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We compare the magnitudes of CP effects in $K^{\pm} \to \pi^{\pm}\pi^{\pm}\pi^{\pm}$ decays caused by different combinations of the superweak, Kobayshi-Maskawa and spontaneous CP violation. It is found that the last mechanism could be the main source of these effects.

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1 Introduction

In the Standard Model (SM) CP violation arises due to Kobayashi-Maskawa (K-M) [1] mechanism producing CP-odd parts of transitions $K^0 \to \bar{K}^0$ (indirect CP breakdown) and CP-odd parts of transitions with $|\Delta S| = 1$ (direct CP breakdown). In the case of the $\{K^0\bar{K}^0\}$ system decaying in 2π, a magnitude of the direct CP violation is characterized by the parameter ϵ'. The existing data

$$\epsilon'/\epsilon = \begin{cases} (2.3 \pm 0.7)10^{-3} & [2] \\ (0.6 \pm 0.7)10^{-3} & [3] \end{cases} \tag{1}$$

do not exclude that $\epsilon' = 0$, of very small, as it takes place when the mechanism of superweak interaction [4] is the main source of CP violation. As the modern calculations in the framework of SM give

$$\epsilon'/\epsilon = few \times 10^{-4} \tag{2}$$

and a sensitivity of the planned experiments for measurement of ϵ'/ϵ is expected at the same level, it may occure that we shall not be able to distinguish between the above mentioned mechanisms of CP breakdown.

But if it will be confirmed that ϵ'/ϵ is as large as $few \times 10^{-3}$, it would be an evidence [5] of existence of one more mechanism of CP breakdown – spontaneous CP violation in sector of the scalar fields incorporating more than two Higgs doublets [6].

Such a mechanism could originate the direct CP violation with a magnitude considerably larger than of SM [5], and it is interesting to understand how large could be CP effects in $K^\pm \to 3\pi$ decays where only the direct CP violation takes place. These effects are

$$\Delta\Gamma/\Gamma = \left[\Gamma(K^+ \to \pi^+\pi^+\pi^-) - \Gamma(K^- \to \pi^-\pi^-\pi^+)\right]/\Gamma(K^+ \to \pi^+\pi^+\pi^-) \tag{3}$$

and

$$\Delta g/g = (g^{(+)} - g^{(-)})/g^{(+)} \tag{4}$$

where $g^{(\pm)}$ are defined by the relation

$$|M(K^\pm(k)) \to \pi^\pm(p_1)\pi^\pm(p_2)\pi^\mp(p_3)| \sim 1 + g^{(\pm)}(s_3 - s_0)/m_\pi^2 + \ldots$$

and

$$s_3 = (k - p_3)^2, \;\; s_0 = \frac{1}{3}m_k^2 + m_\pi^2.$$

Their magnitudes originated by K-M mechanism were estimated in refs. [7, 8, 9]. Here we investigate the role of mechanism of spontaneous CP violation.

2 CP effects in presence of K-M and Weinberg mechanisms of CP breakdown

To calculate the effects arising in SM we shall use the effective lagrangian for $|\Delta S| = 1$ non-leptonic transitions in the form proposed in ref. [10]:

$$L(\Delta S = -1) = -\sqrt{2} G_F \sin\theta_c \cos\theta_c \sum_i c_i O_i \tag{5}$$

where O_{1-6} are the different four-quark operators represented in [10] and $O_{7,8...}$ correspond to so-called electroweak penguin diagram "ewp", box diagram and so on [11]. Of all c_i, only $c_i \geq 5$ have the imaginary parts arising due to complex couplings of c-and t-quarks inside the loop diagrams. In this paper we confine ourself by consideration of the role of operators O_{1-6} entering eq.(5), not neglecting, however, a contribution of the other operators when they play a considerable role.

The spontaneous CP breakdown in the Higgs sector of the theory originates the additional operators [12]

$$O_{W_1}^{CP=-1} = \frac{g_s B \xi}{3 m_G^2} [M_s \bar{s}(1-\gamma_5) q \cdot \bar{q}(1+\gamma_5) d + M_d \bar{s}(1+\gamma_5) q \cdot \bar{q}(1-\gamma_5) d] \tag{6}$$

and

$$O_{W_2}^{CP=-1} = \frac{g_s B I \zeta}{3 m_K^2} (G_{\mu\nu}^a)^2 \bar{s}(1+\gamma_5) d. \tag{7}$$

These operators incorporate the short-distance and long-distance interactions, so that , a question on their renormalization is not simple. A possible renormalization is taken into account by introduction of the parameters ξ and ζ which can be different due to long-distance effects.

In eqs.(6) and (7), M_s and M_d are the constituent masses of s-and d-quarks, B is the imaginary part of the effective coupling constant in $\bar{s} d G^a$ vertex

$$V(sdG) = B \bar{s}(1+\gamma_5) \sigma_{\mu\nu} \frac{\lambda^a}{2} d G_{\mu\nu}^a \tag{8}$$

$$I \approx 1.54 \quad and \quad m_G^2 = 0.64 m_K^2 \qquad (see\ [12]) \tag{9}$$

The diquark combinations can be replaced by the mesonic fields using the relations [13]

$$\bar{q}_j (1+\gamma_5) q_k = -\frac{1}{\sqrt{2}} f_\pi r (U - \frac{1}{\Lambda^2} \partial^2 U)_{kj} \tag{10}$$

$$\bar{q}_j\gamma_\mu(1+\gamma_5)q_k = i\left\{(\partial_\mu U)U^+ - U(\partial_\mu U^+) - \frac{rf_\pi}{\sqrt{2}\Lambda^2}[m(\partial_\mu U^+) - (\partial_\mu U)m]\right\}_{kj} \tag{11}$$

where $f_\pi \approx 93$ MeV, $r = 2m_\pi^2/(m_u + m_d) \approx 3.25$ GeV. $\Lambda^2 = 0.94\ GeV^2$,

$$U = \frac{f_\pi}{\sqrt{2}}\left[1 + \frac{i\sqrt{2}}{f_\pi}\hat{\pi} + \left(\frac{i\hat{\pi}}{f_\pi}\right)^2 + a_3\left(\frac{i\hat{\pi}}{\sqrt{2}f_\pi}\right)^3 + 2(a_3-1)\left(\frac{i\hat{\pi}}{\sqrt{2}f_\pi}\right)^4 + \ldots\right] \tag{12}$$

and

$$\hat{\pi} = \begin{bmatrix} \frac{\pi_0}{\sqrt{3}} + \frac{\pi_8}{\sqrt{6}} + \frac{\pi_3}{\sqrt{2}} & \pi^+ & K^+ \\ \pi^- & \frac{\pi_0}{\sqrt{3}} + \frac{\pi_8}{\sqrt{6}} - \frac{\pi_3}{\sqrt{2}} & K^0 \\ K^- & \bar{K}^0 & \frac{\pi_0}{\sqrt{3}} - \frac{2\pi_8}{\sqrt{6}} \end{bmatrix} \tag{13}$$

As the results for the mesonic amplitudes on mass shell do not depend of a_3, it is convenient to take $a_3 = 0$. Such a choice corresponds to the simple form of PCAC relations [14]. Then we get

$$< \pi^+(p_1)\pi^+(p_2)\pi^-(p_3)|L(\Delta S = -1)|K^+(k) >=$$

$$= \frac{G_F}{2\sqrt{2}}\sin\theta_C\cos\theta_C\left[(c_1 - c_2 - c_3 - c_4 + 4\beta\tilde{c}_5)(s_1 + s_2 - 2m_\pi^2) + 9c_4(s_0 - s_3)\right] \tag{14}$$

where $\tilde{c}_5 = c_5 + \frac{3}{16}c_6$, $\beta = 2m_\pi^4/[(m_u + m_d)^2\Lambda^2]$.

In eq.(14), the part proportional to $(c_1 - c_2 - c_3 + 4\beta Re\tilde{c}_5)$ corresponds to CP-even $ReA_{1/2}$ amplitude, the part proportional to c_4 corresponds to $ReA_{3/2}$ amplitude and the part $4\beta Im\tilde{c}_5$ corresponds to CP-odd $ImA_{1/2}$ amplitude.

The operators (6) and (7) give the additional CP-odd part of $A_{1/2}$ amplitude:

$$< \pi^+(p_1)\pi^+(p_2)\pi^-(p_3)|O_{W1}^{CP=-1}|K^+(k) >= -\frac{ig_sB\xi}{6m_G^2}\frac{r^2(M_s + M_d)}{\Lambda^2}(s_1+s_2-2m_\pi^2) \tag{15}$$

$$< \pi^+(p_1)\pi^+(p_2)\pi^-(p_3)|O_{W2}^{CP=-1}|K^+(k) >= -\frac{i4\pi g_sBIr\zeta}{27\alpha_s m_K^2}(s_1+s_2-2m_\pi^2) \tag{16}$$

To get the last relation, we have used the result of ref.[15]:

$$< \pi^+(p)\pi^-(p')|\frac{\alpha_s}{\pi}G^a_{\mu\nu}G^a_{\mu\nu}|0 >= -\frac{8}{9}(p+p')^2 \tag{17}$$

improved by replacement $(p+p')^2 \Rightarrow [(p+p')^2 - m_\pi^2]$ required by the Adler selfconsistency rule [16]. The CP-odd part of $A_{1/2}$ amplitude in presence of $K-M$ and Weinberg's CP violation acquires the form

$$ImA_{1/2} = \left[\frac{G_F}{2\sqrt{2}}\sin\theta_C\cos\theta_C \cdot 4\beta Im\tilde{c}_5 - \frac{4\pi g_s BIr\zeta}{27\alpha_s m_K^2}(1+\alpha_s\xi/\zeta)\right](s_1+s_2-2m_\pi^2) \tag{18}$$

To estimate a magnitude of CP effects in $K^+ \to 3\pi$ decay one needs to use the known data on $K \to 2\pi$ decays.

Our technics gives

$$\begin{aligned} &< \pi^+\pi^-|L(\Delta S = \pm 1)|K_s >= \kappa(a_{1/2}e^{i\delta_0} + a_{3/2}e^{i\delta_2}) \\ &< \pi^0\pi^0|L(\Delta S = \pm 1)|K_s >= \kappa(a_{1/2}e^{i\delta_0} - 2a_{3/2}e^{i\delta_2}) \\ &< \pi^+\pi^0|L(\Delta S = -1)|K^+ >= -\kappa\frac{3}{2}a_{3/2}e^{i\delta_2}) \end{aligned} \tag{19}$$

where $\kappa = \frac{G_F}{\sqrt{2}}\sin\theta_C\cos\theta_C f_\pi(m_K^2 - m_\pi^2)$ and

$$\begin{aligned} a_{1/2} &= c_1 - c_2 - c_3 + 4\beta\tilde{c}_5 \\ a_{3/2} &= -c_4 \end{aligned} \tag{20}$$

The data on $K \to 2\pi$ decay probabilities give us:

$$a_{1/2} = -10.295; \quad a_{3/2} = -0.335; \quad \delta_0 - \delta_2 = 53.7^0 \tag{21}$$

Using also the estimates

$$c_1 = -2.75; \quad c_2 = 0.06; \quad c_3 = 0.08 \qquad [17] \tag{22}$$

we find

$$4\beta\tilde{c}_5 = -7.4 \tag{23}$$

Now we are able to estimate the relative contribution of two mechanisms of CP violation basing on the data on $K \to 2\pi$ decays. Experimentally [18]

$$(\epsilon'/\epsilon)_{exp} = (1.5 \pm 0.8)10^{-3} \equiv 2 \cdot 10^{-3}\rho, \qquad \rho \leq 1 \tag{24}$$

The theoretical prediction is

$$(\epsilon'/\epsilon)^{th} = -\frac{a_{3/2}}{2.23 \cdot 10^{-3}(a_{1/2})^2}\{4\beta(Im\tilde{c}_5)\eta -$$

$$-\frac{2\sqrt{2}}{G_F \sin\theta_C \cos\theta_C}\left[\frac{4\pi g_s BIr\zeta}{27\alpha_s m_K^2} + \frac{g_s Br^2(M_s - M_d)}{6\Lambda^2 m_G^2}\cdot\xi\eta\right]\Bigg\} \qquad (25)$$

The parameter η in (25) takes into account the effects of "ewp" and box diagrams not considered above, but significant for $K_L \to 2\pi$ decay. It is of the form

$$\eta = \left(1 - \omega^{-1}\frac{Ima_{3/2}}{Ima_{1/2}}\right) \qquad (26)$$

where

$$\omega^{-1} = \frac{c_1 - c_2 - c_3 + 4\beta\tilde{c}_5}{(-c_4)} \approx 31 \qquad (27)$$

and $Ima_{3/2}/Ima_{1/2} \sim \alpha_{em}/\alpha_s$.

As it was first noticed in [19], at $m_t \sim 180$ GeV the second term in r.h.s. of eq.(26) is big enough to produce $\eta \sim 0.1$. Then, in SM

$$(\epsilon'/\epsilon)^{th}_{SM} = -\frac{a_{3/2}}{2.23 \cdot 10^{-3}(a_{1/2})^2}4\beta(Im\tilde{c}_5)\eta = (few)_{SM} \times 10^{-4} \qquad (28)$$

where

$$(few)_{SM} = \begin{cases} 6.7 \pm 0.7 & [20] \\ 3.1 \pm 2.5 & [21] \\ 4 \pm 5 & [22] \end{cases} \qquad (29)$$

Using eq.(24) and neglecting the contribution (28) we come to the result

$$\frac{-2\sqrt{2}\cdot 4\pi g_s BIr\zeta}{27\alpha_s G_F \sin\theta_C \cos\theta_C m_K^2}[1 + 0.19\alpha_s\xi\eta/\zeta] \approx 1.4 \cdot 10^{-3}\rho \qquad (30)$$

where the magnitudes of the parameters were taken from ref.[12].

From eq. (23) and (28), it follows too that

$$\frac{-Im\tilde{c}_5}{Re\tilde{c}_5} = (few)_{SM} \cdot 10^{-5}\eta^{-1} \sim (few) \cdot 10^{-4} \qquad (31)$$

Now we are able to estimate a value of CP effects in $K^{\pm} \to 3\pi$ decays caused by $K - M$ and Weinberg CP breakdown using eq.(18) and the results (30) and (31). But first of all let's notice that eq.(18) does not affected by

the "ewp" and box contribution in any considerable way. The reason is that the parameter ω^{-1} in eq.(26) is replaced in this case by the parameter

$$\omega_{3\pi}^{-1} = \frac{c_1 - c_2 - c_3 - c_4 - 4\beta\tilde{c}_5}{-c_4} \approx 3.6 \tag{32}$$

and the related effect is negligible.

Then, a magnitude of CP effects in $K^{\pm} \to \pi^{\pm}\pi^{\pm}\pi^{\mp}$ decays produced by KM and Weinberg mechanisms of CP violation can be evaluated using the results [7, 8] for SM and making the replacement

$$\begin{aligned} \left|\frac{Im\tilde{c}_5}{Re\tilde{c}_5}\right| \Rightarrow Z = \left|\frac{Im\tilde{c}_5}{Re\tilde{c}_5}\right| + 10^{-3}\rho\eta^{-1}\frac{1+0.985\zeta/\alpha_s\xi}{1+5.24\zeta/\alpha_s\xi\eta} = \\ = \left|\frac{Im\tilde{c}_5}{Re\tilde{c}_5}\right| + 1.88\cdot 10^{-4}\rho\frac{1+\alpha_s\xi/\zeta}{1+0.19\alpha_s\xi\eta/\zeta}. \end{aligned} \tag{33}$$

In the case of combination of the superweak and Weinberg CP violation, the part $Im\tilde{c}_5/Re\tilde{c}_5 = 0$.

The parameter α_s is related to the momentum region of order $p = \mu \sim m_\pi$ and can be estimated inserting the value $\tilde{c}_5 = -0.33$ following from eq. (23) into the relation

$$\tilde{c}_5 \approx -\frac{\alpha_s}{6\pi}\ln(m_c/\mu) \qquad [17] \tag{34}$$

giving then $\alpha_s(\mu) = 2.7$ at $\mu = m_\pi$.

It should be stressed that a non-coincidence of our magnitude of $\tilde{c}_5$ with the values obtained with the aid of group-renormalization technics [10, 11] means only that this technics misses a considerable contribution into $\tilde{c}_5$ arising from the regin $p < 1$ GeV where such technics is not applicable.

The replacement (33) can be represented in the form

$$\left|\frac{Im\tilde{c}_5}{Re\tilde{c}_5}\right| \Rightarrow \left|\frac{Im\tilde{c}_5}{Re\tilde{c}_5}\right|\left[1 + \frac{1.88\rho}{(few)_{SM}}(1+2.5\xi/\zeta)\right] \tag{35}$$

showing that even at $\xi/\zeta = 1$ and $(few)_{SM} = 4$, the spontaneous CP violation doubles the effect, if $\rho \approx 1$.

At $\xi/\zeta \gg 1$ and $\rho \sim 1$, the CP effects could be considerably larger than that predicted by SM. The last ones are [7, 8]

$$|\Delta\Gamma/\Gamma|_{SM} = 2.5\cdot 10^{-3}Z(SM) \tag{36}$$

$$|\Delta g/g|_{SM} = 3.8\cdot 10^{-2}Z(SM) \tag{37}$$

where

$$Z(SM) = |Im\tilde{c}_5/Re\tilde{c}_5| = (few)_{SM} \cdot 10^{-4} \tag{38}$$

As it follows from eq.(33),the largest increasing of CP effect by the Weinberg mechanism takes place at $\zeta/\xi \to 0$.

Then

$$Z = 10^{-3}\rho\eta^{-1} \sim 10^{-2} \cdot \rho, \qquad \rho \leq 1 \tag{39}$$

and $|\Delta g/g|$ could by of order $4 \cdot 10^{-4}$ instead $2 \cdot 10^{-5}$ in SM.

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