \times \qquad (2.42)$$
$$\times \bar{\varphi}(z - R^{(k)}) \cdot \bar{\varphi}^+(z' - R_k) \Phi(R^{(k)}, z') i\hat{D}\Phi(z', R^{(i)}) \bar{\varphi}(z' - R(i))$$

We note that d^{ik}, f^{ik} and V^{ik} as well as $\bar{d}^{ik}$, $\bar{f}^{ik}$, $\bar{V}^{ik}$ are gauge covariant and transform as

$$(f^{ik}, V^{ik}, d^{ik}) \to U(R^{(i)})(f^{ik}, V^{ik}, d^{ik}) U^+(R^{(k)}) \qquad (2.43)$$

In contrast to that δ and ε are gauge invariant, as can seen in (2.37) and (2.42).

The measurable physical quantity associated with f, d is the heavy-light Green's function G_{HL} and we use (2.10), (2.13), (2.29) to write it in the form similar to that of Eq.(40) from [10] (we remind the reader that D^{ik} and F^{ik} are defined without returns to the centers i and k and therefore the returns, i.e. factors like $\frac{1}{1-im\delta}$, should be inserted in (2.10)).

$$< G_{HL}(x,y) >_{B,A^{(i)}} = < \Phi(y,x)\Gamma S_0(x,y)\Gamma > -$$
$$-\frac{N}{2VN_c}(\frac{1}{i\bar{m}} + \frac{\delta}{1 - i\bar{m}\delta})$$
$$\times < \Gamma\Phi(y,x)\Gamma\bar{\varphi}(x - R_i)\Phi(x, R_i)\Phi(R_i, y)\bar{\varphi}(y - R_i) >_{B,R_i}$$

$$-(\frac{N}{2VN_c})^2(\frac{1}{1-i\bar{m}\delta})^2 \tag{2.44}$$
$$\times < \Gamma\Phi(y,x)\Gamma\bar{\varphi}(x-R_i)\Phi(x,R_i)d_{ik}(R_i,R_k)\Phi(R_k,y)\bar{\varphi}(y-R_k) >_{B,R_i,R_k}$$
$$+(inst. \rightarrow antiinst.)$$
$$-(\frac{N}{2VN_c})^2(\frac{1}{1-i\bar{m}\delta})^2$$
$$\times < \Gamma\Phi(y,x)\Gamma\bar{\varphi}(x-R_i)\Phi(x,R_i)f_{ik}(R_i,R_k)\Phi(R_k,y)\bar{\varphi}(y-R_k) >_{B,R_{ij},R_k}$$
$$-(inst. \rightarrow antiinst.)$$

When the field B_μ is put equal to zero, one comes back to the situation studied in [10]. In this case it is useful to work in the momentum space and one obtains as in [10]

$$S(p) \sim < G_{HL}(p) >= \frac{i\hat{p}+M(p)}{p^2+M^2(p)} \tag{2.45}$$

with

$$M(p) = \frac{\varepsilon N}{2VN_c}p^2\bar{\varphi}^2(p) \tag{2.46}$$

and the equation for ε ("consistency equation", which one can call also the "gap equation") obtains from (2.40) putting all $\Phi = 1$,

$$\frac{4VN_c}{N}\int\frac{d^4p}{(2\pi)^4}\frac{M^2(p)}{M^2(p)+p^2} = 1 \tag{2.47}$$

This is the equation (36) of ref. [10] in the limit $m \rightarrow 0$.

The corresponding partition function can be written as [21]

$$Z_{QCD}(N_f=1) = const\int D\psi D\psi^+ exp[-\int\frac{d^4p}{(2\pi)^4}\psi^+(p)(\hat{p}-iM(p))\psi(p)] \tag{2.48}$$

Note that the resulting theory of massive quarks with the chiral mass $M(p)$ is not gauge invariant, as is clearly seen in (2.48). This serious defect is cured in (2.44) which is fully gauge invariant, but the notion of the quark Green's function $S(p)$ and the chiral mass can not be made gauge invariant in contrast to G_{HL}.

3 Method of effective action -the case of one flavour

In this section we adopt another procedure – the method of effective action suggested in [21-24]. One can prove [24] that this method yields the same Green's function as in the more direct but tedious method of [10], which we have used in the previous section.

The proof given in [24] contains intermediate formulas which are diverging for $m \to 0$, i.e. the partition function $Z \sim m^{-NN_f}$. Therefore we give a different derivation in Appendix C and check at each point the accuracy of physical approximations. In particular we show in Appendix C that there is no divergency for $m \to 0$ and that leading term in the limit $m\rho \ll 1$ is the same as obtained in [24] when $B_\mu = 0$.

As shown in Appendix C the QCD partition function for quarks in the field (2.1) can be written as (in the limit $m \to 0$)

$$Z_{QCD} = \int D\psi D\psi^+ D\mu(B) e^{\int d^4x \psi_f^+ i\hat{D}(B)\psi_f} \times$$

$$\times < \prod_{f=1}^{N_f} \prod_{i=1}^{N} (2im_f - \int d^4x \psi_f^+ i\hat{D} u_0^{(i)}(x) \int d^4y u_0^{(i)+}(y) i\hat{D}\psi_f(y)) >_{R_i,\Omega_i} \quad (3.1)$$

Here f refers to the flavour (total number of flavours is N_f) the meaning of $2im_f$ is explained in Appendix C eq. C.8, and $u_0^{(i)}(x)$ is defined in (2,23) and we did not specify the difference between instantonic and antiinstantonic zero modes, which should be kept in mind in (3.1).

We now consider systematically the cases of $N_f = 1$ in this section and the case of $N_f = 2$ in the next section.

The case of $N_f = 1$.

In this case the averaging procedure over R_i, Ω_i factorizes in (3.1) and one can introduce a two-fermion vertex (to stress the analogy with [21] we keep notations of that paper)

$$Y_\pm = \int dR_i d\Omega_i \int \psi^+(x) i\hat{D} u_0^{(i)}(x) d^4x \int d^y u_0^{(i)^+}(y) i\hat{D}\psi(y) =$$

$$= \int dR_i \psi^+(x) i\hat{D}\bar{\varphi}(x - R_i) \frac{1}{2}(1_\mp \gamma_5) \Phi(x, R_i, y) \bar{\varphi}^+(y - R_i) i\hat{D}\psi(y) dy dx \quad (3.2)$$

where $\Phi(x, R_i, y) \equiv \Phi(x, R_i)\Phi'(R_i, y)$

Following [21] we introduce identically integrations over $\lambda_+, \lambda_-, \Gamma_+, \Gamma_-$ to obtain

$$Z_{QCD} = \int D\mu(B) D\psi D\psi^+ \int_{-\infty}^{\infty} \frac{d\lambda_+}{2\pi} \int_{-\infty}^{\infty} \frac{d\lambda_-}{2\pi} \int_{-\infty}^{\infty} d\Gamma_+ \int_{-\infty}^{\infty} d\Gamma_- exp W \quad (3.3)$$

$$W = \int d^4x\psi^+ i\hat{D}\psi + i\lambda_+(Y_+ - \Gamma_+) + i\lambda_-(Y_- - \Gamma_-) + N_+ \ln\frac{\Gamma_+}{V} + N_- \ln\frac{\Gamma_-}{V}$$

Integrating over Γ_+, Γ_- by the steepest descent method in the thermodynamical limit $N_\pm \to \infty, V \to \infty, \frac{N_\pm}{V} = const.$, one obtains

$$Z_{QCD} = \int D\mu(B)\frac{d\lambda_+}{2\pi}\frac{d\lambda}{2\pi}D\psi D\psi^+ exp\bar{W} \tag{3.4}$$

$$\bar{W} = N_+ \ln\frac{N_+}{i\lambda_+ V} - N_+ + N_- \ln\frac{N_-}{i\lambda_- V} - N_- + \int d^4x\psi^+ i\hat{D}\psi + i\lambda_+ Y_+ + i\lambda_- Y_-$$

The integation over $d\lambda_+ d\lambda_-$ can be also done by the steepest descent method when $N_\pm$ is large [21].

Before doing that we integrate over ψ, ψ^+; using notation

$$Y_\pm = \psi^+ \frac{1 \mp \gamma_5}{2}\bar{Y}_\pm \cdot \psi \tag{3.5}$$

we obtain

$$Z_{QCD} = \int D\mu(B)\frac{d\lambda_+}{2\pi}\frac{d\lambda_-}{2\pi}exp(W' + W''), \tag{3.6}$$

$$W' = N_+ \ln\frac{N_+}{i\lambda_+ V} - N_+ + N_- \ln\frac{N_-}{i\lambda_- V} - N_- \tag{3.7}$$

$$W'' = \ln det\begin{pmatrix} i\hat{D} & i\bar{Y}_+\lambda_+ \\ i\bar{Y}_-\lambda_- & i\hat{D} \end{pmatrix} = \tag{3.8}$$

$$Tr\ln(-\hat{D}^2 + \bar{Y}_+\bar{Y}_-\lambda_+\lambda_-),$$

where Tr means trace over coordinates, color, and Lorentz indices. Integrating now over $d\lambda_+ d\lambda_-$ one finds λ_+, λ_- in the extremum of $(W' + W'')$ to satisfy equations

$$\frac{N_+}{\lambda_+} = Tr\{\bar{Y}_+\bar{Y}_-\lambda_-(-\hat{D}^2 + \bar{Y}_+\bar{Y}_-\lambda_+\lambda_-)^{-1}\} \tag{3.9}$$

$$\frac{N_-}{\lambda_-} = Tr\{\bar{Y}_+\bar{Y}_-\lambda_+(-\hat{D}^2 + \bar{Y}_+\bar{Y}_-\lambda_+\lambda_-)^{-1}\}$$

For $N_+ = N_- = \frac{N}{2}$, one finds $\lambda_+ = \lambda_- = \frac{\varepsilon N}{2VN_c}$, where ε satisfies the "gap equation"

$$\frac{N}{2} = (\frac{\varepsilon N}{2VN_c})^2 Tr(\frac{\bar{Y}_+\bar{Y}_-}{-\hat{D}^2 + \bar{Y}_+\bar{Y}_-(\frac{\varepsilon N}{2VN_c})^2}) \tag{3.10}$$

Since according to (3.2), (3.5), one has for $\bar{Y}_\pm$,

$$\bar{Y}_\pm = \int dR_i i\hat{D}\bar{\varphi}(x - R_i)\Phi(x, R_i, y)\bar{\varphi}(y - R_i)i\hat{D} \tag{3.11}$$

the r.h.s. of (3.10) and hence ε is gauge invariant (only closed "contours" of $\Phi(x, R_i, y)$ enter under the sign of Tr).

In case when $B_\mu = 0$, $\Phi = 1$, $\hat{D} \to \hat{\partial}$, (3.11) reduces to

$$Y_\pm(x,y) = \int \frac{dp}{(2\pi)^4} e^{ip(x-y)} (\hat{p}\bar{\varphi}_\pm(p))^2 \tag{3.12}$$

and (3.10) becomes the same as (2.47)

$$\int \frac{d^4p}{(2\pi)^4} \frac{M^2(p)}{p^2 + M^2(p)} \frac{4VN_c}{N} = 1 \tag{3.13}$$

with $M(p)$ given by (2.46).

Now back to the case $B_\mu \neq 0$. We integrate in (3.4) over $d\lambda_+ d\lambda_-$ first, replacing in $\bar{W}$ the $\lambda_\pm$ by their extremal values (3.9) We obtain the effective action for quarks in the form ($N_f = 1$)

$$Z_{QCD} = const \int D\mu(B) D\psi D\psi^+ exp \int dx dy \psi^+(x) [i\hat{D} \cdot \delta(x-y) + iM(x,y)] \psi(y) \tag{3.14}$$

where the nonlocal mass operator is

$$M(x,y) = \frac{\varepsilon N}{2VN_c} \int dR_i i\hat{D}\bar{\varphi}(x - R_i)\Phi(x, R_i, y)\bar{\varphi}^+(y - R_i) i\hat{D} \tag{3.15}$$

and ε is to be defined from (3.10).

Note, that the effective action in (3.14) is now gauge invariant in contrast to (2.48), since from (3.15) one deduces that under gauge transformations $M(x,y)$ changes as

$$M(x,y) \to U(x) M(x,y) U^+(y) \tag{3.16}$$

$M(x,y)$ as given by (3.15) is what one may call the chiral mass operator. Inclusion of background field B_μ makes it gauge covariant, but now $M(x,y)$ is dependent on the confining forces, more explicitly, $M(x,y)$ contains the field B_μ and therefore depends on the string which connects quark with antiquark (or with the string junction in baryon).

To study this dependence more explicitly consider again the heavy-light Green's function, calculated with the help of (3.14):

$$< G_{HL}(x,y) >_B = \int D\psi D\psi^+ D\mu(B) \psi(y) \Phi(y,x) \psi^+(x) e^{\psi^+(\hat{D}+\hat{M})\psi} = \tag{3.17}$$

$$= tr_c(\Phi(y,x)(i\hat{D} + i\hat{M})^{-1}_{xy})$$

We have neglected additional $q\bar{q}$ pairs (quenched approximation or large N_c limit). Using the Feynman-Schwinger representation [14] (3.17 can be rewritten as

$$< G_{HL}(x,y) >_B = < tr_c(i\hat{D} - i\hat{M}) \int_0^\infty ds Dz e^{-K} \psi_z(y,x)\Phi(y,x) >_B \quad (3.18)$$

where

$$\psi_z(x,y) = P(e^{-s(\hat{M}^2 - \Sigma F)}\Phi_z(x,y)) \ , \ K = \int_0^s \frac{\dot{z}^2(\lambda)}{4} d\lambda \ , \ \Sigma F = \frac{1}{4}\sigma_{\mu\nu}F_{\mu\nu} \quad (3.19)$$

and the ordering operator P ensures the proper insertion of the mass operator $\hat{M}^2$ into the phase factor $\Phi_z(x,y)$ where the subscript z refers to the contour of Φ_z taken along the quark path.

In (3.18) the field B_μ enters $\hat{M}$ and the closed contour C formed by the straight line and the quark path between x and y, as shown in Fig.1, together with insertions of the mass operator.

The dynamics in (3.18) is defined by the Wilson loop with mass insertions, shown in Fig. 1, which obeys the area law:

$$< \psi_z(x,y)\Phi(y,x) > \equiv < W_M(C) > \sim exp(-S_M \cdot \sigma) \quad (3.20)$$

where S_M is the minimal area of the contour C with mass insertions. Eq. (3.20) means that a string is formed between light and heavy quark trajectories and this string is influenced by the insertions of the mass operator.

To understand better these insertions one may look at the quark propagator and expand it in powers of the mass operator:

$$(i\hat{D} + i\hat{M})^{-1}_{xy} = (i\hat{D})^{-1}_{xy} - (i\hat{D})^{-1}_{xu} d^4u i\hat{M}(u,v) d^4v (i\hat{D})^{-1}_{xy} \quad (3.21)$$

$$+(i\hat{D})^{-1}_{xu} d^4u i M(u,v) d^4v (i\hat{D})^{-1}_{vt} d^4t i M(t,w) d^4w (i\hat{D})^{-1}_{wy} - ...$$

The nonlocality of $M(x,y)$ as can be seen in (3.15) is of the order of the ρ - average size of (anti)instantons (the integrand in (3.15) behaves as $\frac{1}{|x-R_i|^4}\frac{1}{|y-R_i|^4}$ at $(|x-R_i|, |y-R_i| \gg \rho)$. At the same time the average size of a hadron in (3.18) - the width of the contour C in Fig. 1 - is of the order of the confinement radius $R_c \sim 0.5fm$ for lowest states or larger than R_c for excited states.

Therefore we have to distinguish two cases.

i)$\rho \geq R_c$. In this case the nonlocality of $\hat{M}$ is strongly influenced and interrelated with the dynamics of the string - one cannot separate effects of chiral

symmetry braking (the chiral mass $\hat{M}$) and confinement.
ii)$\rho \ll R_c$. In this case we can effectively replace in (3.15) $\Phi(x, R_i, y)$ by $\Phi(x, y)$ and hence rewrite $\hat{M}$ as

$$M(x,y) = \Phi(x,y) \int M(p) e^{ip(x-y)} \frac{d^4p}{(2\pi)^4} \tag{3.22}$$

Furthermore we can neglect nonlocality of $M(x,y)$ on the scale of $(x-y) \sim R_c$ and replace the operator $M(x,y)$ by

$$M(x,y) \approx \delta(x-y) \int d(x-y) M(x,y) = \delta(x-y) M(0) \tag{3.23}$$

Introduction of (3.23) into (3.14) yields now a gauge-invariant expression with constant chiral mass and eq.(3.18) assumes the form

$$< G_{HL}(x,y) >= tr(i\hat{D} - iM(0)) \int ds Dz e^{-K - sM^2(0)} e^{-\sigma S} \tag{3.24}$$

where the effects of CSB and confinement are separated. The first produced the chiral mass $M(0)$ which enters instead of current mass, and the latter ensures the string dynamics between the now massive light quark and heavy antiquark. We conclude this Section with discussion of the quark condensate. From (3.14) we have

$$< \bar{q}q >_M = itr < (-i\hat{D} - i\hat{M})^{-1}_{xx} >_B \tag{3.25}$$

In the case $B_\mu = 0$ (3.25) reduces to

$$< \bar{q}q >_M = - \int \frac{d^4p}{(2\pi)^4} \frac{M(p)}{p^2 + M^2(p)} \tag{3.26}$$

where $M(p)$ is the Fourier transform of (3.15) when $\Phi \equiv 1$, and is the same as in (2.46). We note that the form (3.26) coincides with that given in [10,21].

When $B_\mu \neq 0$ one should calculate the original expression (3.25) which can be rewritten with the help of (3.24) as

$$< \bar{q}q >_M = - < \int_0^s tr\hat{M} Dz e^{-K} \psi_C(x,x) >_B \tag{3.27}$$

where the contour C in ψ_C is a closed trajectory of the quark, to be integrated over in Dz. From (3.26) one can notice that the integral is effectively defined by the distances of the order of the radius of instanton ρ. When this radius is assumed to be much smaller than the confinement radius R_c (the case ii) above), then the effects of confinement are unimportant and give only a small

correction to (3.26). Thus we have two different situations: in computing $q\bar{q}$ system (wave functions and masses) one can use the inequality $\rho \ll R_c$ and keep only $M(0)$ instead of $M(p)$ in the first approximation in Green's function; in computing $< \bar{q}q >$ on the other hand one should keep dependence $M(p)$ and can neglect in the first approximation effects of confinement.

4 The case of two flavours

In the general case of N_f flavours in the effective action in (3.1) there enters a $2N_f$ vertex [21,24]

$$Y_{\pm} = (-)^{N_f} \int d^4R_i d\Omega_i \prod_{f=1}^{N} \int d^4x \psi_f^{+} i\hat{D}u^{(i)} \int d^4y u^{(i)}(y) i\hat{D}\psi_f(y) \tag{4.1}$$

Integrating over $d\Omega_i$ for $N_f = 2$ and taking the limit $N_c \to \infty$ one obtains

$$Y_{\pm} = \int d^4R det J_{\pm}(R) \tag{4.2}$$

where

$$(J_{\pm}(R))_{fg} = \int dx dy \psi_f^{+}(x) \frac{1}{2}(1 \mp \gamma_5) K(x,y,R) \psi_g(y) \tag{4.3}$$

and

$$K(x,y,R) = i\hat{D}\bar{\varphi}(x-R)\Phi(x,R,y)\bar{\varphi}^{+}(y-R)i\hat{D} \tag{4.4}$$

The interaction (4.2) is reminiscent of the 'tHooft determinantal interaction [7], and can be also compared to the similar term deduced in [28].

The difference in our case, as also in the $B_\mu = 0$ case of [21,28], is that our interaction is nonlocal with nonlocality of the order of ρ – size of instanton. In addition, in contrast to [21,24] the term (4.2) is gauge invariant and takes into account effects of confinement.

The effective partition function similarly to (3.4) is given by (up to unessential factors and replacing $i\lambda_{\pm} \to g_{\pm}$, $R \to u$

$$Z_{QCD} \sim \int dg_{+} \int dg_{-} D\psi D\psi^{+} exp W_2$$

$$W_2 = \int \{ \sum_{f=1}^{2} \psi_f^{+} i\hat{D}\psi_f(u) + g_{+} det J_{+}(u) + g_{-} det J_{-}(u) \} du \tag{4.5}$$
$$- N_{+} ln g_{+} - N_{-} ln g_{-}$$

One can introduce as in [21] the 2×2 flavour matrices τ_a^-, $\tau_a^- = (\vec{\tau}, i), a = 1, ...4$, and use the identity

$$(\tau_a^-)_{fg}(\tau_a^-)_{f'g'} = -2\varepsilon_{fg'}\varepsilon_{gf'} \tag{4.6}$$

One can exploit the Hubbard-Stratonovich tranformation and write Z_{QCD} through additional functions L_a, R_a [21] to obtain

$$Z_{QCD} = \int D\mu(B) dg_+ \int dg_- \int D\psi D\psi^+ DL_a DR_a exp\bar{W}_2 \tag{4.7}$$

$$\bar{W}_2 = -N_+ lng_+ - N_- lng_- + \int d^4x [\psi_f^+ i\hat{D}\psi_f + \\ + g_+ L_a^2(x) + g_- R_a^2(x) + 2g_+ L_a(x) J_+^a(x) + \\ + 2g_- R_a(x) J_-^a(x)]$$

where

$$J_\pm^a(x) = (\tau_a^-)_{fg}(J_\pm(x))_{fg} \tag{4.8}$$

The effective action $\bar{W}_2$ contains both fermionic degrees of freedom ψ, ψ^+ and bosonic L_a, R_a, and global parameters g_+, g_-. Integrating over $D\psi D\psi^+$ we get a fully bosonic effective action

$$e^{-W(L,R)} = \int dg_+ dg_- e^{-N_+ lng_+ - N_- lng_- + \int d^4y (g_+ L_a^2(y) + g_- R_a^2(y))} exp X; \tag{4.9}$$

$$X = ln\ det \begin{pmatrix} i\hat{D} & 2g_+ \hat{L}K \\ 2g_- \hat{R}K & i\hat{D} \end{pmatrix}, \tag{4.10}$$

where $\hat{L} = L_a \tau_a^-$, $\hat{R} = R_a \tau_a^-$, and e.g.$(\hat{L}K)_{xy} = \int \hat{L}(u) K(x, y, u) du$.

We shall now study following [21] the phenomenon of CSB using variables L_a, R_a. We shall look for the condensate of L_4, R_4, namely introducing

$$\sigma(x) = L_4(x) + R_4(x), \tag{4.11}$$

we show that

$$< \sigma > \neq 0, < L_4 - R_4 > = < L_i > = < R_i > = 0 \tag{4.12}$$

Since $\hat{L}$ and $\hat{R}$ transform under $SU_L(N_f) \times SU_R(N_f)$ as

$$\hat{L} \to U_L \cdot \hat{L} U_R^+, \quad \hat{R} \to U_R \hat{R} U_L^+ \tag{4.13}$$

the nonzero value of $< \sigma >$ signals CSB.

The insertion of (4.11) , (4.12) into (4.10) yields

$$X = Tr\ ln(-\hat{D}^2 + g_+g_-\sigma^2K^2) \tag{4.14}$$

The integration over $dg_+dg_-d\sigma$ via the steepest descent method yields equations for determination of $g_+^{(0)}, g_-^{(0)}, \sigma^{(0)}$

$$\frac{\partial W}{\partial \sigma} = -\frac{g_+ + g_-}{2}V\sigma - 2\sigma g_+g_-Tr(\frac{K^2}{-\hat{D}^2 + g_+g_-\sigma^2K^2}) = 0 \tag{4.15}$$

$$\frac{\partial W}{\partial g_\pm} = +\frac{N_\pm}{g_\pm} - \frac{V\sigma^2}{4} - \sigma^2 g_\mp Tr(\frac{K^2}{-\hat{D}^2 + g_+g_-\sigma^2K^2}) = 0$$

From (4.15) we find that $g_0 = -\frac{N}{V\sigma_0^2}$, and

$$g_0^2\sigma_0^2Tr(\frac{K^2}{-\hat{D}^2 + g_0^2\sigma_0^2K^2}) = +\frac{N}{2} \tag{4.16}$$

Comparing with the gap equation (3.10) we see that $\sigma_0 = \frac{2N_c}{\varepsilon}$ (our normalization differs from that of [21], note also misprints in numerical coefficients in Eqs. (27-29) of [21]).

We can now again define the quark effective action if we insert in W_2 in (4.7) the extremal values of L_a, R_a, g_+, g_- found above.

We obtain as before the effective action (3.14) when we express σ_0 and g_0 in terms of ε.

Thus the one-quark situation in case of $N_f = 2$ is the same as in the case $N_f = 1$: the effect of CSB is to create the chiral quark mass.

This is true however only in the approximation (4.12) when all the effects of bosonization reduce to the creation of nonzero σ_0, and no boson exchanges (quantum boson fields L_a, R_a) are allowed. In the next section we shall discuss these effects and derive effective chiral Lagrangian with confinement taken into account.

5 Effective chiral Lagrangian and quarks

We are now in position to calculate the "bosonic" effective Lagrangian $W(L,R)$ in (4.9), inserting there extremal values of $g_+ = g_- = g_0$ and $<\sigma>=\sigma_0$,
namely

$$g_0\sigma_0 = -\frac{\varepsilon N}{2VN_c}, \quad \sigma_0 = \frac{2N_c}{\varepsilon} \tag{5.1}$$

To parametrize the quantum bosonic fields in $\hat{L}, \hat{R}$ we introduce as in [21] the forms

$$\hat{L} = i\sigma_0 \frac{1+\sigma+\eta}{2} UV \ , \quad \hat{R} = i\sigma_0 \frac{1+\sigma-\eta}{2} VU^+ \tag{5.2}$$

where $U = exp(i\pi_i\tau_i)$, $V = exp(i\sigma_i\tau_i)$, $i = 1,2,3$. The eight bosonic fields $\pi_i, \sigma_i, \sigma, \eta$ correspond to eight field L_a, R_a, $a = 1, ...4$.

Insertion of (5.2) into (4.9) yields

$$W(L,R) = \frac{N}{2V} \int d^4x(\sigma^2(x) + \eta^2(x)) - \tag{5.3}$$

$$Tr\ ln\{i\hat{D} + i(1+\sigma+\eta)UV\hat{M}_+ + i(1+\sigma-\eta)VU^+\hat{M}_-\}$$

where $\hat{M}_\pm = \hat{M}\frac{1\pm\gamma_5}{2}$ and $\hat{M}$ is defined in (3.15) and Tr is taken over coordinates, color and Lorentz indices. One should keep in mind that terms linear in fields should be suppressed since they vanish due to the steepest descent condition, yielding $< \eta >=< \pi_i >=< \sigma_i >= 0$.

It is instructive to expand the effective action (5.3) in $\pi_i, \sigma_i, \sigma, \eta$ and to find the corresponding quadratic terms yielding masses of mesons.

For pions this procedure looks like

$$-W(\pi) = Tr\ ln(i\hat{D} + iU\hat{M}_+ + iU^+\hat{M}_-) =$$

$$Tr\ ln(i\hat{D} + i\hat{M} + \Delta) = Tr\ ln(i\hat{D} + i\hat{M}) + Tr((i\hat{D} + iM)^{-1}\Delta) - \tag{5.4}$$

$$-\frac{1}{2}Tr[(i\hat{D} + iM)^{-1}\Delta(i\hat{D} + iM)^{-1}\Delta] \equiv (-W^{(1)} + W^{(2)} + W^{(3)}).$$

where

$$\Delta = i(e^{i\pi\tau} - 1)\hat{M}_+ + i(e^{-i\pi\tau} - 1)\hat{M}_-, \hat{M} = \hat{M}_+ + \hat{M}_- \tag{5.5}$$

The first term, $W^{(1)}$, contains no pions and describes contribution of $q\bar{q}$ pairs interacting with background field B_μ, the quark being already massive due to appearence of chiral mass M.

The second and third term, $W^{(2)}$ and $W^{(3)}$ are depicted in Fig.2 (a) and (b) respectively.

Expanding in (5.4) Δ in powers of π_i and keeping only quadratic terms one obtains

$$+ W^{(2)}(\pi) + W^{(3)}(\pi) = \tag{5.6}$$

$$\int \frac{dk\ dk'}{(2\pi)^8} \pi_a(k)\pi_a(k')N(k,k')$$

where

$$N(k,k') = \frac{1}{2}Tr[\frac{1}{i\hat{D}+i\hat{M}}i\tilde{M}(k+k') + \frac{1}{i\hat{D}+i\hat{M}}i\tilde{M}(k)\frac{1}{-i\hat{D}+i\hat{M}}i\tilde{M}(k')]$$

and

$$< x|\tilde{M}(k)|y >= \frac{\varepsilon N}{2VN_c}\int K(x,y,u)e^{iku}du\ ;\quad \tilde{M}(0) = M\ .$$

It is easy to see in (5.6) and we show it explicitly in Appendix D that for $\pi_a = const$ i.e. for $k = k' = 0$ the sum in the square brackets on the r.h.s. of (5.6) vanishes, signalling vanishing of the pion mass, as it should be for the spontaneous CSB with zero current quark masses.

A similar analysis for other mesons can be done and results are similar to those of [21] . We note that masses of all mesons, other than π_i, do not vanish and are of the order of typical hadron mass.

We turn now to the final topic of this section; effective chiral Lagrangian for pions. In this case we integrate out all meson degrees of freedom exept that of the pion, since pion is the lightest particle dominating at small momenta. From (5.4) we obtain

$$W(\pi) = -Tr\ ln(i\hat{D} + i\hat{M}\hat{U}_5) \tag{5.7}$$

where

$$\hat{U}_5 = U\frac{1-\gamma_5}{2} + U^+\frac{1+\gamma_5}{2} = e^{i\pi_i\tau_i\gamma_5} \tag{5.8}$$

The expression (5.7) reduces to the chiral Lagrangian obtained in [21,16] when $\hat{M} \to M(0)$ and $B_\mu \to 0$, so that $i\hat{D} \to i\hat{\partial}$.

In the form given in (5.7) the gauge invariance is seen explicitly (for that the factor $\Phi(x,R,y)$ should be kept in $\hat{M}$ as is given in (3.15)).

The effective action (5.7) describes quark pairs (integrated out) propagating in the confining gluon field B_μ and the chiral field $\hat{U}_5$ - the remaining degrees of freedom – those of pions (π_i) and gluons (B_μ).

We can compare (5.7) with the standard chiral Lagrangian [2,3]

$$W_{eff}(\pi) = \frac{F_\pi^2}{4}\int d^4x Tr\Lambda_\mu\Lambda_\mu + \frac{N_c}{240\pi^2}\int d^5x\varepsilon_{\alpha\beta\gamma\delta\epsilon}Tr\Lambda_\alpha\Lambda_\beta\Lambda_\gamma\Lambda_\delta\Lambda_\epsilon + ... \tag{5.9}$$

where

$$\Lambda_\mu = U^\dagger i\partial_\mu U.$$

Comparing (5.6) and (5.7) with (5.9) we deduce the expression for the pion decay constant F_π. To this end one should average in Z_{QCD} (4.7) over fields

B_μ, which in the leading N_c order results in averaging over B_μ the effective action $W(\pi)$ (5.6–5.7). Since $N(k,k')$ includes trace over coordinates, one has $< N(k,k') >_B \equiv (2\pi)^4\delta(k+k')N(k)$, and $N(k) = k^2 N_0(k)$. Finally comparing (5.6) to (5.9) we have

$$\frac{1}{2}F_\pi^2 = N_0(k=0) \tag{5.10}$$

When B_μ is absent, we come back to Eq.(68) of [10]. For details see Appendix D.

6 Correlation functions and the double nature of pion

Our starting point now is the QCD partition function where we keep quark and meson degrees of fredom (in addition to B_μ). To obtain it , we integrate (4.7) over $dg_+ dg_-$, which effectively reduces to insertion into $\bar{W}_2$ in (4.7) $g_+ = g_- = g_0$. We obtain

$$W(L,R,\psi) = \int d^4x d^4x' \psi_f^+(x)[i\hat{D} + i(1+\sigma+\eta)UV\frac{1-\gamma_5}{2}\hat{M} + \tag{6.1}$$

$$+(1+\sigma-\eta)VU^+\frac{1+\gamma_5}{2}\hat{M}]_{fg,xx'}\psi_g(x') + \frac{N}{2V}\int d^4x(\sigma^2(x)+\eta^2(x))$$

where U, V are given in (5.2).

We now can calculate the $q\bar{q}$ correlation function

$$\Pi^\Gamma(x,y) = < \psi^+(x)\Gamma\psi(x)\psi^+(y)\Gamma\psi(y) > \tag{6.2}$$

where $\Gamma = 1, \gamma_5, \gamma_\mu, \gamma_\mu\gamma_5, \sigma_{\mu\nu}$. The result is

$$\Pi^\Gamma(x,y) = \frac{1}{Z}\int D\Phi_i D\mu(B) e^{-W(L,R)}\{Tr(S(x,y;\Phi,B)\Gamma S(y,x,\Phi,B)\Gamma) - \tag{6.3}$$

$$Tr\ S(x,x;\Phi,B) Tr\ S(y,y;\Phi,B)\}$$

Here $W(L,R)$ is the effective action (5.3); $\Phi_i(x)$ $i = 1,2,...8$ denotes the set of meson variables; $\sigma(x), \eta(x), \sigma_a(x), \pi_a(x)$. $S(x,y;\Phi,B)$ is the quark propagator in the external field of mesons $\{\Phi\}$ and confining gluons $\{B_\mu\}$.

$$S(x,y;\Phi,B) = [i\hat{D} + i(1+\sigma+\eta)UV\frac{1-\gamma_5}{2}\hat{M} + \tag{6.4}$$

$$+i(1+\sigma-\eta)VU^+\frac{1+\gamma_5}{2}\hat{M}]^{-1}$$

Two contributions in eq. (6.3) are shown in Fig.3a,b, we shall call them one (quark) loop and two–(quark) loop respectively. This however is somewhat misleading since the action $W(L,R)$ corresponds to the superposition of any number of quark loops , since

$$W(L,R) = -Tr\ lnS^{-1} + \frac{N}{2V}\int d^4x(\sigma^2(x) + \eta^2(x)) \tag{6.5}$$

Therefore our terminology refers only to the interior of the curly brackets in (6.3) with understanding that these graphs should be integrated with the weight shown in (6.3). Now we discuss the result of integration of diagrams (a) and (b) of Fig. 3 over bosonic fields in (6.3). To this end we expand $W(L,R) \equiv W(\Phi_i)$ in powers of Φ_i up to the quadratic terms [21].

$$W(\Phi_i) = \frac{N}{V}\int dx\ dy\ \Phi_i(x)\mathcal{L}_i(x,y)\Phi_i(y) \tag{6.6}$$

The term $\mathcal{L}_i$ gets contribution from two diagrams depicted in Fig. 2(a,b). For the case of pions $\mathcal{L}_\pi$ can be read off from (5.6). For small values of momenta the Fourner transform $\mathcal{L}_i(k=0)$ plays the role of meson mass. As we discussed in section 5 contributions to $\mathcal{L}_\pi(k=0)$ of Fig. 2(a) and (b) cancel each other establishing in this way the Goldstone theorem. One can easily see that $\mathcal{L}_i(x,y)$ do not depend on N_c (since also $\hat{M}$ does not depend on N_c). On the other hand,

$$\frac{N}{V} = \frac{g^2}{8\pi^2} < F^2 >_{inst} \sim 0(N_c) \tag{6.7}$$

Since $W(\Phi) \sim \frac{N}{V} \sim N_c$, the exchange of a meson yields correction $0(1/N_c)$ – in other words one can tell that the coupling constant of a quark to the meson

$$g_{\Phi q\bar{q}} \sim \frac{M}{F_\phi} \sim 0(N_c^{-1/2}) \tag{6.8}$$

Hence the contribution of the diagram Fig.3(b) is $0(N_c^2 \cdot \frac{1}{N_c}) = 0(N_c)$ just as that of the diagram Fig. 3(a). This is in contrast to the situation with gluon exchanges for the diagram of Fig.4, which contributes $0(N_c^2 \cdot \frac{1}{N_c^2}) = 0(N_c^0)$ – this is the so-called OZI supressed diagram, just as the diagram of Fig. 3(b). However in the vector and tensor channels the latter does not exist (tensor mesons appear in the $1/N_c$ corrections, see [21,24]) and therefore in these channels OZI suppression can be explained by the smallness of the diagram Fig.4. At the same time, in the scalar and pseudoscalar channels there is no

suppression since the diagram of Fig.3(b) provides for the amplitude roughly the same amount as the OZI allowed diagram of Fig. 3(a). This conclusion of no OZI suppression in scalar and pseudoscalar channels has been obtained before in [29] using another arguments.

We now turn to the important question about the double role of the pion – as the Goldstone particle and as the $q\bar{q}$ bound state. Our present formalism is a convenient setting for the study of this question and we shall actually have in mind all scalar and pseudoscalar particles (to be compared later with vector and tensor particles).

Let us start with the effective action (6.1). It contains both quark and meson degrees of freedom, but here they enter not on equal footing. Quarks ψ_f, ψ_f^+ are supplied with kinetic term and are respectable quantum field–theoretical quantities. In contrast to that meson fields Φ_i enter as auxiliary fields, they do not possess kinetic terms and should be considered as external fields to be averaged out with the given weight. When one integrates out quarks, one obtains an effective chiral Lagrangian as in (5.7–5.8), where the chiral pion field now inherits the kinetic term and the full QFT status. But in doing that (integrating out quarks) we average over all quark structure of the pion and "see only its chiral face". Quantitatively both faces of the pion are given by (6.3), or diagrams Fig.3(a,b). While the diagram of Fig.3(a) is the usual one considered in quark models, the diagram Fig.3(b) is pertinent to the chiral degrees of freedom – it contains the chiral propagator $(\mathcal{L}_i)^{-1}_{xy}$.

Now in the limit of large N_c the diagram Fig. 3(a) will contain only gluon (B_μ) exchanges and provides only poles at $p^2 = m_a^2(n), n = 1, 2....$ The chiral propagator $(\mathcal{L}_i)^{-1}_{xy}$ provides a pole at $p^2 = m_b^2$; for pion $m_b^2 = 0$. How these two types of poles coexist?

For pion when p^2 is small, $p^2 \ll R_0^{-2}, \rho^{-2}$ the situation is relatively simple. In this region only the diagram Fig. 3(b) has the pole and it is dominant. When p^2 increases and is close to the pole m_a^2, the same pole appears in $\mathcal{L}_i(p)$. This is the signal that the expansion in powers of Φ_i done in (6.6) and considering the diagrams of Fig. 3(b) is meaningless – the notion of the meson mass is not useful when it is strongly p – dependent and even has a pole at $p^2 = m_a^2$. Formally, the diagrams Fig.3(b) acquire a pole at m_a^2, so that the sum of contributions of Fig.3a and Fig.3b according to (6.3) can be written as

$$\pi_5(k) = \pi_5^{(0)}(k) - \pi_5^{(0)}(k)\frac{M^2}{N(k)}\pi_5^{(0)}(k) \tag{6.9}$$

where $\pi_5^{(0)}(k)$ is the contribution of the quark loop of Fig.3a.

Now $N(k)$ can be written as (in the limit $\rho \to 0$)

$$N(k) = C + M^2(0)\pi_5^{(0)}(k) \tag{6.10}$$

The "quark model poles" appear in $\pi_5^{(0)}(k)$,

$$\pi_5^{(0)}(k) = \frac{\lambda^2}{k^2 + m_a^2} \tag{6.11}$$

One can see in (6.9) (at least in the limit $\rho \to 0$, i.e. for $k\rho \ll 1$) that "quark model poles" (6.11) exactly cancel, and only Nambu-Goldstone pole at $k = 0$ due to $N(k \to 0) = \frac{1}{2}F_\pi^2 \cdot k^2$ survives. Thus the Goldstone theorem manifests itself in our case, as it should be since chiral symmetry breaks spontaneously.

7 Conclusions and outlook

We have shown that the QCD vacuum with confinement and topological charges can be adequately described by the model, in which confining configurations are added to instantons.

On one hand, instantons are stabilized by confinement, and become more dilute, so that they can be properly treated in the instanton gas approximation.

On an other hand, quarks and gluons are confined in the model, and unphysical features of the instanton gas or liquid model, where quarks propagate freely, are now absent.

Therefore in this paper we have obtained a realistic approach, in which quarks, gluons and Goldstone bosons can be treated simultaneously on the fundamental level. The basic effective action is given in (5.3) and can be used for diagrammatic expansion in powers of Goldstone exchanges or integrating over all bosonic fields, in particular when there is an extremum corresponding to a selfconsistent bosonic field of solitonic type.

The latter might be important for baryons.

In the limit when confining configurations vanish we come back to the effective chiral Lagrangian obtained earlier [21],[30].

The framework suggested in the paper can be used for the calculations of all effects where chiral physics and confinement are both important. A systematic quantitative study is planned in subsequent publications.

Figure captions

Fig. 1. Graphical representation of the heavy–light Green's function, eq.(3.17), The straight line from x to y corresponds to the heavy–quark path, the light–quark path is typically away at a distance of $R_c \sim 1fm$ from the heavy quark.

The chiral mass insertion (second term in Eq. (3.21)) is shown near the instanton position R_i with nonlocality of the order of $\rho \sim 0.2\ fm$.

Fig. 2. Graphical representation of two terms in the effective action (5.4). Part (a) corresponds to $W^{(2)}$ and part (b) to $W^{(3)}$, solid lines denote quark propagator $(iD + iM)^{-1}$, broken lines – emitted pion field π_i from expansion of Δ, Eq. (5.5).

Fig. 3. Graphical representation of the $q\bar{q}$ correlator $\Pi^{\Gamma}(x, y)$, Eq. (6.3). Fig. 3a corresponds to the "one–loop term" (first term inside the curly brackets of (6.3)), while Fig. 3b corresponds to the "two–loop term" (second term inside the curly brackets). Broken line denotes the boson propagator which appears when one expands $S(x, x, \Phi, B)$ in powers of Φ.

Fig. 4. The OZI violating two–gluon exchange diagram obtained from expansion in B_μ of the two–loop term of Eq. (6.3).

Appendix A

Quark Green's function in the instanton field

In this appendix we remind the reader the expansion of the quark Green's function g in the pure instanton field $A^{(i)}(x)$ in the unitary gauge, which was found in [31].

$$G(x,y) = (i\hat{D} - im)\Delta\frac{1+\gamma_5}{2} + i\Delta\hat{D}\cdot\frac{1-\gamma_5}{2} + \frac{1}{im}u(x,y) \tag{A.1}$$

where $u(x,y)$ is the contribution of zero modes

$$u(x,y) = u_0(x)u_0^+(y) \tag{A.2}$$

while $\Delta(x,y)$ is the Green's function of scalar particles in the instantonic field in the unitary gauge

$$(-D_\mu^2 + m^2)\Delta(x,y) = \delta^{(4)}(x-y) \tag{A.3}$$

One can obtain expansion of Δ in powers of m [32,29] in the singular gauge

$$\Delta(x,y) = \phi^{-1/2}(x)\tilde{\Delta}(x,y)\phi^{-1/2}(y)\ , \quad \phi = 1 + \frac{\rho^2}{x^2} \tag{A.4}$$

$$\tilde{\Delta}(x,y) = \frac{1}{4\pi^2}\{\frac{1}{(x-y)^2} + \rho^2\frac{(\tau_- x)(\tau_+ y)}{x^2(x-y)^2y^2}\} + 0(m^2) \tag{A.5}$$

Consequently one has for $G(x,y)$, $x \sim y \sim \rho$

$$G(x,y) = \frac{1}{im}u(x,y)(1 + 0(m\rho) \tag{A.6}$$

This can be compared with the spectral decomposition (2.4), with the result that the contribution of the nonzero modes is finite for $m \to 0$.

Now we can see the physical parameter of expansion in the ansatz (2.9) used here and earlier papers [10,21-24].

One can state that the omitted terms in (2.9) are of the order $m\rho$ (since mx and my which can also appear effectively enter our expressions for quark propagator or effective action at distances $x, y \leq \rho$).

Thus $m\rho \ll 1$ is also accuracy of our approximation in (3.1). One can see that for a typical instanton size $\rho \approx 0.2fm$, u, d and s quarks satisfy condition $m\rho \ll 1$, while for c quark this is already violated and all effective actions considered below are not applicable for the c quark.

Appendix B

Shift of eigenvalues due to background B_μ

We study eigenvalues in the field of one instanton A_μ plus background B_μ, which are to be found from the equation [12]:

$$[-(\partial_\mu - ig(A_\mu + B_\mu))^2 - g\vec{\sigma}(\vec{E}(A+B) + \vec{B}(A+B))]\varphi_n = \lambda_n^2\varphi_n \qquad \text{(B.1)}$$

where $A_\mu^a = \frac{2}{g}\eta_{a\mu\nu}\frac{x_\nu}{x^2+\rho^2}\frac{\rho^2}{x^2}$ while $\vec{E}, \vec{B}$ are colorelectric and colormagnetic fields respectively.

Let us concentrate on the shift of the zero eigenvalue which we shall evaluate by perturbation theory

$$\delta\lambda^2 = (\varphi_0 \delta V \varphi_0) \qquad \text{(B.2)}$$

where δV is to be read off from eq. (B.1) and is due to $B_\mu \neq 0$. One can extract the phase factor out φ_n, as is done in (2.23) and the rest is equivalent to the wave function in the Fock-Schwinger gauge for B_μ;

$$B_\mu(x) = \int_0^x \alpha(u) du_\nu F_{\nu\mu}(u) \ , \quad \alpha(u) = \frac{u}{x} \qquad \text{(B.3)}$$

In δV there are three terms: (i) linear in B_μ (ii) quadratic in B_μ , $g^2 B_\mu^2$ (iii) proportional to $\vec{E} + \vec{B}$. Due to symmetry reasons only quadratic term contributes to $\delta\lambda^2$.

We have

$$\delta\lambda^2 = 2g^2 \int_0^\infty \frac{r^3 dr \rho^2 B_\mu^2}{(\rho^2 + r^2)^3} = \qquad \text{(B.4)}$$

$$= 2g^2 \int_0^\infty \frac{y^3 dy}{(1+y^2)^3} \int_0^{\rho y} \alpha(u)\alpha(u') du_\nu du'_\nu < F_{\nu\mu}(u) F_{\nu\mu}(u') >$$

Here we have introduced for an estimate the average value of $< F(u)F(u') >$ which at $u = u'$ should be less or equal to the gluonic condensate of [33]:

$$\frac{\alpha_s}{\pi} < F_{\mu\nu}^a F_{\mu\nu}^a >= 0.012 GeV^4 \qquad \text{(B.5)}$$

Taking into account that $< F(u)F(u') >$ falls off at distances $|u - u'| \sim T_g \approx 0.2 fm$ [34], one can compute the integrals in (B.4) to obtain

$$\delta\lambda^2 \cong \frac{\rho^2}{8}\frac{\alpha_s}{\pi} < F^a F^a > \leq 0.0015 Gev^4 \qquad \text{(B 6)}$$

In getting upper bound on the r.h.s. of (B.6) we use the fact that part of gluonic condensate should be due to instantons and in our case only that part of $< F^a F^a >$ enters in (B.6) which is due to confining background B_μ. Hence we have average shift of the zero eigenvalue $< |\delta\lambda| > \leq 40 MeV$. This is small as compared to $\rho^{-1} \approx 1 Gev$, and therefore $|\delta\lambda|\rho \ll 1$.

Appendix C

Derivation of the effective action (3.1)

We start with the general form of the partion function for the quarks of flavour $f = 1, ...N_f$ in the field (2.1)

$$Z = const \int D\mu(B) D\psi D\psi^+ exp - \int \psi_f^+ S^{-1} \psi_f dx \qquad (C.1)$$

where the action of gluonic field is included in $D\mu(B)$.

We now transform S^{-1} in (C.1) to make explicit contribution of zero modes

$$S^{-1} \equiv -i\hat{\partial} - g\hat{B} - g\sum \hat{A}^{(i)} - im = S_0^{-1} + \sum_{i=1}^{N} [(S^{(i)})^{-1} - S_0^{-1}] \qquad (C.2)$$

Using (2.5) and (2.8) after some algebra one obtains

$$S^{-1}(x,y) = S_0^{-1}(x)\delta(x-y) + \sum_{i=1}^{N} \sum_{n,q=0}^{K} S_0^{-1}(x) u_n^{(i)}(x) (im - \hat{\lambda} - \hat{V})^{-1}_{ii,nq} u_q^{(i)+}(y) S_0^{-1}(y) \qquad (C.3)$$

where matrix elements of $\hat{V}$ are

$$V_{nq,ii} = \int u_n^{(i)+}(z)(-i\hat{D}(B) - im) u_q^{(i)}(z) dz \qquad (C.4)$$

It is easy to check that inverion of S^{-1} given by (C.3) yields (2.10).

For $n = q = 0$ $u_0^{(i)}$ has a definite chirality and therefore $V_{00,ii} = -im$. Hence, keeping only zero models in (C.3), as it is done e.g.in [21–24], one obtains a term $0(\frac{1}{2im-\lambda})$ in S^{-1} in (C.3) which was not present from the beginning in (C.2)

The reason for this apparent paradox can be identified as an improper omission of terms with $n, q \neq 0$. Indeed, matrix elements $V_{11,ii}$ and $V_{10,ii}$ are of the order of $0(\frac{1}{\rho})$, since eigensolutions of (2.3) with $n \neq 0$ have no definite chirality and have nonzero matrix elements even in the limit $m = 0$.

Therefore $(im - \hat{\lambda} - \hat{V})^{-1}_{nq} \sim (det(im - \hat{\lambda} - \hat{V}))^{-1}$ is finite for $m \to 0, \lambda_0 \to 0$, and this fact resolves the apparent paradox occurring in derivation in [21, 24].

Insertion of (C.3) into (C.1) yields

$$Z = const \int D\mu(B) D\psi D\psi^+ exp(-\int \psi_f^+ S_0^{-1} \psi_f dx) exp \int \mathcal{L} dx dy \qquad (C.5)$$

where

$$\mathcal{L} = -\sum_f \psi_f^+(x) \sum_{i=1}^{N} \sum_{n,q} S_0^{-1}(x) u_n^{(i)}(x) (im - \hat{\lambda} - \hat{V})^{-1}_{ii,nq} u_q^{(i)+}(y) S_0^{-1}(y) \psi_f(y) \qquad (C.6)$$

One can notice that $\int \mathcal{L} dx\, dy$ contains fermionic operators of the type:

$$\int \psi_f^+(x) S_0^{-1}(x) u_n^{(i)}(x) dx \equiv \Psi_n^{(i)}(f) \tag{C.7}$$

Due to anticommutativity of ψ_f^+ the product $\Psi_n^{(i)}(f)\Psi_n^{(i)}(f)$ vanishes. Therefore in expansion of $exp\mathcal{L}$ only finite number of terms survives. Namely, if we for simplicity keep only zero modes, $n = q = 0$ in (C.6) then one can write.

$$exp \int \mathcal{L} dx dy = \prod_{i,f}^{N,N_f} (1 - \Psi_0^{(i)}(f)(im - \hat{\lambda}_0 - \hat{V})^{-1} \Psi_0^{(i)+}(f)) \tag{C.8}$$

This form coincides with that obtained in [21–24]. Our derivation which has used (C.3) is more direct than prosented in [24]. Eq. (C.8) also coincides with the form originally suggested in [21] up to a change $im_f \to (im - \lambda_0 - \hat{V})^{-1}_{00}$. This overall factor can be taken out to redefine a normalization of the partition function and one comes to (3.1).

One can also show that nonzero modes can be neglected since the ratio $(im - \hat{\lambda}_n - \hat{V})^{-1}_{nn}/(im - \lambda_0 - \hat{V})^{-1}_{00}$ is of the order $0(m\rho)$. Namely for the two-channel situation, $n = 0, 1$ one has

$$(im - \hat{\lambda} - \hat{V})^{-1}_{00} = \frac{im - \lambda_1 - V_{11}}{det\Delta}\ , \quad (im - \hat{\lambda} - \hat{V})^{-1}_{11} = \frac{2im - \lambda_0}{\det \Delta} \tag{C.9}$$

Now take into account that $\lambda_1 \sim V_{11} \sim \rho^{-1}$, while m and λ_0 are much smaller (see Appendix B).

Appendix D

Study of the pion inverse propagator, $N(k,k')$ in (5.6)

We first prove that $N(k=k'=0)=0$. From (5.6) one has

$$N(0,0)=\frac{1}{2}Tr[\frac{1}{i\hat{D}+iM}iM+\frac{1}{i\hat{D}+iM}M\frac{1}{-i\hat{D}+iM}M]= \tag{D.1}$$

$$=\frac{1}{2}Tr[\frac{1}{i\hat{D}+iM}M\frac{1}{-i\hat{D}+iM}\hat{D}]=$$

$$=\frac{1}{4i}Tr(\frac{1}{-i\hat{D}+iM}\hat{D}+\frac{1}{i\hat{D}+iM}\hat{D})$$

Consider the tranformation of inversion P of all coordinates, $x_\mu \to -x_\mu$, $B_\mu \to -B_\mu$.

From (D.1) one can see that under P transformation $N(0,0)$ changes sign. Therefore, when $N(0,0)$ is integrated over all gluonic fields and is a number, it should vanish

$$< N(0,0) >_B = 0 \tag{D.2}$$

From the invariance properties of the vacuum with respect to the shift of coordinates, one can deduce that $< N(k,k') >_B$ written as

$$< N(k,k') >_B = \int du\, du' e^{iku+ik'u'} < \frac{1}{2}Tr[\frac{1}{i\hat{D}+iM}im(u)\delta(u-u')+ \tag{D.3}$$

$$+\frac{1}{i\hat{D}+im}m(u)\frac{1}{-i\hat{D}+iM}m(u')] >_B$$

where

$$< x|m(u)|y >= \frac{\epsilon N}{2VN_c}K(x,y,u), \tag{D.4}$$

has the property

$$< N(k,k') >_B = (2\pi)^4\delta(k+k')N(k). \tag{D.5}$$

Expanding now $N(k)$ and again using invariance of the vacuum we get

$$N(k)=k^2N_0(k). \tag{D.6}$$

Finnaly we report in this Appendix the calculation of $< N(k,k') >_B$ in the limit $\rho \to 0$. We have for the first term in (5.6)

$$N^{(1)} =< \frac{1}{2}tr\int dx\, dy\, du \tag{D.7}$$

$$\cdot e^{i(k+k')u} S(x,y,B) \frac{\varepsilon N}{2VN_c} i\hat{D}\bar{\varphi}(y-u)\Phi(y,u,x)\bar{\varphi}^{+}(x-u)i\hat{D} >_B$$

When $\rho \to 0$, both x and y tend to u and one can replace $S(x,y;B) \to S(u,u;B)$. Integration over $D(x-u)$ and $(y-u)$ yields

$$N^{(1)} = (2\pi)^4 \delta(k+k') const \tag{D.8}$$

Analysis of the second term on the r.h.s. of (5.6) yields in the limit $\rho \to 0$

$$N^{(2)} = \int du\, dw\, e^{iku+ik'w} < \frac{1}{2} tr\, S(w,u;B) M(0) S(u,w;B) M(0) >_B \tag{D.9}$$

Finally we can rewrite the sum $N^{(1)} + N(2)$ as

$$< N(k,k') >_B = N^{(1)} + N^{(2)} = \tag{D.10}$$

$$= (2\pi)^4 \delta(k+k') const + M^2(0)\pi_5(k))$$

This justifies eq. (6.10) used in the text.

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