INSTITUTE OF THEORETICAL
AND EXPERIMENTAL PHYSICS

ITEP 55–94

A.V.Barkhudaryan, P.R.Zenkevich

FOCUSING OF IONS NEAR THE NEUTRALIZATION ELECTRODE IN ELECTRON COOLING SYSTEM

Moscow, ITEP

MOSCOW 1994

UDK 621.384.663:537 M-16

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A.V.[illegible], [illegible] - M., 1994 - 15p.

For the electron cooling system a stationary longitudinal density of ions near the neutralization electrode has been investigated in one-dimensional approximation. It is discovered that if a transversal field of the electrode penetrates inside the ion beam, then defocusing radial electric field may appear near the border of the beam.

Fig. - 5, ref. - 3

1 Introduction

A previous theory of equilibrium state in electron cooling system was derived for a case when neutralization degree is near unity [1],[2]. In this case the ion energy appearing due to Coulomb interaction with fast electrons is extremely small ($\sim eV$). For such cold ions the electric field of the neutralization electrode is screened by the self Coulomb forces, and the external electric field influences on the ion motion only in a very small region with dimension about Debye radius ("sharp edge model").

However, the recent mesurements of neutralization in LEAR electron cooling system have shown that the real observed neutralization degree is far enough from unity. For such situation ions are born with the appreciable potential energy in the Coulomb field of the electron beam. Thus, without maxvellization the transversal ion energy may be essentially larger than the longitudinal one, and the transversal electric field of the neutralization electrode may penetrate inside the beam and disturb the ion motion.

In our paper we have considered the following model: a) the longitudinal components of the external electric field are screened by the self Coulomb forces of the ion beam; b) the corresponding transversal screening is absent; c) for simplification of the electric field calculations we have limited ourselves by the following geometry: the left part of the infinite cylindrical chamber has uniform potential equal to U_0, the right part has the potential equal to zero.

Such simple model permits us to estimate the effect analitically and to discuss its qualitative features. Really, of course, it is necessary to examine the problem by numerical methods with account of the self-consistent potential of the ion beam.

2 Investigation of Self-consistent State

A condition of the longitudinal screening has the following form:

$$U_{ext}(z) + U_i(z) = 0 \tag{1}$$

Here $U_{ext}(z)$ is a longitudinal potential of the neutralization electrode, $U_i(z)$ is a longitudinal potential of the ion beam (we have supposed that both potentials are calculated for $r = 0$), r, z are, respectively, transversal and longitudinal coordinates. The potentials are normalized by use of the same condition:

$$U_{ext}(\infty) = U_i(\infty) = 0 \tag{2}$$

The equation (1) with account of the normalization condition (2) may be written in the following form:

$$\int_{x_0}^{\infty} G(x, x')\Lambda(x')dx' = 1 - \widetilde{U}F(x) \tag{3}$$

The Green function $G(x, x')$ for ion beam Coulomb potential is determined in Appendix by Eq(17), $F(x) = U_{ext}(0, z)/U_0$, $U_{ext}(r, z)$ is the external potential (see (20)), $\widetilde{U} = U_0/U_\infty$, U_∞ is the potential of the uniform beam, $x = z/b$, where b is the chamber radius.

In Eq(3) $\Lambda(x)$ is a longitudinal linear density of the ions divided by a corresponding density of the uniform beam. We see that formally Eq(3) is an integral Fredholm equation of the second kind for unknown function $\Lambda(x)$. It is convenient for calculations to write this integral equation for argument $u = x - x_0$ in the form:

$$\int_0^{\infty} K(u, u')\Lambda(u')du' = 1 - \frac{F(x_0 + u)}{F(x_0)} \tag{4}$$

where the kernel $K(u, u')$ is determined by:

$$K(u, u') = G(u, u') - \frac{F(x_0 + u)}{F(x_0)}G(0, u') \tag{5}$$

The value of x_0 is connected with $\widetilde{U}$ by the equation:

$$1 - \widetilde{U}F(x_0) = \int_0^{\infty} G(0, u)\Lambda(u)du \equiv \varphi(x_0) \tag{6}$$

We shall consider x_0 as a free parameter of our problem. If x_0 and the corresponding function $\Lambda(u)$ (and, therefore, $\varphi(x_0)$) are known, then the dimensionless potential $\widetilde{U}$ may be expressed as follows:

Table 1: Dependence $\widetilde{U}$ upon x_0

	$a/b = 1/3$	$a/b = 1/2$
x_0	$\widetilde{U}$	$\widetilde{U}$
0.05	0.94	0.79
0.10	1.25	1.10
0.20	1.69	1.54
0.60	3.52	3.19
1.00	8.33	7.48

$$\widetilde{U} = \frac{1 - \varphi(x_0)}{F(x_0)} \tag{7}$$

Corresponding value of the potential U_0 is determined by the following expression:

$$U_0 = \widetilde{U} \cdot U_\infty \tag{8}$$

U_∞ is given in Appendix by Eq(18), dependence $\widetilde{U}$ upon x_0 for different values of a/b is given in Table 1.

The results of calculation $\Lambda(u)$ are presented in Fig.1-Fig.4. We see that charge density of the beam is nonuniform near the boundary of the reflecting field of the electrode. This nonuniformity increases with decrease of the voltage U_0 and with growth of the beam radius (i.e., parameter a/b). If a transversal electric field of the electrode penetrates inside the beam, then its gradient (calculated according to (20)) changes the complete radial focusing gradient acting on the ions as follows:

$$\begin{gathered} \frac{g(u)}{g_0} = 1 - \eta\Lambda(u) - \alpha\widetilde{U}\left(\frac{a}{b}\right)^2 \Sigma \frac{\lambda_k}{J_1(\lambda_k)} exp(-\lambda_k u) \\ \alpha = 2\frac{b}{a} \Sigma \frac{J_1(\lambda_k \frac{a}{b})}{\lambda_k^3 J_1^2(\lambda_k)} \end{gathered} \tag{9}$$

Here η is the coefficient of neutralization, a is the beam cross section radius. $J_n(x)$ is Bessel function of the n-th index, λ_k are roots of $J_0(x)$, the summation is made for all values of k. Values of $g(u)/g_0$ are calculated for the different parameters η, x_0 and a/b and presented in Fig.5-Fig.8.

We see that without transversal screening the gradient of the external field may significantly influence on the ion focusing, moreover, a dependance of this effect on parameters U_0 and a/b is similar with described above dependence of ion longitudinal density. Let us underline that for $x_0 < 0.1$ (i.e. for $U_0/U_\infty < 1.25$) near the edge of the ion beam appears a defocusing radial gradient ($g(u) < 0$), which may result in particle losses.

3 Discussion

Of course, our note does not exhaust this very complicated problem. Particularly, we have not considered the influence of a dipole component of the neutralization electrode field. This dipole component may also penetrate into the ion beam and to affect on ion-dynamics (these both effects diminish with increase of the neutralization electrode voltage). Therefore, such edge effects may help us to explain the influence of this voltage on the neutralization degree, which was found in recent LEAR experiments.

Moreover, it is necessary to underline that straight numerical calculation of the edge effects obligatory has to include the analysis of self-consistent state of the ion beam. Such analysis (especially in three-dimensional case) is not a trivial problem and needs in a tremendous work, but without such work the results of simple-hearted numerical calculations are not reliable.

4 Appendix

The potential of the uniform charged disk in the chamber of circular section is derived as followes. The solution of the Laplace equation with the zeroth boundary condition for a potential on the chamber wall is of the form:

$$U(r,z) = \sum c_k J_0\left(\frac{\lambda_k r}{b}\right) exp\left(-\frac{\lambda_k |z|}{b}\right) \tag{10}$$

Here a,b, consequently, are the radii of the disk and the chamber cross section. The boundary condition is $U(b,z) = 0$.

The boundary conditions on the disk are the following:

$$\frac{dU}{dz} = \begin{cases} 2\pi\sigma & r \leq a \\ 0 & r > a \end{cases} \tag{11}$$

Here σ is the surface density of the charge. Performing the differentiation of (10) we obtain:

$$(\frac{dU}{dz})_{z=0} = \sum \frac{c_k \lambda_k}{b} J_0(\frac{\lambda_k r}{b}) \qquad (12)$$

When multiplied by the $rJ_0(\frac{\lambda_k r}{b})$ and integrated over r from 0 to b this equation in accordance with (12) becomes the form:

$$\frac{c_k \lambda_k}{b} \int_c^b r J_0^2(\frac{\lambda_k r}{b}) dr = 2\pi\sigma \int_0^a r J_0(\frac{\lambda_k r}{b}) dr \qquad (13)$$

Using the expressions for these integrals given in [3] we obtain:

$$c_k = 4\pi\sigma a \frac{J_1(\lambda_k \frac{a}{b})}{\lambda_k^2 J_1^2(\lambda_k)} \qquad (14)$$

To derive the Green function for the paraxial beam we take the expression $\sigma = \rho dz = \frac{I}{\pi a^2 \beta c} dz$, where ρ is the charge density of particles, I is the beam current, βc is the velocity of particles, and obtain from (10) and (14):

$$U(r,z) = \frac{4}{ac} \int_{-\infty}^{\infty} \frac{I(z')}{\beta(z')} dz' \sum \frac{J_1(\lambda_k \frac{a}{b})}{\lambda_k^2 J_1^2(\lambda_k)} J_0(\lambda_k \frac{r}{b}) exp(-\lambda_k \frac{|z-z'|}{b}) \qquad (15)$$

Introducing $\Lambda(z) = \frac{\rho(z)}{\rho(\infty)}$ and taking $r = 0$ we derive:

$$U(0,z) = \frac{I_\infty}{b\beta_\infty c} \int_{-\infty}^{\infty} \Lambda(z') G(z-z') dz' \qquad (16)$$

where the Green function $G(z-z')$ is defined by:

$$G(z-z') = 4\frac{b}{a} \sum \frac{J_1(\lambda_k \frac{a}{b})}{\lambda_k^2 J_1^2(\lambda_k)} exp(-\lambda_k \frac{|z-z'|}{b}) \qquad (17)$$

Here I_∞, β_∞ are the values of I, β on the infinity. It follows from foregoing that the potential on the end of the uniformly charged beam is expressed by the equation:

$$U_\infty = \frac{I_\infty}{\beta_\infty c b} \int_0^\infty G(z) dz = \frac{4 I_\infty b}{\beta_\infty c a} \sum \frac{J_1(\lambda_k \frac{a}{b})}{\lambda_k^3 J_1^2(\lambda k)} \qquad (18)$$

We evaluate also the potential of the half cylinder that has on its left bound the condition $U = U_0$. Taking $z = 0$ in the Eq(10) we obtain:

$$U_0 = \sum c_k J_0(\lambda_k \frac{r}{b}) \qquad (19)$$

Performing the same procedure as in Eq(13) we evaluate c_k and the desired potential:

$$U_{an}(r,z) = U_0 \sum \frac{1}{\lambda_k J_1(\lambda_k)} J_0(\lambda_k \frac{r}{b}) \exp(-\lambda_k \frac{z}{b}) \tag{20}$$

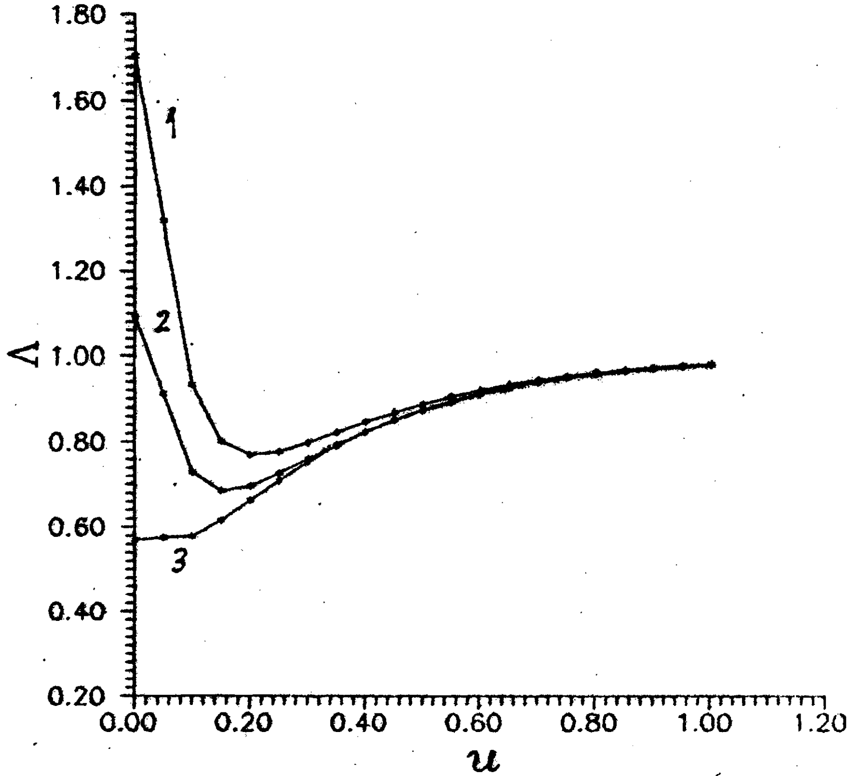

Fig. 1 Linear charge density

$a/b = 1/3$

1 $x_0 = 0.05$
2 $x_0 = 0.10$
3 $x_0 = 0.20$

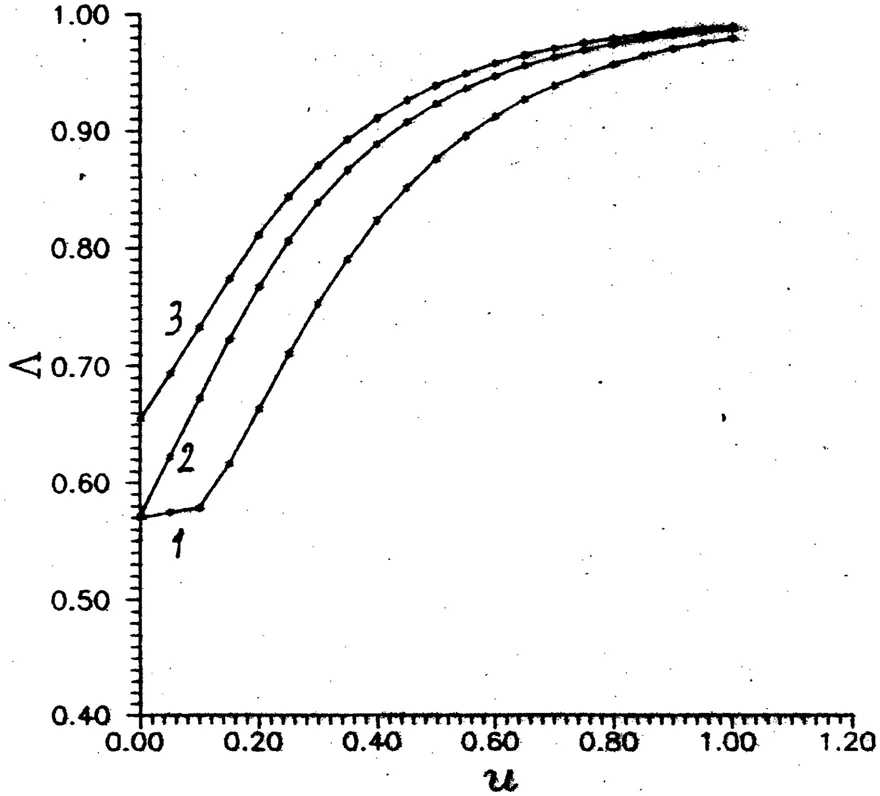

Fig.2 Linear charge density *a/b = 1/3*

1 $x_0 = 0.20$
2 $x_0 = 0.60$
3 $x_0 = 1.00$

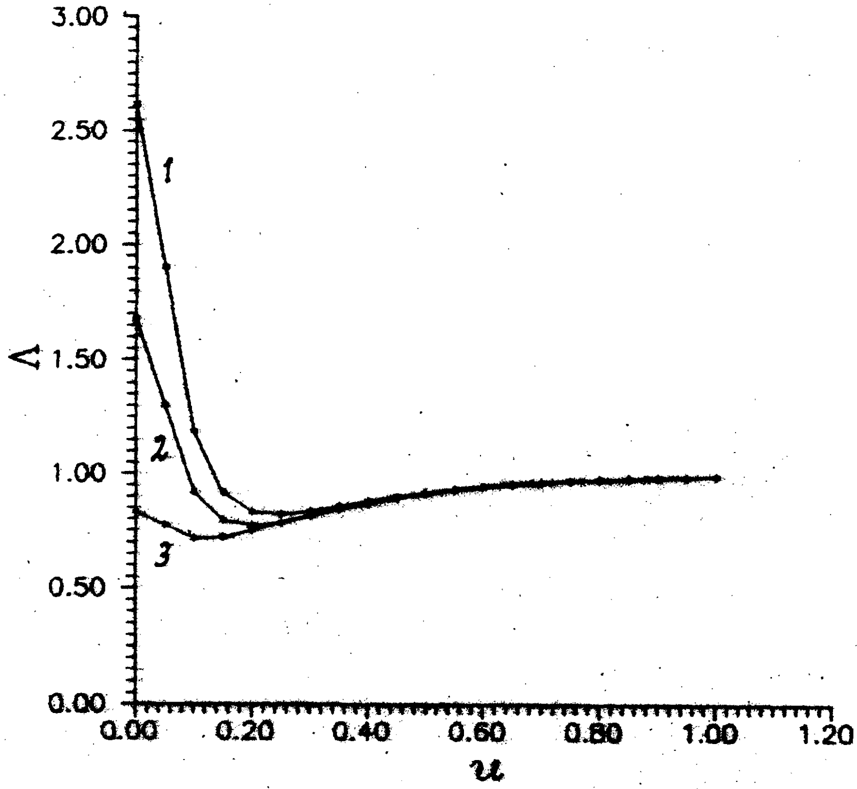

Fig.3 Linear charge density

$a/b = 1/2$

1 $x_p = 0.05$
2 $x_p = 0.10$
3 $x_p = 0.20$

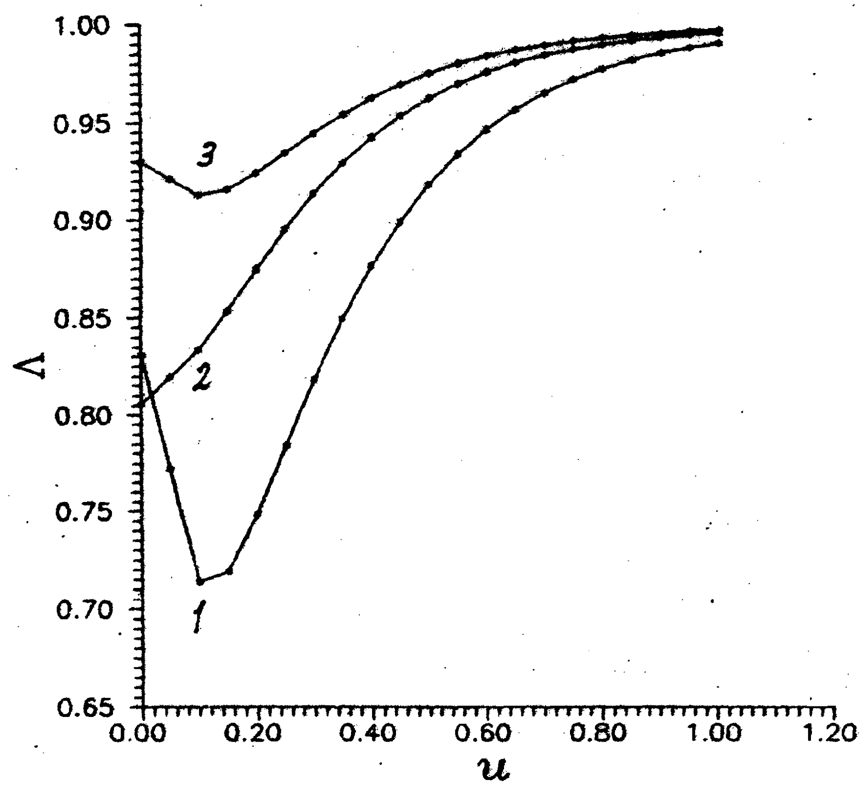

Fig.4 Linear charge density

a/b = 1/2

1 $x_0 = 0.20$
2 $x_0 = 0.60$
3 $x_0 = 1.00$

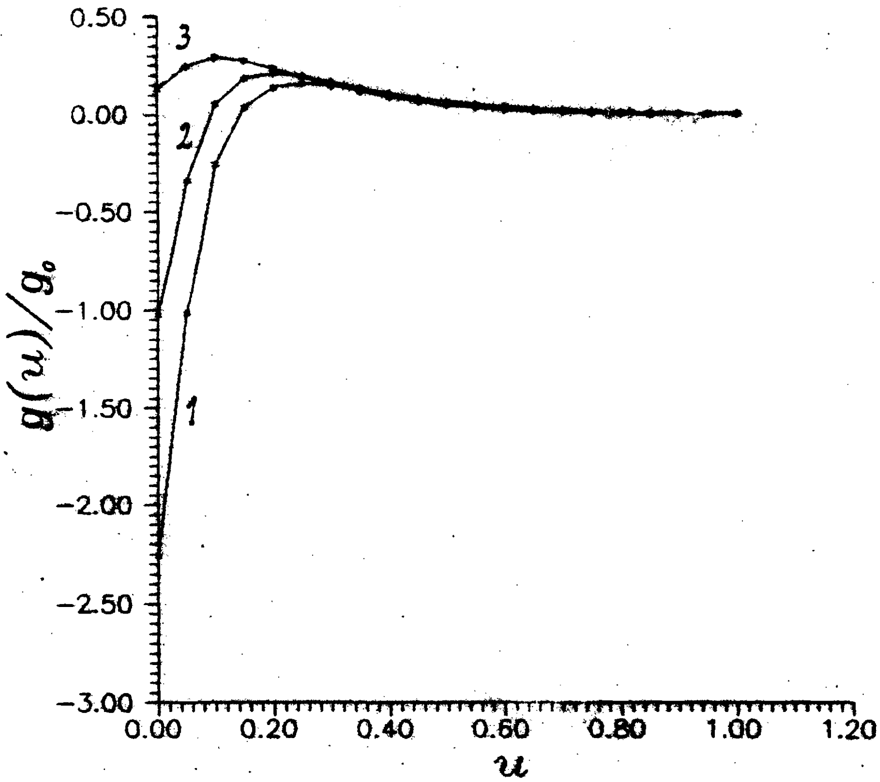

Fig.5 Radial gradient

a/b = 1/3

1 $x_0 = 0.05$
2 $x_0 = 0.10$
3 $x_0 = 0.20$

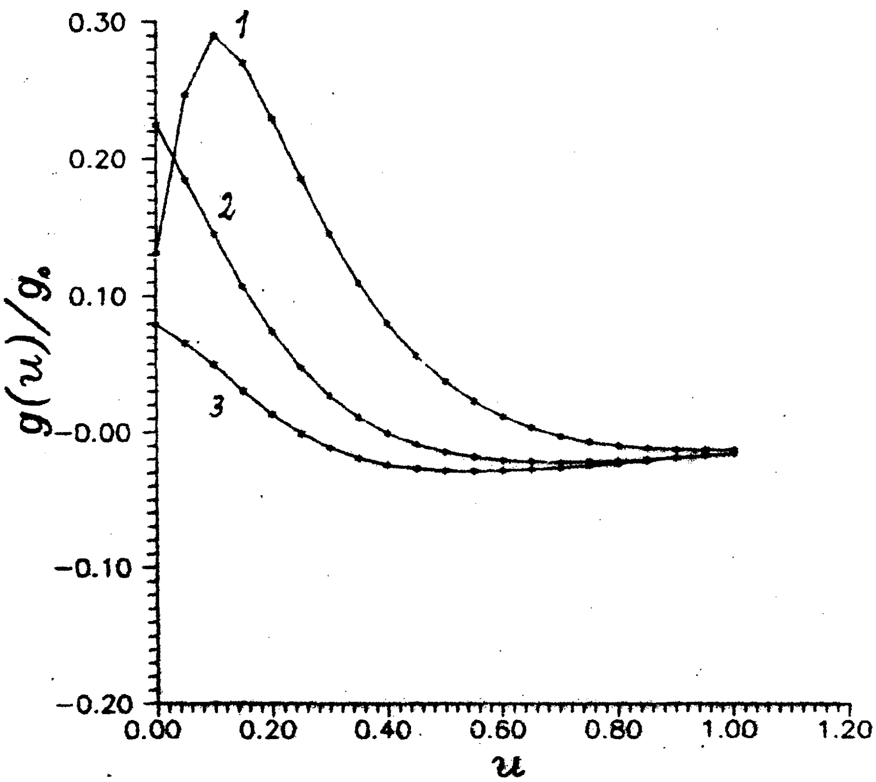

Fig.6 Radial gradient

a/b = 1/3

1 $x_o = 0.20$
2 $x_o = 0.60$
3 $x_o = 1.00$

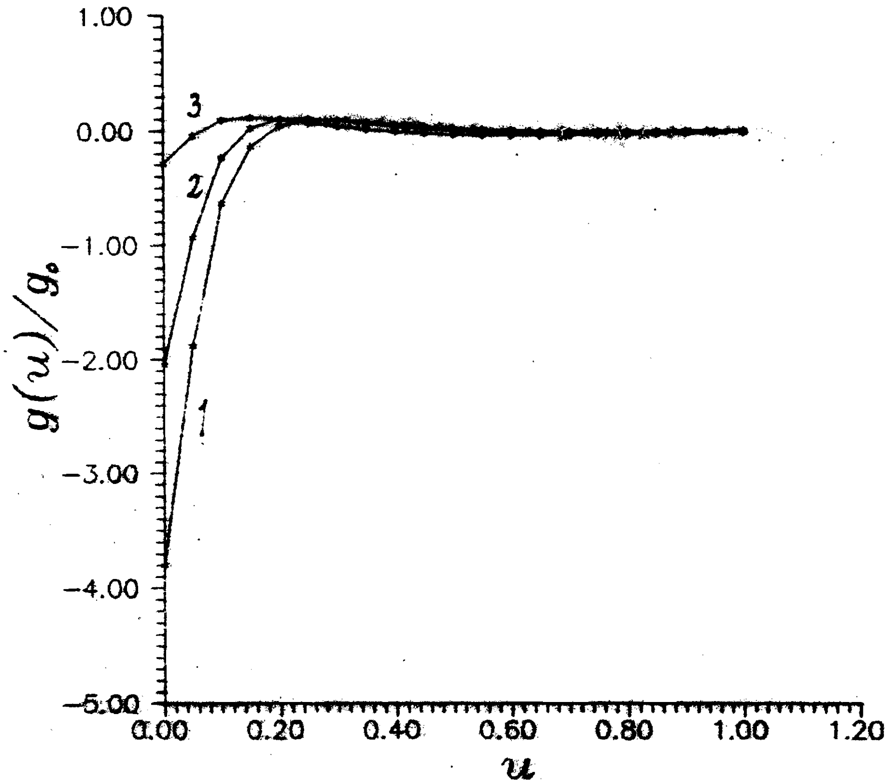

Fig. 7 Radial gradient

$a/b = 1/2$

1 $x_o = 0.05$
2 $x_o = 0.10$
3 $x_o = 0.20$

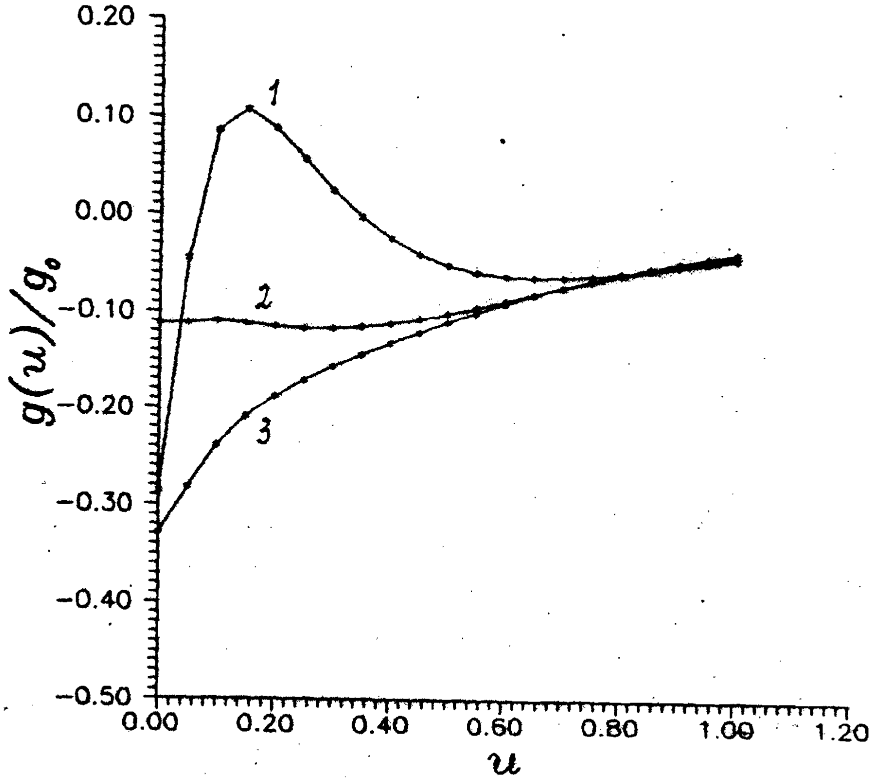

Fig.8 Radial gradient

$a/b = 1/2$

1 $x_0 = 0.20$
2 $x_0 = 0.60$
3 $x_0 = 1.00$

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