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Bubble evolution in the Theory with Degenerate Vacuum for the Friedmann and de Sitter space-times.

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BUBBLE EVOLUTION IN THE THEORY WITH DEGENERATE VACUUM FOR THE FRIEDMANN AND DE SITTER SPACE-TIMES: Preprint ITEP 7-95/

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Equations describing the bubble evolution in the theory with degenerate vacuum in external gravitational fields with the Friedmann and de Sitter metrics were received. Here we show that the bubble does not collapse in the de Sitter space-time.

Fig. - 3, ref. - 8.

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Bubbles determed in \1\ for the theory with the degenerated vacuum by the Lagrangian:

$$\mathcal{L}(\phi)=1/2(\partial\phi)^2-\lambda^2(\phi^2-\eta^2)^2 , \qquad (1)$$

(here λ and η are constant) were regarded later in the theory with the metastable vacuum \2\, and they were investigated rather carefully in papers \3\ with the same Lagrangian (1). A number of papers \4\ examined the influence of the own gravitational field on the bubble dynamics. But the problem of the gravitation influence on the bubble evolution may be coinsidered in a different way. We can imagine the case when the influence of the own gravitational field may be neglected in the background of a strong external field .

Here we investigate the bubble evolution in the theory described by the Lagrangian (1) in the background of the external metric of the type: a) the expanding Friedmann space-time \5\ (here the cosmological constant $\Lambda=0$ and the pressure of matter $p=0$):

$$ds^2=a^2(\eta)(d\eta^2-d\chi^2-sh^2\chi\ (d\theta^2+\sin^2\theta\ d\phi^2)), \qquad (2)$$

here $a(\eta)=a_0(ch\ \eta\ -1)$; b) the de Sitter space-time \6\ ($\Lambda=3H^2$, and H is the Hubble constant)

$$ds^2=dt^2-a^2(t)[dr^2+ r^2(d\theta^2+\sin^2\theta\ d\phi^2)], \qquad (3)$$

here $a(t)=a_0\exp(Ht)$. There is exist a large number of papers investigated the behavior of physical fields in the de Sitter metric background taken as a vacuum (see, for example, \7\ and references there). The scalar field action for (1) ($\lambda=\sqrt{2}^{-1}$, $\eta=1$) with the minimal interaction in gravitational field has the standard form:

$$S=\int dx\ \sqrt{-g}\ [\ g^{ik}\ \phi_{,i}\phi_{,k}\ -(\phi^2-1)^2] . \qquad (4)$$

Whence we get the following equations of motion for the scalar field in metrics (2) and (3):

$$\phi_{\eta\eta} + 2\, a_\eta a^{-1}\phi_\eta - \phi_{\chi\chi} - 2\, \mathrm{ch}\chi / \mathrm{sh}\chi\, \phi_\chi + 2a^2(\eta)\, \phi(\phi^2-1) = 0, \qquad (5)$$

here the initial condition is : $\phi(\eta_0) = \mathrm{th}\,[a(\eta_0)(\chi - \chi_0)]$; $\phi_\eta(\eta_0) = 0$, as they correspond to the rest bubble for the moment $\eta = \eta_0$ when χ_0 is large enough. By the analogy to the previous case the equation in the de Sitter space-time is:

$$\phi_{tt} + 3\, a_t a^{-1}\phi_t - 2r^{-1}a^{-2}\phi_r - a^{-2}\phi_{rr} + 2\phi(\phi^2-1) = 0 \qquad (6)$$

with initial conditions: $\phi(t_0) = \mathrm{th}\,[a(t_0)(r - r_0)]$, $\phi_t(t_0) = 0$.

The equation describing the bubble radius evolution can be written using two methods: the first one is the "contracted action" method, first regarded in the paper \2\. Following to this method the action for the Friedmann space-time was obtained using the expression (4):

$$S = \int d\eta\; a^3(\eta)\; \mathrm{sh}^2\chi \sqrt{1 - \chi_\eta^2} \qquad (7)$$

and the corresponding equation of motion is:

$$\chi_{\eta\eta} + 2\, \mathrm{ch}\chi / \mathrm{sh}\chi(1-\chi_\eta^2) + 3a_\eta / a\; \chi_\eta(1-\chi_\eta^2) = 0, \qquad (8)$$

(here $\chi(\eta)$ is the bubble radius). By the analogy for the de Sitter:

$$S = \int dt\; a^2(t) R^2 \sqrt{1 - a^2\dot{R}^2} \qquad (9)$$

and

$$\ddot{R} + 2\, a^{-2}R^{-1}(1 - a^2\dot{R}^2) + 3\, \dot{a}a^{-1}\dot{R}(1 - a^2\dot{R}^2) + \ddot{a}a^{-1}\dot{R} = 0, \qquad (10)$$

here $R = R(t)$ is the bubble radius.

It is possible directly get the bubble radius equation of motion from eqations (5) and (6) using the second method . It was developed for the flat space-time in the paper \8\. Here we describe this method when applying it to the curved space-time of metrics

(2) and (3). We shall search for the solution of the following equation:

$$1/\sqrt{-g}\ (\sqrt{-g}\ g^{ik}\ \phi_i)_k + 2\phi(\phi^2-1)=0$$

in the form $\phi = \mathrm{th}\alpha$, here α is the new variable . When passing to the limit of infinitely thin wall this exact equation disintegrates into two equations:

$$\begin{aligned} &g^{ik}\alpha_i\alpha_k+1=\alpha^2 f \\ &1/\sqrt{-g}\ (\sqrt{-g}\ g^{ik}\alpha_i)_k=\alpha f \end{aligned} \qquad (11)$$

here f is some function which has not influence on the form of the bubble radius equation of motion. Next transformations depend on the concrete type of background metrics. For the space-time (2) we insert the parametrization resolving the first equation (11):

$$\begin{aligned} &\alpha_\chi=a\ (1-\alpha^2 f)^{1/2}(1-v^2)^{-1/2} \\ &\alpha_\eta=-va(1-\alpha^2 f)^{1/2}(1-v^2)^{-1/2} \end{aligned} \qquad (12)$$

The function v is the velocity of the surface- (α=const)- motion: $v=-\alpha_\eta/\alpha_\chi$. As the bubble radius is the radius of the surface α=0 in this terms, we get the equation for the radius using the formula (11), and this equation coincides with the term (8). Besides the equation(8) the given method allows to get the additional information about the bubble behavior due to the integrability condition $\alpha_{\eta\chi}=\alpha_{\chi\eta}$. By analogy this construction may be done for the de Sitter space-time:

$$\begin{aligned} &\alpha_\tau=a(1-\alpha^2 f)^{1/2}(1-v^2a^2)^{-1/2} \\ &\alpha_\ell=-va(1-\alpha^2 f)^{1/2}(1-v^2a^2)^{-1/2} \end{aligned} \qquad (13)$$

The resulting equation for the bubble radius R (the surface α=0) coincides with the term (10).

Equations (8) and (10) were solved numerically using the Runge-Kutta method for initial conditions of the rest bubble at the time t_0 : $\chi(0)=R_0$ и $\dot{\chi}(0)=0$. Computed results for the eq.(8) are shown at Fig.1. Here we see that for any initial values R_0 the bubble collapses. The eq. (10) has more complicated behavior than the eq.(8). For $R_0 < R_{cr} \approx 1.22$ the bubble has time for collapsing. But for the initial radius R_0 which is more than the critical radius R_{cr}, the bubble does not collapse (see Fig.2). After the primary small compression it tends to the steady asymptotical value R_{as}. There is the picture of the dependence of the asymptotical radius R_{as} on the initial radius R_0 at Fig.3. Thus in the region of applying of the equation (10), that is the region where the bubble radius $R \gg \ell \sim 1$, here ℓ is the width of the wall, the bubble never collapses. Notice here that the equation (10) has the exact solution :

$$R=C_0 \pm a_0^{-1} H^{-1} \exp(-Ht)$$

(C_0 is the arbitrary constant), but unfortunately it does not satisfy to the initial condition $\dot{R}(0)=0$. For obtaining the solution satisfying to this condition it is need to expand the function R on powers of $\exp(-2Ht)$:

$$R=\sum_{n=0}^{\infty} C_n \exp(-2nHt)$$

The resolution on non collapsing of the bubble in the de Sitter space-time is confirmed by the computation of the exact equation (6), these results (together with computer investigations of the eq.(5)) will be publicized later.

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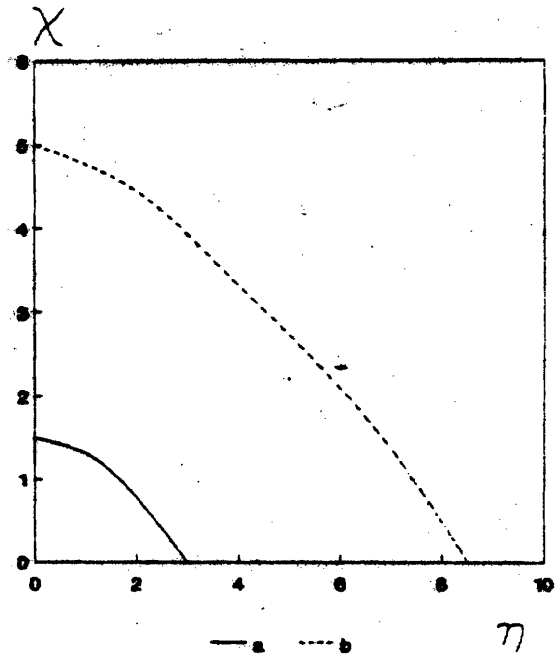

Fig. 1

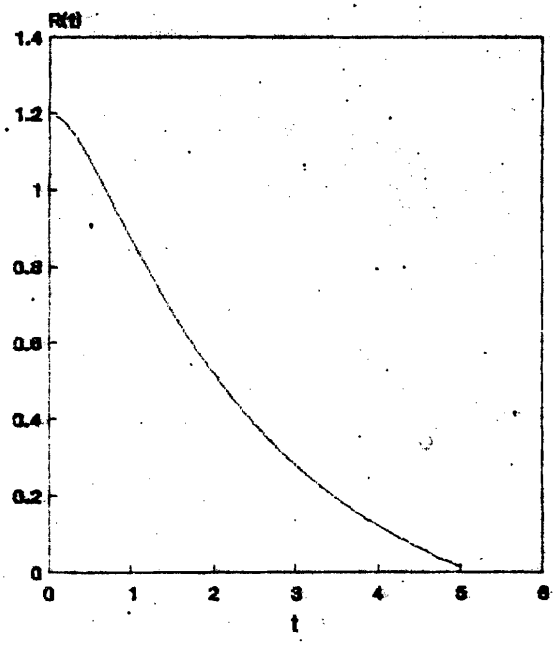

Fig. 2a

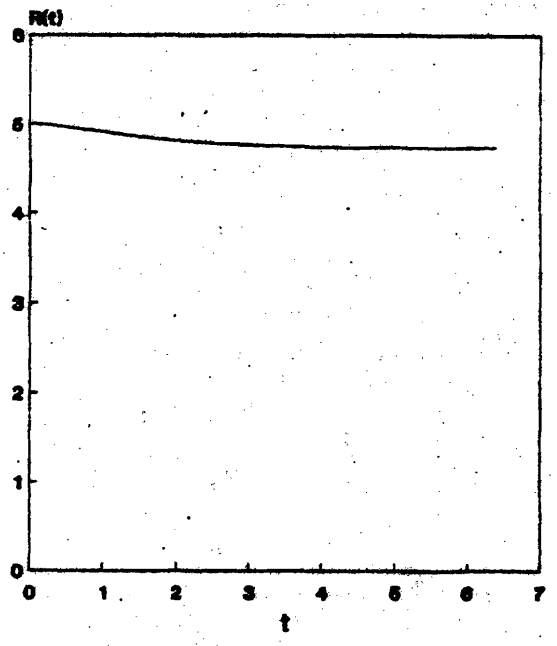

Fig. 2b.

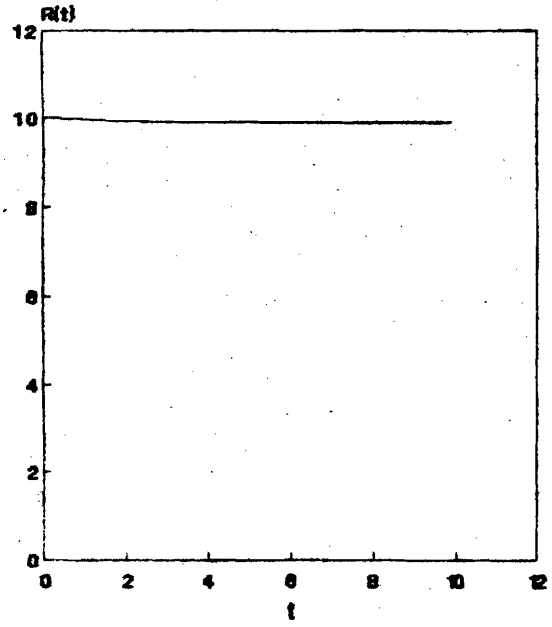

Fig. 2c

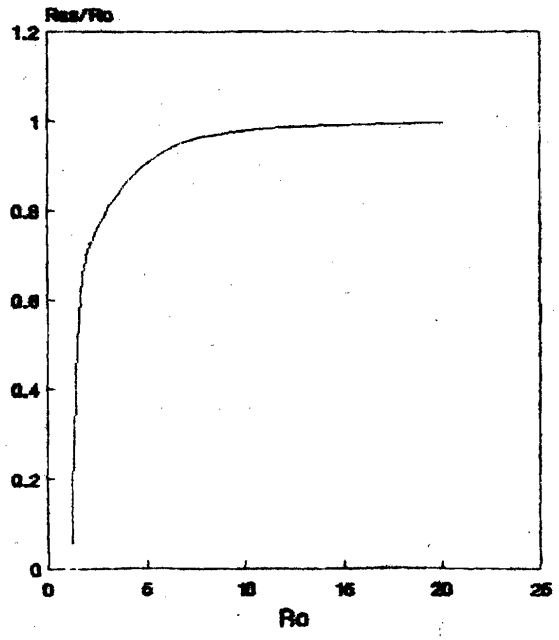

Fig. 3

FIGURE CAPTIONS

Fig.1 The dependence of the bubble radius χ on conform time η for initial conditions: a) $\chi(0)=1.5$; b) $\chi(0)=5.0$ in the Friedmann space-time metric.

Fig.2 a,b,c The dependence of the bubble radius R on time t for initial conditions: a) $R_0=1.2$ (collapse); b) $R_0=5.0$; c) $R_0=10.$ in the de Sitter space-time metric (H=0,5)

Fig.3 The dependence of the asymptotical $t\to\infty$ value of the bubble radius on its initial value for the de Sitter space-time metric (H=0.5).

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