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ELECTROMAGNETIC NUCLEON FORM FACTORS

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VMD models for nucleon form factors are considered in the no-width limit with constraints imposed to reproduce the quark counting rules. It is shown that the scaling relations for Sachs form factors can be satisfied identically for zero couplings of the ϕ-mesons with nucleons and degeneracy of the 1^{--} states of the ω- and ρ-mesons. This model is used as a zeroth approximation to construct realistic VMD model for nucleon form factors at spacelike and timelike momentum transfers with account of the finite meson widths, correct analytical properties, and the quark counting rules. The ρ-meson contributions to nucleon form factors are controlled by the low-energy unitarity in the isovector channel.

Fig. - 2, ref. - 12

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1. Introduction

In this talk I shall report some results of our investigations of the nucleon form factors within the VMD model framework[1].

The experimental progress in studying the electromagnetic nucleon form factors[2] stimulates development of the phenemenological models[3,4] capable of the detailed description of the nucleon form factors at low and intermediate energies and having correct QCD asymptotic behavior. The experimental data on the nucleon form factors provide us with the important information about internal structure of nucleons.

In the spacelike region Sachs form factors obey scaling relations

$$G_{Ep}(t) = G_{Mp}(t)/\mu_p = G_{Mn}(t)/\mu_n \qquad (1.1)$$

with μ_p and μ_n being the proton and neutron magnetic moments (in n.m. units). With good accuracy $G_{Ep}(t)$, $G_{Mp}(t)/\mu_p$, and $G_{Mn}(t)/\mu_n$ are described by the dipole function $G_D(t) = (1 - t/.71)^{-2}$ (t in GeV^2), the form factor $G_{En}(t)$ is positive but small. The physical origin and justification of these equations remain still mysterious. In the non-relativistic quark models, the densities of charge and spin coincide, and the 10% deviations from scaling relations arise already at $|t| = 0.1(2m_N)^2 = 0.35$ GeV^2. Despite Eqs.(1.1) are useful, simple, and known for many years, they are considered usually as being incidental.

In Sect. 2 we show that in the VMD models scaling relations can be satisfied identically to zeroth order in parameters $O(\Gamma_k/m_k)$, $O(g_{\phi NN}/g_{\omega NN})$, and $O(\Delta m_k/m_k)$ where Γ_k and m_k are widths and masses of the vector mesons, $g_{\phi NN}$ and $g_{\omega NN}$ coupling constants of the ϕ- and ω-mesons with nucleons, and Δm_k mass differences of the ω- and ρ-mesons. The no-width models are useful as zeroth approximations in constructing realistic VMD models for nucleon form factors. In Sect. 3 the ρ-meson contributions to nucleon form factors are evaluated from the low-energy unitarity. We give the one-dimensional integral representations for the low-energy contributions taking into account existence of the eikonal vertex function in a way consistent with the low-energy unitarity. In Sect. 4 we describe the model and compare predictions of this model with the data at spacelike and timelike momentum transfers.

2. Scaling Relations in No-Widths VMD Models

The electric $G_{EN}(t)$ and magnetic $G_{MN}(t)$ Sachs form factors (N = p,n) are defined in terms of the Dirac and Pauli form factors by

$$G_{EN}(t) = F_{1N}(t) + tF_{2N}(t)/(4m_N^2),$$
$$G_{MN}(t) = F_{1N}(t) + F_{2N}(t) \qquad (2.1)$$

where m_N is the nucleon mass. The isoscalar (s) and isovector (v) components are given by

$$F_{is}(t) = (F_{ip}(t) + F_{in}(t))/2,$$
$$F_{iv}(t) = (F_{ip}(t) - F_{in}(t))/2 \qquad (2.2)$$

(i = 1,2). The quark counting rules[5] imply

$$F_{1N}(t) = O(1/t^2),$$
$$F_{2N}(t) = O(1/t^3) \qquad (2.3)$$

when $t \to \infty$. The additional power of $1/t$ in $F_{2N}(t)$ occurs due to suppression of the quark spin-flip amplitude. It follows that

$$G_{EN}(t) \sim G_{MN}(t) = O(1/t^2) \qquad (2.4)$$

when $t \to \infty$. The asymptotic behavior of the dipole function is in agreement with the quark counting rules.

2.1. Additive representation for Sachs form factors

In the no-width limit, the VMD expressions for Sachs form factors with account of the eikonal vertex function $F^{ej}(t)$ arising due to summation of the soft ω-meson exchanges can be written in the form

$$G_{TN}(t) = [\sum_{l=0}^{j-2} b^{l}_{TN} t^{l} + \sum_{k=1}^{n} c^{k}_{TN} \frac{m_k^2}{m_k^2 - t}] F^{ej}(t) \tag{2.5}$$

where n is the number of vector mesons with different masses, b^{l}_{TN} are coefficients of the polynomials, c^{k}_{TN} are residues describing contribution of the k-th meson to the form factors, and T = E,M. The vertex function

$$F^{ej}(t) = \exp(i\chi(t)) \tag{2.6}$$

is defined by its eikonal[5]

$$i\chi(t) = \frac{\gamma t}{2} \int_{t_0}^{+\infty} \frac{dt'}{(t' - t)} \frac{2t' - 4m_N^2}{t'^{3/2}(t' - 4m_N^2)^{1/2}} \tag{2.7}$$

where $t_0 = 4m_N^2$,

$$\gamma = \frac{g^2_{\omega NN}}{8\pi^2} \ln(1 + \frac{\Lambda_c^2}{m_\omega^2}),$$

m_ω is the ω-meson mass, Λ_c is a cutoff parameter. We have $i\chi(t) \to \gamma(1 - \ln(-t/m_N^2))$ as $t \to \infty$. The quark counting rules require $F^{ej}(t) = O(1/t^j)$ as $t \to \infty$, so $\gamma = j$ must be integer. The eikonal $i\chi(t)$ increases with t up to t_0, at $t > t_0$ $\mathrm{Re}\, i\chi(t) = i\chi(t_0 - t)$.

The threshold conditions $G_{EN}(t_0) = G_{MN}(t_0)$ arising from the definition of Sachs form factors, give constraints for residues of the form

$$\sum_{l=0}^{j-2} (b^{l}_{EN} - b^{l}_{MN}) t_0^{l} + \sum_{k=1}^{n} (c^{k}_{EN} - c^{k}_{MN}) \frac{m_k^2}{m_k^2 - t_0} = 0. \tag{2.8}$$

For $j \geq 1$ the quark counting rules are fulfilled. For $j = 0$ ($F^{ej}(t) = 1$) additional constraints must be imposed

$$\sum_{k=1}^{n} m_k^2 c_k^{TN} = 0. \tag{2.9}$$

2.1. Multiplicative representation for Sachs form factors

The additive representation (2.5) can be considered as an expansion of the rational function

$$G_{IN}(t) = \frac{P^{IN}_{n+j-2}(t)}{\prod_{k=1}^{n}(1 - t/m_k^2)} F^{ei}(t), \tag{2.10}$$

over its elementary ratios. For polynomials $P^{IN}_{n+j-2}(t)$ of power $n + j - 2$ the quark counting rules are fulfilled.

2.3 Scaling relations and the OZI rule

In order to satisfy scaling relations and threshold relations $G_{EN}(t_0) = G_{MN}(t_0)$, Sachs form factors should vanish at $t_0 = 4m_N^2$. Extracting from the polynomials the factor $1 - t/(4m_N^2)$, one can write the following representations

$$G_{Ep}(t) = G_{Mp}(t)/\mu_p = G_{Mn}(t)/\mu_n = (1 - t/(4m_N^2))\frac{P^{Ep}_{n+j-3}(t)}{\prod_{k=1}^{n}(1 - t/m_k^2)} F^{ei}(t),$$

$$G_{En}(t) = (1 - t/(4m_N^2))\frac{P^{En}_{n+j-3}(t)}{\prod_{k=1}^{n}(1 - t/m_k^2)} F^{ei}(t) \tag{2.12}$$

which satisfy identically scaling relations, threshold relations, and the quark counting rules. The normalization conditions for the polynomials are $P^{Ep}_{n+j-3}(0) = 1$, $P^{En}_{n+j-3}(0) = 0$. In general case, there are $n + j - 3$ parameters (in addition to masses) fitting the dipole curve and $n + j - 3$ parameters fitting $G_{En}(t)$.

The isotopic components can be found to be

$$F_{1I}(t) = 0.5\,\frac{(1 - t(\mu_p \pm \mu_n)/(4m_N^2))P^{Ep}_{n+j-3}(t) \pm P^{En}_{n+j-3}(t)}{\prod_{k=1}^{n}(1 - t/m_k^2)} F^{ei}(t),$$

$$F_{2I}(t) = 0.5\frac{(-1+\mu_p \pm \mu_n)P^{Ep}_{n+j-3}(t) \mp P^{En}_{n+j-3}(t)}{\prod_{k=1}^{n}(1-t/m_k^2)}F^{ej}(t). \qquad (2.13)$$

The upper and lower signs refer, respectively, to I = s,v. These form factors contain in general extra poles corresponding to mesons from opposite isotopic channel. In order to eliminate these poles, their residues must be set equal to zero. Trivial solutions $P^{Ep}_{n+j-3}(m^2_k) = P^{En}_{n+j-3}(m^2_k) = 0$ would imply that meson with mass m_k is absent in the both channels. It is necessary, therefore, to look for non-trivial solutions which exist if

$$\det\begin{pmatrix} 1-(\mu_p \pm \mu_n)m_k^2/(4m_N^2) & \pm 1 \\ -1+\mu_p \pm \mu_n & \mp 1 \end{pmatrix} = 0. \qquad (2.14)$$

The determinant vanishes for $m_k = 2m_N$. In such a case $G_{TN}(t_0) \neq 0$, however. We thus conclude that it is impossible to eliminate extra poles.

There is still one possibility. In order to accompany the VMD model with scaling relations, it is necessary to assume that

(i) the numbers of vector mesons in the both channels are equal,
(ii) these mesons are degenerate in mass.

The first assumption is equivalent to the OZI rule: strange vector mesons are decoupled from the nonstrange nucleons. In such a case the isosinglet channel is represented by the ω-mesons only. The second assumption implies degeneracy in mass of the ground and exited states of the ω- and ρ-mesons. The decoupling from nucleons and the degeneracy imply in turn absence of the φ-ω mixing, which is again in agreement with the OZI rule. The approximate character of the OZI rule restricts accuracy of scaling relations by values $O(g_{\phi NN}/g_{\omega NN})$ and $O(\Delta m_k/m_k)$.

For zero coupling of the φ-mesons with nucleons and degeneracy of the 1^- states of ω and ρ, the dipole function $G_D(t)$ can be fitted with any fixed accuracy: n = 3, j = 0 ($F^{ej}(t) = 1$), $m_1 = m_2 = \sqrt{.71}$ GeV, $m_3 \to 2m_N$. The limit cannot be reached, since equality $m_3 = 2m_N$ violates threshold relations $G_{EN}(t_0) = G_{MN}(t_0)$. The model n = 3, j = 0 predicts $G_{En}(t) = 0$. The first resonance is identified naturally with the ground-state 1^- mesons.

The equality $m_1 = m_2$ should be treated as an evidence for existence of light exited 1^- mesons. In papers[6] existence of the candidates with masses in the interval 1.1 ÷ 1.3 GeV are reported (see also[7] and earlier issues of PDG). The third resonance can be identified with one of the well established mesons of masses 1.4 GeV or 1.6-1.7 GeV.

It is clear also that for arbitrary set of the meson masses $G_D(t)$ can be fitted with any precision in the limit $n \to \infty$.

3. Low-Energy Unitarity in Isovector Channel

The nucleon form factors are analytical functions in the complex t-plane with cuts $(t_0, +\infty)$ where t_0 is two- or three-pion threshold. The form factors can be restored in the complex t-plane through dispersion relations, given that their imaginary parts at the cuts are known. From the point of view of the analyticity, the region $t > 0$ ($\mathrm{Im}\,t = +0$) is of the extreme importance, since it contains information about behavior of the form factors in the whole complex t-plane.

The behavior of nucleon form factors is governed by nearest singularities, so in isovector channel the dominant effects come from the ρ-meson pole. The dispersion theory based on the analyticity, unitarity, and crossing symmetry of the scattering amplitudes gives a possibility to evaluate the low-energy contribution to the discontinuities of the scattering amplitudes and of the form factors in a model independent way.

3.1 Unitarity relations for nucleon form factors

The unitarity relations for isovector nucleon form factors $F_{1v}(t)$ and $F_{2v}(t)$ are the two-body ones. They can be used for computing imaginary parts of the form factors in terms of the pion form factor and the p-wave πN-scattering amplitudes below the NN-threshold in the t-channel[8].

The higher partial wave contributions to the backscattering amplitudes near the two-pion threshold can apparently be neglected, in which case the unitarity relations take the form[9]

$$\mathrm{Im}F_{1v}(\nu) = \frac{k^3}{12\pi\sqrt{t}}|F_\pi(\nu)|^2(4m_N\alpha(\nu,-1) + B^{(-)}(\nu,-1))/F_\pi(\nu),$$

$$\mathrm{Im}F_{2\nu}(\nu) = -\frac{k^3}{3\pi\sqrt{t}}|F_\pi(\nu)|^2 m_N \alpha(\nu,-1)/F_\pi(\nu) \qquad (3.1)$$

where $k = (t/2 - \mu^2)^{1/2}$, μ the pion mass, $\alpha(s,u,t) = A^{(-)}(s,u,t)/(s-u)$, $A^{(-)}(s,u,t)$ and $B^{(-)}(s,u,t)$ are invariant pion-nucleon scattering amplitudes whose analytical properties are fixed on the basis of the double spectral representation. The function $\alpha(s,u,t)$ is regular at $s = u$, since the amplitude $A^{(-)}(s,u,t)$ is antisymmetric with respect to the interchange $s \leftrightarrow u$. The backscattering amplitudes $\alpha(\nu,-1)$ and $B^{(-)}(\nu,-1)$ ($\cos\theta = -1$ in the s- and t-channels) as functions of the s-channel c.m. momentum squared ν are analytical functions in the complex ν-plane with two cuts $(-\infty, -\mu^2)$ and $(0, +\infty)$. In the backscattering case $t = -4\nu$. The unitarity relations show that phases of the amplitudes and of the pion form factor coincide, so the ratios $\alpha(\nu,-1)/F_\pi(\nu)$ and $(4m\alpha(\nu,-1) + B^{(-)}(\nu,-1))/F_\pi(\nu)$ have only the one right cut. These ratios are expressible in terms of the observable πN-scattering amplitudes with the use of the one-subtracted dispersion relations

$$(4m_N\alpha(\nu,-1) + B^{(-)}(\nu,-1))/F_\pi(\nu) = \frac{4\pi}{\mu}a_s^{(-)} + \frac{8\pi f^2}{\mu^2}\frac{\nu}{\nu-\nu_0}\frac{1}{F_\pi(\nu_0)} + \frac{8\nu}{3}\int_0^{+\infty}\frac{\mathrm{Im}f_{33}(\nu')}{\omega'\nu'(\nu'-\nu)}\frac{d\nu'}{F_\pi(\nu')},$$

$$\alpha(\nu,-1)/F_\pi(\nu) = -\frac{2\pi}{\mu^2}(a_{p1/2}^{(-)} - a_{p3/2}^{(-)}) - \frac{2\nu}{3}\int_0^{+\infty}\frac{\mathrm{Im}f_{33}(\nu')}{\nu'^2(\nu'-\nu)}\frac{d\nu'}{F_\pi(\nu')}, \qquad (3.2)$$

where $\nu_0 = -\mu^2 + \mu^4/(4m_N^2)$, $f^2 = 0.09$ the pion-nucleon coupling constant, $\omega = (\mu^2 + \nu)^{1/2}$ the pion energy, $a_s^{(-)}$, $a_{p1/2}^{(-)}$, $a_{p3/2}^{(-)}$ scattering lengths, $f_{33}(\nu)$ the pion-nucleon scattering amplitude in the $J = I = 3/2$ channel. The second term in the brackets of first of the equations comes from the pole Born terms. It is assumed that the pion form factor has no zeros in the complex ν-plane.

3.2 Pion form factor in FFGS model

The Frazer-Fulco-Gounaris-Sakurai (FFGS) model[8,10] satisfies analyticity, low-energy unitarity for pion form factor, and crossing symmetry for the $\pi\pi$-scattering amplitudes, providing a simple relation between imaginary part and absolute square of the pion form factor

$$\frac{k^3}{\sqrt{t}}|F_\pi(t)|^2 = \mathrm{Im}F_\pi(t)\frac{k_\rho^3}{m_\rho^2\Gamma_\rho}D(0) \tag{3.3}$$

where $k_\rho = (m_\rho^2/4 - \mu^2)^{1/2}$, m_ρ and Γ_ρ the ρ-meson mass and width, $D(0) = m_\rho^2 + O(\Gamma_\rho/m_\rho)$ the value of the D-function at the origin. This equation is valid at $|t| < 10^6$ GeV^2 where the pion form factor falls off like 1/t in agreement with the quark counting rules.

From the unitarity relations it follows then that imaginary parts of the nucleon form factors are proportional to imaginary part of the pion form factor multiplied by the ratios $\alpha(\nu,-1)/F_\pi(\nu)$ and $(4m\alpha(\nu,-1) + B^{(-)}(\nu,-1))/F_\pi(\nu)$, and so like in the case of the kaon form factor[11] the dispersion integrals can be evaluated explicitly. The pion form factor in the FFGS model has no zeros in the complex ν-plane, so the pole terms in Eqs.(3.2) do not exist and the dispersion relations are valid.

3.3 How to treat eikonal vertex function keeping low-energy unitarity?

In nucleon form factors, however, there is an additional cut starting from the two-nucleon threshold. We wish to take into account it explicitly assuming the cut is determined by the eikonal vertex function (2.6). In such a case, the low-energy unitarity relations and dispersion relations should be written for the ratios $F_{i\nu}(t)/F^{ej}(t)$ in which the cut at $t > t_0$ no longer exists.

In the no-width limit, the right sides of Eqs.(3.1) are proportional to $\delta(t - m_\rho^2)$, and so in the one-pole approximation the unitarity relations for $F_{i\nu}(t)/F^{ej}(t)$ take the form

$$\mathrm{Im}\{F_{i\nu}(t)/F^{ej}(t)\} = c_{i\nu}\pi m_\rho^2\delta(t - m_\rho^2) \tag{3.4}$$

where $c_{i\nu}$ are the ρ-meson residues. The factor $1/F^{ej}(m_\rho^2)$ is absorbed by the residues. The ρ-meson contributions to the nucleon form factors can be evaluated in the no-width limit with the use of the dispersion relations to give

$$F_{i\nu}(t) = [\sum_{l=0}^{j-i-1} b_{i\nu}^l t^l + c_{i\nu}\frac{m_\rho^2}{m_\rho^2 - t}]F^{ej}(t) \tag{3.5}$$

The degree of the polynomial is determined by the quark counting rules. The residues $c_{i\nu}$ can in principle be computed, whereas the coefficients $b_{i\nu}^l$ are free parameters.

The effects of the finite width can be taken into account by expanding $F^{ej}(t)$ in powers of $(t - m_\rho^2)$ up to $O((t - m_\rho^2)^2)$ in terms $\mathrm{Im}F_\pi(t)/F^{ej}(t)$ entering the right side of the unitarity relations. We define

$$R_\rho(\nu) = \frac{1}{\pi}\int_{-\infty}^{-\mu^2} \frac{\mathrm{Im}F_\pi(\nu')}{\nu' - \nu}\frac{d\nu'}{H(\nu')} = \frac{F_\pi(\nu)(\nu_1 - \nu_2) - F_\pi(\nu_1)(\nu - \nu_2) + F_\pi(\nu_2)(\nu - \nu_1)}{(\nu_1 - \nu_2)\,H(\nu)}$$

where $H(\nu) = F^{ej}(\nu_\rho) + F^{ej}(\nu_\rho)'(\nu - \nu_\rho) + 0.5F^{ej}(\nu_\rho)''(\nu - \nu_\rho)^2$, ν_1 and ν_2 are roots of equation $H(\nu_1) = H(\nu_2) = 0$, $\nu_1 = \nu_2^*$, and $m_\rho^2 = -4\nu_\rho$. Notice that $\mathrm{Im}R_\rho(\nu) = \mathrm{Im}F_\pi(\nu)/H(\nu)$.

The function $R_\rho(t)$ is analytical function in the complex t-plane with a cut $(4\mu^2, +\infty)$. In general case $R_\rho(t) = O(1/t)$ as $t \to \infty$. In the no-width limit, $F_\pi(t) = m_\rho^2/(m_\rho^2 - t)$ and $R_\rho(t) = F_\pi(t)/F^{ej}(m_\rho^2)$.

The unitarity relations can now be written in the form

$$\mathrm{Im}\{F_{1\nu}(\nu)/F^{ej}(\nu)\} = \frac{k_\rho^3}{3\pi m_\rho^2\Gamma_\rho} D(0)\mathrm{Im}R_\rho(\nu)\Big(\frac{\pi}{\mu}a_s^{(-)} + \frac{2\pi f^2}{\mu^2}\frac{\nu}{\nu - \nu_0}\frac{1}{F_\pi(\nu_0)} + \frac{2\nu}{3}\int_0^{+\infty}\frac{\mathrm{Im}f_{3\nu}(\nu')}{\omega'\nu'(\nu' - \nu)}\frac{d\nu'}{F_\pi(\nu')}\Big),$$

$$\mathrm{Im}\{F_{2\nu}(\nu)/F^{ej}(\nu)\} = \frac{k_\rho^3}{3\pi m_\rho^2\Gamma_\rho} D(0)2m_N\mathrm{Im}R_\rho(\nu)\Big(\pi\,(a_{p1/2}^{(-)} - a_{p3/2}^{(-)}) + \frac{\nu}{3}\int_0^{+\infty}\frac{\mathrm{Im}f_{3\nu}(\nu')}{\nu'^2(\nu' - \nu)}\frac{d\nu'}{F_\pi(\nu')}\Big). \qquad (3.6)$$

The nucleon form factors in the dispersion theory have normally two-dimensional integral representations. The first integral comes from the dispersion relation for the nucleon form factors, while the second one comes from the dispersion relation for the πN-scattering amplitude through Eqs.(3.2). Using Eq.(3.3) and analytical properties of the integrands in the complex ν-plane, it becomes possible to evaluate one integral explicitly, as a result of which the one-dimensional integral representations can be derived:

$$F_{1\nu}(\nu) = \Big[\sum_{l=0}^{j-2} b^{l}_{1\nu}\nu^{l} + \frac{k_\rho^3}{3\pi m_\rho^2\Gamma_\rho} D(0)\Big(\frac{\pi}{\mu} a_s^{(-)} R_\rho(\nu) + \frac{2\pi f^2}{\mu^2}\frac{\nu R_\rho(\nu) - \nu_0 R_\rho(\nu_0)}{\nu - \nu_0}\frac{1}{F_\pi(\nu_0)} +$$

$$\frac{2}{3}\int_0^{+\infty}\frac{\mathrm{Im} f_{33}(\nu')}{\omega'\nu'}\frac{\nu R_\rho(\nu) - \nu' R_\rho(\nu')}{\nu' - \nu}\frac{d\nu'}{F_\pi(\nu')}\Big)\Big]F^{ej}(\nu),$$

$$F_{2\nu}(\nu) = \Big[\sum_{l=0}^{j-3} b^{l}_{2\nu}\nu^{l} + \frac{k_\rho^3}{3\pi m_\rho^2\Gamma_\rho} D(0) 2m_N\Big(\pi\,(a^{(-)}_{p1/2} - a^{(-)}_{p3/2}) R_\rho(\nu) +$$

$$\frac{1}{3}\int_0^{+\infty}\frac{\mathrm{Im} f_{33}(\nu')}{\nu'^2}\frac{\nu R_\rho(\nu) - \nu' R_\rho(\nu')}{\nu' - \nu}\frac{d\nu'}{F_\pi(\nu')}\Big)\Big]F^{ej}(\nu). \qquad (3.7)$$

In the no-width limit, Eqs.(3.7) take the form of Eq.(3.5).

In derivation of Eqs.(3.7) following ideas are combined: (i) low-energy unitarity, (ii) FFGS model for the pion form-factor, and (iii) existence of the eikonal vertex function.

The one-dimensional integral representations (3.7) simplify treatment of the ρ-meson contribution to nucleon form factors. In particular, it becomes evident that asymptotics of the non-polynomial part is determined by and the same as asymptotics of the function $R_\rho(t)$. In representations (3.7), there are no restrictions with respect to t for validity of the quark counting rules, while in the FFGS model such restrictions exist.

We thus demonstrated that the ρ-meson contribution to the nucleon form factors can be computed in a model independent way using the experimental data on the phase shifts of the ππ- and πN-scatterings. The ππ-scattering phases enter Eqs.(3.7) through the pion form factor. In isoscalar channel, the unitarity conditions are much more complicated, and so this channel should be treated more phenomenologically within VMD approach.

The final-state NN-interaction is important at sufficiently high t (in comparison with the region just above the two-pion threshold where the low-energy unitarity is exact). The high energy cuts in the nucleon form factors insert in estimated based on the low-energy unitarity and the dispersion relations ambiguities which should be controlled separately. We assumed the ratios $\alpha(\nu,-1)/F_\pi(\nu)$ and $(4m\alpha(\nu,-1) + B^{(-)}(\nu,-1))/F_\pi(\nu)$ are regular at $\nu < -\mu^2$. The same suggestion for the ratios $\alpha(\nu,-1)/(F_\pi(\nu)R_\rho(\nu))$ and $(4m\alpha(\nu,-1) + B^{(-)}(\nu,-1))/(F_\pi(\nu)R_\rho(\nu))$ is, however, equally well justified.

4. The Model and Results

The low-energy unitarity in isovector channel is effective for computing the ρ-meson contribution to nucleon form factors. These computations are in excellent qualitative agreement with VMD models. However, the ρ-meson case is exceptional one. In isosinglet channel the low-energy unitarity relations are already three-body ones, and so they are not so efficient like for the ρ-meson. The ω-meson contributions to nucleon form factors should be treated in the spirit of VMD models. The higher radially excited states of the vector mesons can also contribute to imaginary parts of nucleon form factors.

4.1 The ρ-meson dominance and low-energy unitarity

A suggestion that the ratios $\alpha(\nu,-1)/F_\pi(\nu)$ and $(4m\alpha(\nu,-1) + B^{(-)}(\nu,-1))/F_\pi(\nu)$ represent slowly variable functions is equivalent to a hypothesis of the ρ-meson dominance in the πN-scattering amplitudes, since the pion form factor is determined by the ρ-meson pole. In this approximation

$$\mathrm{Im}\{F_{i\nu}(t)/F^{c)}(t)\} = c_{i\nu}\mathrm{Im}R_\rho(t) \tag{4.1}$$

where $c_{i\nu}$ are "residues". The ρ-meson contributions to nucleon form factors can be reconstructed from Eqs.(4.1) using the dispersion relation

$$F_{i\nu}(t) = \{\sum_{l=0}^{j-i-1} b^l_{i\nu}t^l + c_{i\nu}R_\rho(t)\}F^{c)}(t). \tag{4.2}$$

The function $R_\rho(t)$ is essentially determined by the ρ-meson pole. Assuming the the ρ-meson dominance for the πN-scattering amplitudes, we reproduce the ρ-meson dominance for the nucleon form factors. The low-energy unitarity allows therefore to justify qualitatively the ρ-meson dominance. It gives also predictions for the ρ-meson residues $c_{i\nu}$. The representation (4.2) is appropriate for extension of the ρ-meson dominance to other vector mesons.

4.2 VMD model for nucleon form factors

We consider the model discussed in Sect. 2 as a zeroth approximation, namely, no strange mesons are introduced ($g_{\phi NN} = 0$) and the numbers of almost degenerate ω- and ρ-mesons are equal. The nonstrange ω- and ρ-mesons with masses of about 1.25 GeV are included to the fit for reasons explained in Sect. 2. The representation for the nucleon form factors of the form

$$F_{ij}(t) = [\sum_{l=0}^{j-i-1} b_{ij}^{l} t^{l} + \sum_{k=1}^{n} c_{ij}^{k} R_k(t)] F^{ei}(t) \qquad (4.3)$$

represents a natural extension of Eq.(4.2). The k-th meson contribution to the form factors looks like the contribution of the ρ-meson for the constant ratios $\alpha(\nu,-1)/F_\pi(\nu)$ and $(4m\alpha(\nu,-1) + B^{(-)}(\nu,-1))/F_\pi(\nu)$. The functions $R_k(t)$ are defined like the function $R_\rho(t)$ with the substitutions $m_\rho, \Gamma_\rho \to m_k, \Gamma_k$. The ρ-meson is treated separately as described in Sect. 3.

In the no-width limit the functions $R_\rho(t)$ have monopole form. Notice smoothness of the no-width limit. It ensures the existence in model (4.3) of solutions satisfying scaling relations with precision $O(\Gamma_k/m_k) + O(\Delta m_k/m_k)$ and gives therefore an opportunity to avoid of doing a black fit.

4.3 Results and discussion

We fitted first the experimental data at $t < 0$ using the simple polynomial parametrizations of Sect. 2. In this way several models have been selected: with $n = 3, 4$, and 5 and $j = 2, 3$. We then continued search for the realistic models, capable of describing the data at $t < 0$ and $t > t_0$, within that restricted class of the models.

Table 1. Masses and widths of the mesons (in GeV).

k	1	2	3
ω-mesons	0.78 - i0.01	1.15 - i0.10	1.39 - i0.22
ρ-mesons	0.77 - i0.15	1.25 - i0.20	1.50 - i0.24

One of the simplest models is the model with $n = 3$ and $j = 3$. The ωNN coupling constant for $j = 3$ and $\Lambda_c = 2m_\omega$ can be evaluated to be $g^2_{\omega NN}/(4\pi) = 12$ in qualitative agreement with the NN-scattering data. The meson masses and widths are shown in Table

1. The first and the last two mesons are well established, their parameters are taken from PDG[8]. The evidence for existence of the second pair of mesons is strongly supported by the fit.

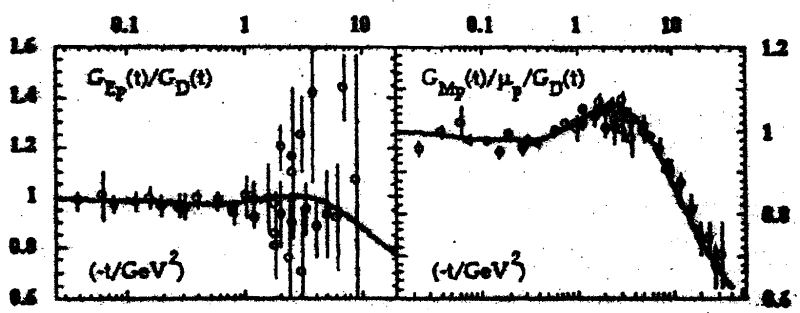
Table 2. The coefficients b_{il}^{l} of the polynomials together with the mesons residues c_{il}^{k}.

il	1s	1v	2s	2v
b_{il}^{0}	-0.18	0.90	0.21	-0.10
b_{il}^{1}	-0.03	-0.02		
c_{il}^{1}	0.28	1.00[a)]	0.29	2.00[a)]
c_{il}^{2}	-0.99	2.09	0.69	-4.74
c_{il}^{3}	0.77	-3.89	-1.59	5.19

[a)]The multipliers of the non-polynomial parts of Eqs.(3.7). The contribution to the $F_{1v}(t)$ is given exactly by first of the Eqs.(3.7), whereas the contribution to the $F_{2v}(t)$ is given by the second one multiplied by a factor two.

We have not found sufficiently good fit with the ρ-meson contribution to the $F_{2v}(t)$ given by Eqs.(3.7) and increased it by a factor two. The estimates (3.7) can in principle be improved by including the higher partial waves to the t-channel backscattering πN-amplitudes.

The coefficients of the polynomials and values of the residues are listed in Table 2. The model curves together with the experimental data are shown in Figs.1-2. The experimental values $|G_{EN}(t)|$ and $|G_{MN}(t)|$ are extracted from the data at $t \geq 4m_N^2$ assuming equality $|G_{EN}(t)| = |G_{MN}(t)|$. This equality is correct at the two-nucleon threshold and no



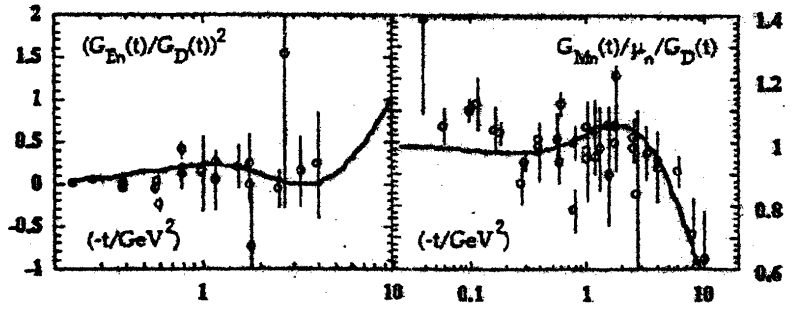


Fig.1. Nucleon form factors at $t \leq 0$. The neutron charge radius is fixed before doing the fit.

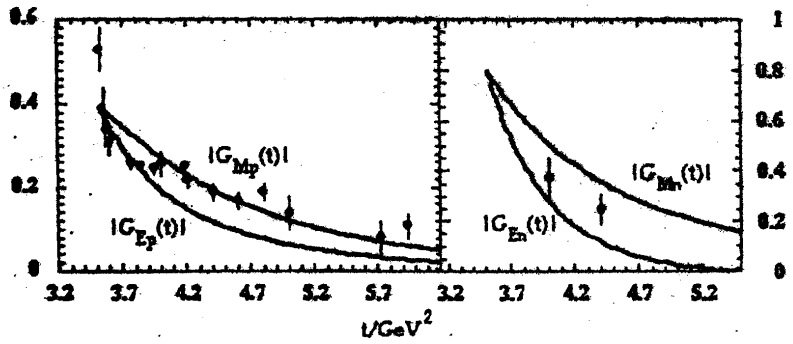


Fig.2. Nucleon form factors at $t \geq t_0$. The experimental values $|G_{EN}(t)|$ and $|G_{MN}(t)|$ are extracted from the data assuming $|G_{EN}(t)| = |G_{MN}(t)|$.

longer valid above the threshold. In the fit presented here, we did not pose any constraints for these form factors. It is seen from Fig.2 that splitting between $|G_{EN}(t)|$ and $|G_{MN}(t)|$ sometimes exceeds the experimental errors. In view of these uncertainties, it is more reasonable to fit directly cross sections, not the data extracted with the use of the crude assumption $|G_{EN}(t)| = |G_{MN}(t)|$. The results of such a procedure will be reported elsewhere[1].

We did not try to reproduce the fast threshold form factor behavior. The lowest point at $t = t_0$ is really very specific one. It refers to annihilation of antiprotons at rest. It means that the stopped antiproton is captured by the proton to form a hydrogen-like atom. The antiproton capture is accompanied by the X-ray radiative transition to lower energy levels and finally by annihilation of the antiproton. The annihilation goes from the subthreshold. The nucleon form factors contain Gamov's factor which describes the Coulomb effects. It has poles corresponding to the bound states. The nucleon form factors thus have complicated comb-like structure just below the threshold. The lowest pole rests at the ground $p\bar{p}$-atom state, while $t = t_0$ is a point of accumulation of the singularities. The meaning of the experimental form factor value at $t = t_0$ is not transparent. These poles are shifted to the lower half-plane, the shifts are determined both by electromagnetic and strong interactions, so the nucleon form factors are not products of the electromagnetic and strong parts.

5. Conclusion

We showed that scaling relations for Sachs form factors can be satisfied identically in the no-width VMD model for zero couplings of the ϕ-mesons with nucleons and degeneracy of the 1^- states of the ω- and ρ-mesons. Using this model as a zeroth approximation, we constructed the VMD model describing successfully the experimental data at spacelike and timelike momentum transfers. This model takes into account finite meson widths, correct analytical properties, and quark counting rules. The ρ-meson contributions to nucleon form factors are controlled with the use of the low-energy unitarity, however, we found it necessary to increase the ρ-meson contribution to $F_{2V}(t)$ in comparison to the estimate (3.7).

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