ITEP-7-93

INSTITUTE OF THEORETICAL
AND EXPERIMENTAL PHYSICS

7–93

25

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Moscow, ITEP

MOSCOW 1993

UDK 530.1 И-16

ISOSPIN VIOLATION IN QCD SUM RULES FOR BARYONS: Preprint ITEP 93-7/

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We thoroughly analyse isospin violating effects in QCD sum rules for the masses of nucleons, Σ and Ξ hyperons. After comparing with experimental mass splitting in isotopic multiplets we obtain for the isospin breaking in quark condensate $< 0 \mid \bar{u}u - \bar{d}d \mid 0 > / < 0 \mid \bar{u}u \mid 0 > 0 >= (2 \pm 1).10^{-3}$,a value significantly smaller than the one usually adopted. We presented arguments in favour of our result and critically analyse previous estimates. The value of the quark mass difference $m_d - m_u = 3.0 \pm 1.0 MeV$ at normalization point $\bar{\mu} = 0.5 GeV$ was also determined.

Fig. -4, ref. - 25

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1 Introduction

Starting from Gasser and Leutwyler pioneer paper [1] it becomes clear that the difference of the current masses of u and d quarks is non-zero even in the absence of electromagnetic interactions and is comparable with the masses of u and d quarks themselves. Weinberg in his famous paper [2] demonstrated that the values of u and d quark masses can be determined from the masses of pseudoscalar mesonic octet in the model independent way and found $m_u = 4.2MeV, m_d = 7.5MeV$ and

$$\mu = m_d - m_u = 3.3MeV \tag{1.1}$$

The non-zero value of μ causes the difference between the values of QCD condensate of u and d-quarks. The parameter

$$\gamma = \frac{<0 \mid \bar{d}d \mid 0>}{<0 \mid \bar{u}u \mid 0>} - 1 \tag{1.2}$$

characterizes the isospin violation in quark condensates.

The knowledge of the numericall value of γ is important as it enters along with the mass difference μ in the determination of the value of isospin splitting in hadronic multiplets, the violation of isospin in various decays etc. The magnitude of γ is also interesting from the viewpoint of nuclear physics. Indeed, it enters in recent attempts to explain the discrepancy between the theoretical and experimental results on the difference of mirror nuclei masses known as Nollen-Schiffer(NS) anomaly [3]. The idea behind the explanation put forward recently [4],[5] is based on the reasonable assumption that quark condensate in nuclei are suppressed compared to their vacuum values and as a consequence the neutron-proton mass difference in nuclei, entering in the formula for the mass difference of mirror nuclei is smaller than that for free protons and neutrons.

The parameter γ was calculated in a number of papers using different approaches. Gasser and Leutwyler [6] carried out the calculations in the framework of chiral perturbation theory. Paver et al. [7] considered the constituent quark model,whereas the Nambu-Jona-Lasinio model was used in [4],[5]. In several papers the parameter γ was obtained from the mass splittings in the framework of QCD sum rules [8-13] with results ranging from -3.10^{-3} and -1.10^{-2}.

We see certain shortcomings in at least part of the above mentioned calculations. For this reason we made a new attempt at extracting the parameter γ from the values of the mass splittings in the baryonic octet based on the QCD sum rule technique (for discussion of previous calculations and a comparison with ours see Sec.5).

From our point of view this way to extract the γ parameter is one of the most promising.The reasons are the following. Experimentally the isospin mass splitting in the baryon octet is known with a good accuracy. The electromagnetic contributions to the mass splittings are reliably estimated [14] and they are rather-small, especially for hyperons.The QCD sum rule method of mass determination works well in the case of baryonic octet: three terms of the operator product expansion (OPE)are calculated and all the selfconsistency checks are fulfilled. Using this method the baryonic masses [15-17], magnetic moments [18,19] and other static parameters were calculated, all in a good agreement with experiment In the baryon octet there are three values of isospin mass splittings which can be used for determination of γ: $n-p, \Sigma^- - \Sigma^+$ and $\Xi^- - \Xi^0$. (The $\Sigma^- - \Sigma^0$ splitting is not suitable for this goal,due to the mixing Σ^0 with Λ^0 via isospin violating interactions). In QCD sum rule approach there are two equations for each mass splitting, corresponding to chiralitiy conserving and chirality violating parts of the polarization operator. Therefore, there are six equations in which γ enters and many checks of self-consistency can be made. An essential feature of these equations is that γ appears with oppossite signs in $n-p$ (or $\Sigma^- - \Sigma^+$) and $\Xi^- - \Xi^0$ splitting while $n-p$ splitting is more sensitve to μ, then to γ. This permits us to obtain reliable upper and lower bounds on γ, while determining μ in an independent way and allowing for a check of Weinberg's prediction (1.1).

2 The Method

In the QCD sum rule method for the baryon case we consider the polarization operator

$$\Pi(q) = i \int d^4x e^{iqx} < 0 \mid T\{\eta(x), \bar{\eta}(0)\} \mid 0 > \tag{2.1}$$

where $\eta(x)$ is the current with the baryon quantum numbers. For the proton

$$\eta(x) = \epsilon^{abc} u^a(x) C\gamma_\mu u^b(x)\gamma_5\gamma_\mu d^c(x) \tag{2.2}$$

Here $u^a(x)$ and $d^c(x)$ stand for the u and d fields,C is charge conjugation matrix, a, b, c - are colour indices. In order to obtain the hyperon current the following substitutions in (4) must be done in (2.2)

$$\Sigma^- : u \to d, d \to s$$

$$\Sigma^+ : d \to s,$$

$$\Xi^- : u \to s,$$

$$\Xi^0 : u \to s, d \to u$$

As discussed in Refs.[15,20], the current as defined in (2.2) seems to be the the most suitable one for the calculation of baryon masses.It results in a relatively small contribution of higher excited states in both chiral structures Π_1 and Π_2 of the polarization operator

$$\Pi(q) = \hat{q}\Pi_1(q^2) + \Pi_2(q^2) \tag{2.3}$$

and in a good convergence of the OPE.

For each structure Π_1, Π_2 we can write the dispersion relation

$$\Pi_{1,2}(q^2) = (1/\pi)\int^{\infty} \frac{\rho_{1,2}(s)}{s - q^2} ds \tag{2.4}$$

The left-hand side (l.h.s.) of eq.(2.4)is calculated in the framework of OPE at large negative values of q^2, i.e.$|q^2| >> R_c^{-2}$ where R_c denotes the confinement radius. In OPE we keep terms up to dimension $d = 7$. As was shown in Refs.[16,17] (see also Appendix B of Ref.[18]),operators of higher dimension ($d = 8$ for Π_1 and $d = 9$ for Π_2) give small contrbutions to the sum rules. We also neglect the perturbative corrections of the order α_s (as can be shown using the results of Ref.[21], they mainly affect the residue at the baryon pole but not the baryon masses).

The r.h.s. of Eq.(2.4) is represented in terms of physical states and modelled in such a way that the lowest energy baryon state is singled out while higher energy states are approximated by a continuum

$$\rho_{1,2}(s) = \lambda^2(1,m)\delta(s-m^2) + \varphi_{1,2}(s)\theta(s-W_{1,2}^2) \quad (2.5)$$

Here λ denotes the overlap

$$< 0 \mid \eta \mid B > = \lambda_B v_B \quad (2.6)$$

between the vacuum and respective baryon, while v_B is the baryon spinor. The functions $\phi_{1,2}$ in the second term of (2.5) are determined as the discontinuities of $\Pi_{1,2}$ at large s

$$\varphi_{1,2}(s) = (\frac{1}{2i})[\Pi_{1,2}(s+i\epsilon) - \Pi_{1,2}(s-i\epsilon)] \quad (2.7)$$

The continuum thresholds $W_{1,2}^2$ (which may be unequal in the general case), the pole position m and the overlap λ^2 will be the variables to be determined from the sum rules.

We apply the Borel transform with the Borel mass M to both sides of eq.(2.4). This procedure is useful for several reasons. It removes the subtraction terms from the dispersion relation and supresses the contribution of excited states in in the r.h.s. of (2.4). It furthermore suppresses the contribution of the next to leading terms in OPE of the l.h.s. of (2.4), thus improving convergence of the series. After the Borel transform, the sum rules appear as equations that hold for a range of values of M, the confidence interval, where the contributions of higher order terms in the OPE are small and the impact of the parametrization of the excited states in the r.h.s. which is model-dependent is minimal and does not exceed the contribution of the pole term.

This method was used in [15-17] to determine octet baryon masses in the absence of isospin violation. The parameters of m, λ and W were obtained with an accuracy of about 10-15%. There, it was shown that in the nucleon case the quark condensate $< 0 \mid \bar{u}u \mid 0 > = < 0 \mid \bar{d}d \mid 0 >$ plays the dominant role. When considering hyperons we must include the strange quark mass m_s which breaks the $SU(3)$ flavour symmetry, as well as the flavour symmetry breaking in the strange condensate

$$\beta = \frac{<0 \mid \bar{s}s \mid 0>}{<0 \mid \bar{u}u \mid 0>} - 1 \quad (2.8)$$

The best fit of hyperon masses to the QCD sum rules calculations is provided by the values $m_s = 150MeV$ and $\beta = -0.2$ - see ref.[19].

3 Sum Rules for Isospin Splitting in Baryon Octet

In order to include the isospin violating effects we need to take into account the non-zero values of the quark masses m_u, m_d. In order to extract the isotopic mass differences, it appears reasonable to consider the difference of the sum rules for baryons that differ by isospin projection only. Thus, we shall arrive at equations for the parameters $\delta m, \delta\lambda^2, \delta W^2$ as well as γ . Since we neglect the electromagnetic effects δm represents a subtracted mass difference

$$\delta m = (\delta m)_{phys} - (\delta m)_{el} \quad (3.1)$$

where $(\delta m)_{phys}$ is the physical (experimental) value of mass splitting and $(\delta m)_{el}$ is the contribution due to electromagnetic interaction. For the latter we take the values presented in Ref.[14].

$$(m_n - m_p)_{el} = -0.76 \pm 0.30MeV \quad (3.2)$$

$$(m_{\Sigma^-} - m_{\Sigma^+})_{el} = 0.17 \pm 0.30MeV \quad (3.3)$$

$$(m_{\Xi^-} - m_{\Xi^0})_{el} = 0.86 \pm 0.30MeV \quad (3.4)$$

Taking the experimental mass differences from Ref.[22]

$$(m_n - m_p)_{phys} = 1.29MeV \quad (3.5)$$

$$(m_{\Sigma^-} - m_{\Sigma^+})_{phys} = 8.09 \pm 0.09MeV, \quad (3.6)$$

$$(m_{\Xi^-} - m_{\Xi^0})_{phys} = 6.4 \pm 0.6 MeV \tag{3.7}$$

we arrive at

$$\delta m_N = 2.05 \pm 0.30 MeV \tag{3.8}$$

$$\delta m_\Sigma = 7.9 \pm 0.33 MeV \tag{3.9}$$

$$\delta m_\Xi = 5.54 \pm 0.67 MeV \tag{3.10}$$

We perform our calculations to linear order in the isospin symmetry violating quantities, i.e. γ and $m_{u,d}$ and in m_s. The polarization operators for Σ^+ and Ξ^0 were calculated in [17] to linear order in the strange quark mass. It is trivial to obtain the proton polarization operator including the contribution from the light quark masses $m_{d,u}$ by simply replacing m_s by $m_d(m_u)$ in the polarization operator for $\Sigma^+(\Xi^0)$. The neutron result is then arrived at by further substituting $m_u \leftrightarrow m_d$. The appearance of the γ factor is also easily understood. In the lowest order diagram for the OPE of the polarization operator of the proton (neutron), it is the $u(d)$ quarks that form a loop. Therefore, for the chosen form of the source current (2.2) for the proton (neutron), the $u(d)$ condensate appears in the chirality conserving structure while the $d(u)$ condensate appears in the chirality violating one (see also [5]). The polarization operators for the Σ and Ξ can be obtained from the corresponding formulae in Ref.[17] in a similar manner.

Thus, using eqs.(14) and (17) of Ref.[17] we obtain the sum rules for nucleon [1].

$$\{2\mu[aM^2E_0(W_N^2/M^2)L^{-1} - \frac{1}{6}m_0^2 aL^{-2}] +$$

$^2 =$

$$+\frac{8}{3}\gamma a^2 L\}e^{m_N^2/M^2} = \delta\bar{\lambda}_N^2 - 2\bar{\lambda}_N^2 \delta m \frac{m}{M^2} -$$

[1]We take this opportunity to correct a misprint in Ref.[17]. The factor 1/2 in front of the first term of eq.(17) for δ_Ξ should be replaced by 2/3.

$$-\frac{1}{2}exp(-\frac{W_N^2-m_N^2}{M^2})L^{-1}(W_N^4+\frac{1}{2}b)\delta W_{1N}^2 \quad (3.11)$$

$$\{2\mu[M^6E_2(W_N^2/M^2)L^{-2}-\frac{4}{3}a^2]+$$

$$+2\gamma a M^4E_1(W_N^2/M^2)\}e^{m_N^2/M^2}=\delta m_N[2\frac{m_N^2}{M^2}-1]\tilde{\lambda}_N^2-$$

$$-\delta\tilde{\lambda}_N^2 m_N+2a\,exp(-\frac{W_N^2-m_N^2}{M^2})W_N^2\delta W_{2N}^2 \quad (3.12)$$

where $\delta f=f(n)-f(p)$. The functions

$$E_0(x)=1-e^{-x} \quad (3.13)$$

$$E_1(x)=1-(1+x)e^{-x} \quad (3.14)$$

$$E_2(x)=1-(1+x+\frac{x^2}{2})e^{-x} \quad (3.15)$$

take into account the continuum. The parameters a, b and m_0^2 are connected with condensates

$$a=-(2\pi)^2<0\mid\bar{u}u\mid 0>_\mu=0.55GeV^3 \quad (3.16)$$

$$b=(2\pi)^2<0\mid(\alpha_s/\pi)G^n_{\mu\nu}G^n_{\mu\nu}\mid 0>=0.5GeV^4 \quad (3.17)$$

$$-g<0\mid\bar{u}\sigma_{\mu\nu}\frac{1}{2}\lambda^n G^n_{\mu\nu}u\mid 0>=m_0^2<0\mid\bar{u}u\mid 0> \quad (3.18)$$

$m_0^2=0.8GeV^2$. (For a discussion of the numerical values used here see Ref.[23].) The factor

$$L = [\frac{ln(M/\Lambda)}{ln(\bar{\mu}/\Lambda)}]^{4/9} \quad (3.19)$$

accounts for anomalous dimensions ($\bar{\mu}$ is the normalization point). In what follows we use the numerical values $\Lambda = 150MeV, \bar{\mu} = 0.5GeV$. Also we take the value of the residue at the nucleon pole and the continuum threshold W^2 obtained from the best fit for the sum rule in the nucleon channel (isospin symmetric case, see Appendix B of Ref.[18])

$$\tilde{\lambda}_N^2 = 32\pi^4\lambda_N^2 = 2.1GeV^6, \quad (3.20)$$

$$W_N^2 = 2.3GeV^2 \quad (3.21)$$

For the sake of generality we have assumed that the values for the continuum threshold differences $\delta W_{1,2N}^2$ in equations (3.11) and (3.12) may be different although it is a simple and plausible assumption to take them to be equal. The sum rules (3.11),(3.12) must hold in the Borel confidence interval [16]

$$0.8GeV^2 \leq M^2 \leq 1.4GeV^2 \quad (3.22)$$

For Σ hyperons we have in a similar way

$$\{-\frac{2}{3}\mu m_0^2 a L^{-1} + \frac{8}{3}\gamma a^2 L\} e^{m_\Sigma^2/M^2} =$$

$$\delta\tilde{\lambda}_\Sigma^2 - 2\tilde{\lambda}_\Sigma^2 \delta m_\Sigma m_\Sigma/M^2 - \frac{1}{2}exp(-\frac{W_\Sigma^2 - m_\Sigma^2}{M^2})L^{-1}[W_\Sigma^4 +$$

$$+b/2 - 4m_s a(1+\beta)]\delta W_{1\Sigma}^2 \quad (3.23)$$

$$\frac{16}{3}a^2(\mu + \gamma m_s)e^{m_\Sigma^2/M^2} = -\delta m_\Sigma(\frac{2m_\Sigma^2}{M^2} - 1)\tilde{\lambda}_\Sigma^2 + \delta\tilde{\lambda}_\Sigma^2 m_\Sigma$$

$$-\frac{1}{2}exp(-\frac{W_\Sigma^2 - m_\Sigma^2}{M^2})W_\Sigma^2[4a(1+\beta) + m_s W_\Sigma^2]\delta W_{2\Sigma} \quad (3.24)$$

where $\delta f = f(\Sigma^-) - f(\Sigma+)$.
The constants $\tilde{\lambda}^2_\Sigma$ and W^2_Σ were determined in [18]

$$\tilde{\lambda}^2_\Sigma = 3.7GeV^2 \tag{3.25}$$

$$W^2_\Sigma = 3.2GeV^2 \tag{3.26}$$

Equations (3.23),(3.24) must be satisfied in the interval

$$1.2GeV^2 \leq M^2 \leq 1.8GeV^2 \tag{3.27}$$

Note that in this case the isospin violating effects manifest themselves only in higher order terms of the OPE; they disappear at high q^2 and do not contribute to discontinuity, i.e. (see eq.(2.7))

$$\delta W^2_{1\Sigma} = \delta W^2_{2\Sigma} = 0 \tag{3.28}$$

The sum rules for Ξ hyperons read

$$-2\mu a[M^2E_0(W^2/M^2)L^{-1} + \frac{1}{6}m_0^2L^{-2}]e^{m_\Xi^2/M^2} =$$

$$= \delta\tilde{\lambda}^2_\Xi - 2\tilde{\lambda}^2_\Xi\delta m_\Xi m_\Xi/M^2 -$$

$$-\frac{1}{2}exp(-\frac{W^2_\Xi - m^2_\Xi}{M^2})L^{-1}(W^4_\Xi + \frac{1}{2}b)\delta W^2_{1\Xi} \tag{3.29}$$

$$\{2\mu[M^6E_2(W^2_\Xi/M^2)L^{-2} + \frac{4}{3}a^2(1+\beta)^2)]+$$

$$+2\gamma a[M^4E_1(W^2_\Xi/M^2) + \frac{4}{3}am_s(1+\beta))]\}e^{m^2_\Xi/M^2} =$$

$$= \delta\tilde{\lambda}^2_\Xi - \delta m_\Xi(2m^2_\Xi/M^2 - 1)\tilde{\lambda}^2_\Xi -$$

$$-2aexp\left(-\frac{W^2_\Xi - m^2_\Xi}{M^2}\right) W^2_\Xi\delta W^2_{2\Xi} \tag{3.30}$$

where $\delta f = f(\Xi^-) - f(\Xi^0)$ and (see [19])

$$\tilde{\lambda}_{\Xi}^2 = 5.0GeV^6 \tag{3.31}$$

$$W_{\Xi}^2 = 3.6GeV^2 \tag{3.32}$$

The sum rules (3.29),(3.31) are expected to be satisfied for Borel masses

$$1.2GeV^2 < M^2 < 1.8GeV^2 \tag{3.33}$$

It is instructive to also consider the linear combinations of Eqs.(3.11) and (3.12),(3.23) and (3.24) and (3.29) and (3.30) which do not contain the unknown constants $\delta\tilde{\lambda}^2$. We present them under assumption $\delta W_1^2 = \delta W_2^2$ for each baryon. Putting the baryon mass splittings on the l.h.s. of the sum rule equations we obtain for the nucleon

$$\delta m_N = e^{m_N^2/M^2}\tilde{\lambda}_N^{-2}\{[\mu[-2M^6E_2L^{-2}+$$

$$+\frac{8}{3}a^2 - 2m_N E_0 a M^2 L^{-1} + \frac{1}{3}m_N m_0^2 a L^{-2}]-$$

$$-2\gamma a(M^4E_1 + \frac{4}{3}amL) - \delta W_N^2 exp(-W_N^2/M^2)\times$$

$$\times[\frac{1}{2}W_N^4 m_N L^{-1} + \frac{1}{4}bm_N L^{-1} - 2aW_N^2]\} \tag{3.34}$$

For Σ-hyperons (assuming again that we can neglect the isospin violating effects in the continuum) we have

$$\delta m_\Sigma = e^{m_\Sigma^2/M^2}\tilde{\lambda}_\Sigma^{-2}\{\frac{16}{3}\mu a[a+$$

$$+\frac{1}{8}m_0^2 m_\Sigma L^{-1}] - \frac{8}{3}\gamma a^2(m_\Sigma L - 2m_s)\} \tag{3.35}$$

The analogous sum rule for Ξ hyperons has the form

$$\delta m_\Xi = e^{m_\Xi^2/M^2}\tilde{\lambda}_\Xi^{-2}\{2\mu[M^6E_2L^{-2}+$$

$$+\frac{4}{3}a^2(1+\beta)^2 + am_\Xi(M^2E_0L^{-1} + \frac{1}{6}m_0^2L^{-2})]+$$

$$+2\gamma a[M^4E_1 + \frac{4}{3}m_s a(1+\beta)] - \delta W_\Xi^2 exp(-W_\Xi^2/M^2)\times$$

$$\times[\frac{1}{2}m_\Xi(W_\Xi^4 + \frac{b}{2})L^{-1} - 2aW_\Xi^2]\} \quad (3.36)$$

4 The Analysis of the Sum Rules

Each of the equations (3.34)-(3.36) can be written in the form

$$\delta m = A(M^2)\mu + B(M^2)\gamma + C(M^2)\delta W^2 \quad (4.1)$$

Of these equations (3.35) is the simplest one as $\delta W_\Sigma^2 = 0$ - see eq.(3.28). For this case straightforward evaluation yields $A_\Sigma = 1.4(1.7)$, $B_\Sigma = -0.85GeV(-0.75GeV)$ for a Borel mass $M^2 = 1.2GeV^2(1.4GeV^2)$. Substituting the values for the quark mass difference and the isotopic mass difference $\mu = 3.3MeV$ (Eq.(1.1)) and δm_Σ (see (3.9) we obtain from (4.1)

$$\gamma = -2.7\times10^{-3}(-4.2\times10^{-3}) \quad (4.2)$$

However, due to the fact that only one term of the OPE contributes to the Σ sum rules in this case, we cannot attach too much significance to this result. Including the uncertainties in (3.9) and an uncertainty of, say, 1 MeV to the quark mass difference, the only safe conclusion we can reach from the Σ- hyperon sum rules is that γ is negatve, and lies somewhere in the interval $-6.10^{-3} < \gamma < 0$.

Let us now turn to nucleons and Ξ hyperons. In this case we need an estimate for the difference in the continuum thresholds for the particles differing only in isospin projection. In the nucleon case for example, we expect a difference in the continuum threshold for the neutron and the proton. It seems reasonable to assume that δW^2 is positive, and that for each baryon

$$\frac{\delta W^2}{W^2} \approx \frac{2\delta m}{m} \quad (4.3)$$

The equation for the Ξ hyperons(3.36) yields an important piece of information as γ enters with a positive sign while $C(M^2)$ is negative. Thus, we are able to obtain the upper bound on $|\gamma|$ by putting $\delta W_{\Xi}^2 = 0$. The numerical values for the coefficients A-C are

$$A_{\Xi} = 2.46(2.75), \quad B_{\Xi} = 1.15(1.16)GeV,$$

$$C_{\Xi} = -0.174(-0.227)GeV^{-1} \tag{4.4}$$

for $M^2 = 1.6(1.2)GeV^2$. The weak dependence of the coefficients on the Borel parameter indicates a certain amount of selfconsistency in the sum rules. Using the numerical values of μ and δm_{Ξ} as given by Eqs.(1.1) and (3.10) we obtain

$$\gamma > -3.5 \times 10^{-3} \tag{4.5}$$

Larger values of $|\gamma|$ are only possible at the expence of larger values of the quark mass difference,e.g.$\gamma = -5.10^{-3}$ requires $\mu \geq 4.5MeV$

For the nucleon Eq.(4.1) is satisfied with the coefficients

$$A_N = -0.46(-0.61), B_N = -1.92(-1.84)GeV$$

$$C_N = 0.046(0.065)GeV^{-1} \tag{4.6}$$

at $M^2 = 1.0(1.2)GeV^2$. From (4.3) we can estimate $\delta W_N^2 < 10^{-2}GeV^2$. and therefore

$$C_N(M^2)\delta W_N^2 << \delta m_N - A_N(M^2)\mu \tag{4.7}$$

Thus, clearly we need a non-zero and negative value of γ to satisfy (4.1). Again,since negative values of γ and positive ones for δW_N^2 contribute with the same sign to $\delta m_N - A_N\mu$, we obtain an upper bound for $|\gamma|$ by setting $\delta W_N^2 = 0$.This turns out to be $|\gamma| = 2.10^{-3}$ for $\mu = 3.3MeV$.More precisely, with $\delta m_N = 2.05MeV, \mu = 3.3MeV$ and $\delta W_N^2 = 8 \times 10^{-3}GeV^2$ Eq.(4.1) yields

$$\gamma = -(1.5 - 2.0).10^{-3} \tag{4.8}$$

for $0.8GeV^2 < M^2 < 1.2GeV^2$. The nucleon sum rule provides a much stronger upper bound on $|\gamma|$ than the Ξ sum rule for the case of large μ: even for $\mu = 5MeV$ we find that $|\gamma| < 3 \times 10^{-3}$. However, it is clear that there is a dependence on the Borel parameter M^2 in these sum rules, which indicates that higher order terms in the OPE are non- negligible and deteriorate the accuracy of the ressult. Therefore, our conservative conclusion from the consideration of the sum rules (3.34)-(3.36) with respect to γ is

$$\gamma = (-2 \pm 1).10^{-3} \tag{4.9}$$

Let us now study the sum rules (3.11),(3.12),(3.23),(3.24),(3.29) and (3.30) which contain more information,as it is possible to extract $\delta\tilde{\lambda}^2$ in two ways from each of pairs of the sum rules and to check if they coincide and depend weakly on M^2.We have plotted in Fig.1 the result for $\delta\tilde{\lambda}_N^2$ as calculated from (3.11) (curves labelled (1)) and (3.12) (curves labelled (2))for $\mu = 3.3MeV, \gamma = -2.10^{-3}$ and $\delta W_1^2 = \delta W_2^2 = 1.10^{-2}GeV^2$ (solid curves). The agreement is satisfactory and the M^2 dependence is weak. In fact, the agreement tends to be more pronounced for slightly smaller values of $|\gamma|$. On the other hand, we also find that the sum rules cannot be made consistent for larger values of $|\gamma|$, such as the value $|\gamma| > 6.10^{-3}$ which was used in [6,7,12]. A closer look at the linear combination of the sum rules (3.11) and (3.12) that eliminates $\delta\tilde{\lambda}_N^2$ with the condition $\delta W_1^2 = \delta W_2^2$ removed, reveals that it can only be satisfied for $\delta W_1^2 >> \delta W_2^2$. This being favoured in the case of large $|\gamma|$, we plot in Fig.1 $\delta\tilde{\lambda}_N^2$ as predicted by each of the equations (3.11) and (3.12) for $\gamma = -6 \times 10^{-3}$ and $\delta W_1^2 = 25 \times 10^{-3}GeV^{-2}, \delta W_2^2 = 0$ (dashed lines). A strong discrepancy of two sum rules is evident. We would like to stress that the unreasonably large value for δW_1^2 used serves to reduce the discrepancy between the curves. Indeed, Eqs.(3.11) and (3.12) show that reducing δW_1^2 or increasing δW_2^2 increases the discrepancy.

In the same way we can investigate the domain of small $|\gamma|$ We find in a similar manner that a value of $\gamma = 0$ can only be tolerated at the expense of a large difference between δW_1^2 and δW_2^2, for instance $\delta W_1^2 = 0$ and $\delta W_2^2 = 10 \times 10^{-3}GeV^2$. This being unreasonable, we can safely exclude $\gamma = 0$.

We present in Fig.2 the results for the analogous investigation of the Ξ

hyperon sum rule. The solid lines correspond to $\gamma = -2.10^{-3}, \delta W_1^2 = \delta W_2^2 = 3.10^{-3} GeV^2$, whereas the dashed lines were produced with $\gamma = -6.10^{-3}$ and $\delta W_1^2 = \delta W_2^2 = 15.10^{-3}$. Again, the values of $\delta W_{1,2}^2$ were chosen such to maximize the agreement between the curves. As before, good agreement is achieved for the first case, disagreement for the second.

Fig.3 shows the result of evaluating Eqs.(3.23) and (3.24) for the Σ hyperon with $\gamma = -2.10^{-3}, \mu = 3.3 MeV$ and $\delta W_1^2 = \delta W_2^2 = 0$

Finally, we would like to investigate the dependence of the sum rules on the quark mass difference. As the Ξ sum rules are most sensitive to this parameter, we shall focus on those. We have plotted in Fig.4 the prediction for $\delta\lambda_{\Xi}^2$ from Eqs.(3.29) and (3.30) labelled (1) and (2) respectively, for $\gamma = -2.10^{-3}$, and $\delta W_1^2 = \delta W_2^2 = 3\times10^{-3} GeV^2$ for two additional values of μ, namely $\mu = 2.3 MeV$ (dashed lines) and $\mu = 4.3 MeV$ (solid lines) (see Fig.2 for the intermediate value of μ). It is clear that both choices lead to a serious disagreement between the curves, that can only be resolved assuming a large difference between δW_1^2 and δW_2^2 which appears unreasonable. Comparing Figs.2 and 4 we conclude that μ should be close to 3 MeV with an error (conservatively) of about 1 MeV.

Our final results then for the quantities γ and μ as implied by the sum rules (3.11),(3.12),(3.23),(3.24),(3.29), and (3.30) are

$$\gamma = (-2 \pm 1).10^{-3} \tag{4.10}$$

$$\mu = m_d - m_u = (3.0 \pm 1.0) MeV \tag{4.11}$$

In order to put this result into perspective, let us comment on the approximations going into it. It is known [16,18] that higher order terms in the OPE neglected here are small for the sum rule for the nucleon mass, and cannot change this value by more than 10% . In the sum rules studied above, the contributions to the sum rule coming from the highest dimension terms in the chirality preserving structure (terms of the order $\sim \mu m_0^2 a$) are always small, smaller than 15% of the main term. We find that contributions of proportional to γa^3 term of dimension $d = 9$ change the value of γ only by a few per cent.

Let us now discuss α_s corrections. The α_s corrections to the main terms (those proportional to $\gamma, \gamma a$ and γa^2) can easily be calculated using the results of Ref.[21]. They turn out to be small ($\leq 10\%$). The α_s corrections to the main term propotional to μ are presently unknown. In the (isospin-symmetric) sum rules for the proton mass the α_s corrections to the main term are relatively large. However, as can be shown using the formulae of Ref.[21], they mainly change the value of the residue $\tilde{\lambda}_N^2$ (increasing it by about 25-20%) while only slightly changing the pole position (diminishing the proton mass by about 5%) [2]. We believe that these conlusions carry over to the sum rules including isospin violaton presented here. In any case we expect the corrections to γ to fall within the conservatively chosen error bars included in the result (4.10). The Ξ sum rules imply that solely increasing $\tilde{\lambda}^2$ is unreasonable as it leads to much smaller value of $|\gamma|$).

5 Discussion and Comparison with Previous Work

In most instances of previous work, values of the order $\gamma = -(6-10)\times 10^{-3}$, significantly different from ours, were obtained. Let us therefore examine some of the earlier results.

In the paper by Paver et al. [7] γ was calculated in a constituent quark model. Hatsuda et al. [4] and Adami and Brown [5] (the latter in one one of their approaches) used the Nambu-Jona-Lasinio model. We believe that values for γ obtained in these approaches are unreliable for the following reasons. In QCD, as well as PCAC-type lagrangians,the value of the current quark masses can be obtained with a rather good accuracy. In QCD furthermore it is obvious that the value of γ is strongly correlated with $\mu = m_d - m_u$. However, in the above mentioned model appoaches, the relation of μ to the other model parameters is obscure. (E.g. in QCD, μ as well as the condensates are renormalization-scale dependent. This concept is absent in quark model and Nambu-Jona-Lasinio approaches).

In Refs.[9-12] γ was obtained by calculating the polarization operators of the divergence of vector and axial currents in the framework of the QCD sum rules. According to current algebra γ is related to $\Pi_V(0)$, the vector-polarization operator at zero momentum transfer, leading to $\gamma = -9.10^{-3}$ [12]. We are rather sceptical towards this approach as it is well known [24] that the QCD sum rule method fails in the scalar and pseudoscalar channels. Indeed,it cannot explain the strong volation of the Okubo-Zweig-

[2]This is the case choosing the normalization point $\bar{\mu} = 0.5GeV$ adopted here (as in Refs.[15-19]) rather than $\bar{\mu} = 0.2GeV$ as in [21]

Iizuka rule in the pseudoscalar channel [25]. Also, there are very serious problems related to subtractions in this approach.

The analysis in [8] and [5] is more closely related to the one presented here. In [8], γ was determined from mass splittings in the baryon octet via the QCD sum rule method, leading to a value $\gamma = -6.10^{-3}$. The approach of Ref.[8] differs from ours in several points (i) A baryon current different from the adopted here was used,(ii) the mixed condensate (3.18) as well as anomalous dimensions were ignored, (iii) a different set of parametrs was used, namely $m_s(0.5GeV) = 260MeV, < \bar{s}s >= 0.5 < \bar{u}u >$ as opposed to our $m_s(0.5GeV) = 150MeV$,$m_s(0.5) = 150MeV, < \bar{s}s >= 0.8 < \bar{u}u >$. We have serious doubts about the procedure to choose a mixing angle t adopted in Ref[8]. On the one hand, t is constrained to be the same for all members of the octet, on the other hand, it is chosen by requiring, that the continuum contributions in the isospin-violating structures vanishes. Also, a vanishing continuum leads to large higher order terms in the OPE side of the sum rule for baryons, since it is impossible to approximate the functional dependence of the exponential $exp(-m^2/M^2)$ on the l.h.s. with only a few terms that have a power-law dependence on $1/M^2$ on the r.h.s.

In [5], the neutron-proton mass difference was considered in the framework of QCD sum rules and the polarization operator was calculated, however without taking into account the continuum. Also, a systematic analysis of the ensuing sum rules was not performed, as its principal aim was to show the mechanism, which made the proton-neutron mass difference vary with density. Nevertheless, moving the mass difference used by Adami and Brown ($\mu = 4MeV$) towards the one adopted here improve the agreement between the latter and present work.

We now turn to chiral perturbation theory, specifically to the results obtained by Gasser and Leutwyler [6] for the parameter γ. Their equation contains an unknown subtraction term, which however can be written in terms of the flavour-SU(3) breaking condensate parameter β defined in (2.8). The final result from Ref.[6] is then

$$\gamma = -\frac{\mu}{m_s - (m_u + m_d)/2}[-\beta +$$

$$+\frac{1}{16\pi^2 F^2}(m_K^2 - m_\pi^2 - m_\pi^2 ln\frac{m_K^2}{m_\pi^2})] \qquad (5.1)$$

where $F \approx F_\pi = 92MeV$; m_K and m_π are the kaon and pion masses. Numerically,

$$\gamma = 2.3.10^{-2}\beta - 3.10^{-3} = -7.6.10^{-3} \tag{5.2}$$

for $\beta = -0.20$. As we argued in section 4, such a large value of $|\gamma|$ is excluded in our QCD sum rule analysis. We do not see any loopholes in our arguments which could possibly accomodate the result (5.2). We should keep in mind however that (5.1) was obtained in first order chiral perturbation theory. The suspicion persists that higher order terms in the series could significantly alter (5.2).. We surmise that the calculation of these terms, as well as those resulting from α_s corrections to the isospin-violating QSD sum rules for baryons will help to resolve this discrepancy.

Our final remark is connected with the proposed [4,5] explanation of the Nollen-Schiffer anomaly. Using eq.(3.34) and our value of γ it is easy to estimate how the neutron-proton mass difference would behave if the value of the quark condensate is reduced by some amount compared to its vacuum value. We find that for $\gamma = -2.10^{-3}$ a 10% reduction of the quark condensate in the nucleus results in a decrease of the neutron-proton mass difference by 1 MeV - just the value needed for a resoluton of the NS-anomaly. A 10% decrease of the quark condensate inside the nucleus appears to be quite reasonable.

Acknowledgements

We are indebted to Prof's.G.E.Brown and H.Leutwyler for useful discussions. Two of us (E.G.D. and B.L.I) use the opportunity to thank Prof.G.Brown for his hospitality during our visit to Stony Brook.

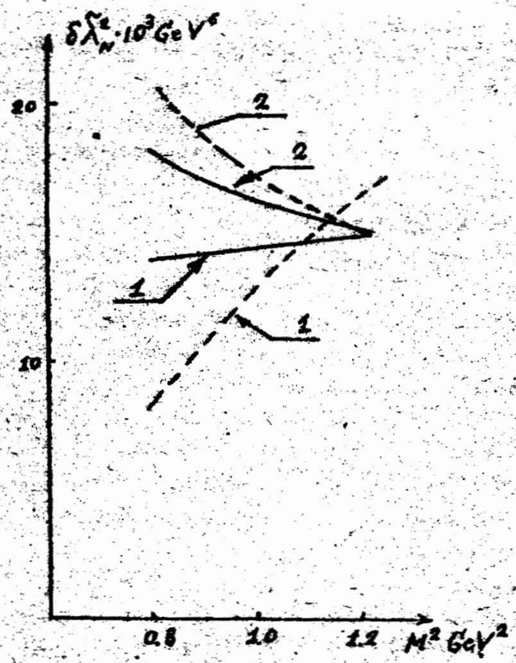

Fig.1. The values of $\delta\bar{\lambda}_N^2$ calculated from the nucleon sum rules (3.11) (labelled as 1) and (3.12) (labelled as 2) at $\mu = 3.3MeV$. The solid curves correspond to $\gamma = -2.10^{-3}$, the dashed ones - to $\gamma = -6.10^{-3}$.

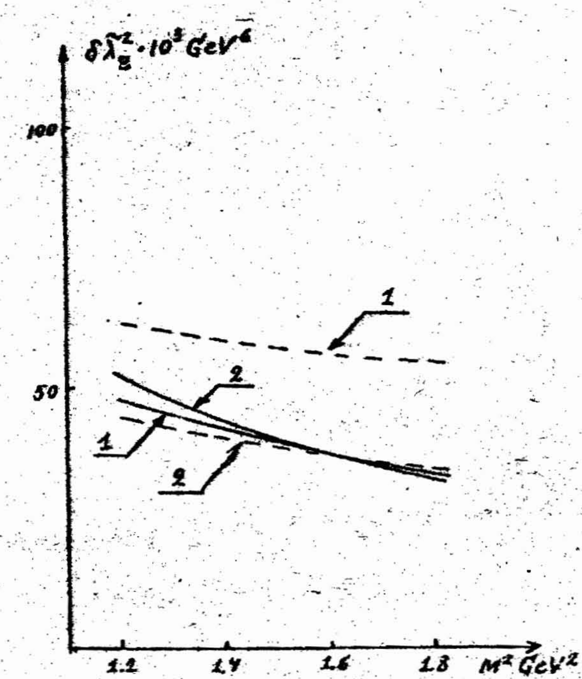

Fig.2. The values of $\delta\tilde{\lambda}^2_\Xi$ calculated from the sum rules (3.29) (labelled as 1) and (3.30) (labelled as 2) at $\mu == 3.3 MeV$. The other notation is the same as in Fig.1.

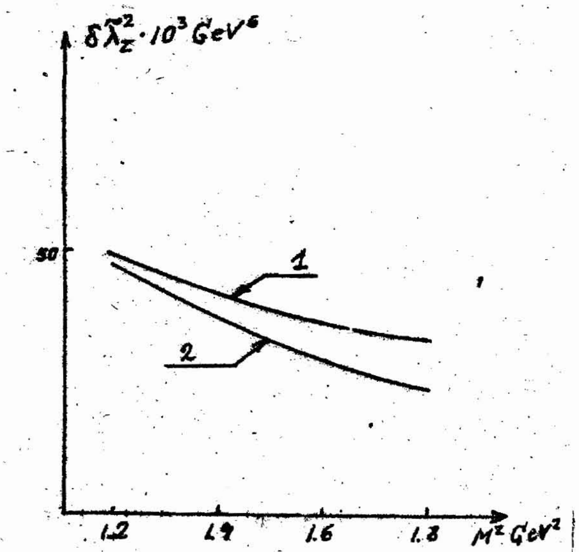

Fig.3. The values $\delta\tilde{\lambda}^2_\Sigma$ calculated from the sum rules (3.23) (labelled as 1) and (3.24) (labelled as 2) at $\mu = 3.3 MeV$, $\gamma = -2.10^{-3}$.

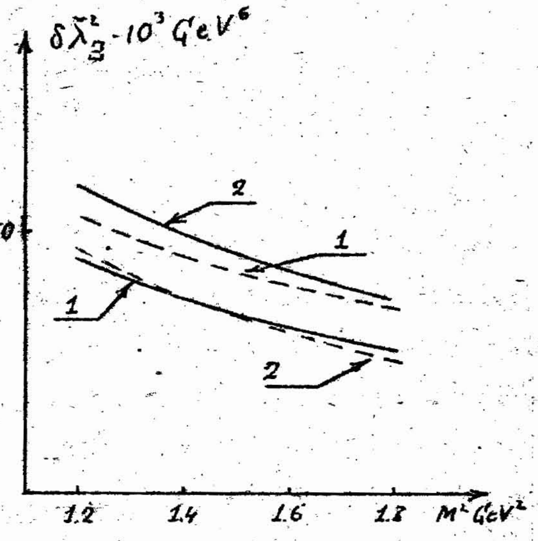

Fig.4. The values $\delta\tilde{\lambda}_3^{2\,\prime}$ calculated from the sum rules (3.29)(labelled as 1) and (3.30) (labelled as 2) at $\mu = 4.3 MeV$ (solid lines) and $\mu = 2.3 MeV$ (dashed lines).

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