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INTERACTION OF WILSON LOOPS IN THE SU(N) GAUGE THEORY

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INTERACTION OF WILSON LOOPS IN THE SU(N) GAUGE THEORY: Preprint ITEP 40-94/

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The average of two Wilson loops is expressed in terms of gauge invariant field strength correlators. Assuming the existence of finite correlation length T_g and taking into account the absence of a fixed direction in colour space, we generalize the area law asymptotics for the case of the average of two Wilson loops embedded into the same plane. The results are presented in terms of averaged single Wilson loop operators in irreducible representations which correspond to the geometry of the contours. A special reduction of our results (obtained for arbitrary dimensionality of space-time) to the case of two-dimensional SU(N) theory is performed and the connection to the results existing in literature is established.

Fig. - 4, ref. - 21

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1 Introduction

To analyse the hadronic spectra and interactions one is to consider the Green functions for many quark system in the confining gluonic fields. Making use of the Feinman–Schwinger proper time representation [1], these Green functions can be expressed in terms of the average of Wilson loops corresponding to the hadrons participating in the process under consideration. In the simplest case of the $q\bar{q}$ system the area law for $< W(C) >$ enables one to study the dynamics of quarks connected by the string. The quantization of the "minimal" QCD string (corresponding to the minimal area asymptotics for $< W(C) >$) with quarks has been done in [2]. For high energy scattering the interaction of two Wilson loops was considered in [3] in eikonal type approach, while for low and intermediate energies the problem of evaluating decay and scattering amplitudes in terms of Wilson loops average is still awaiting for the solution.

The purpose of the present paper is to derive the expressions for the average of two Wilson loops in terms of gauge invariant field strength correlators, and to obtain the generalization of the area law asymptotics for this case.

We use the cluster expansion method [4,5] which enables one to write out the Wilson loops average in terms of irreducible in space correlators (the so-called cumulants). Our basic assumption is the existence of the field strength cumulants with finite correlation length T_g [5,6], which have the structure of Lorentz indexes containing the part expressed only in terms of Kronecker delta functions $\delta_{\mu\nu}$. The second order correlator of this type (defined in the gauge invariant way [5])

$$\ll F_{\mu\nu}(x)F_{\rho\sigma}(y) \gg \sim (\delta_{\mu\rho}\delta_{\nu\sigma} - \delta_{\mu\sigma}\delta_{\nu\rho})$$

was measured recently by lattice simulations [7]. We also make extensive use of the condition that there is no fixed direction in colour space. In the case of one loop the presence of such cumulants leads to the appearence of long range interaction between quarks and allows to obtain the minimal area law asymptotics for large contours [5], which is also observed on the lattice.

We present the generalization of the area law for the case when our loops are embedded into the same plane (Fig.1), or into the planes with distance between them less than the gluonic correlation length T_g. To obtain the generalized area law it appears necessary to reduce the initial SU(N) matrix problem to the effective scalar one with short range scalar cumulants. While a single Wilson loop corresponds to the circulation of colour charge in fundamental representation along the contour, for the case of two loops (with

the geometry of Fig.1) one actually has the circulation of colour charges in various irreducible representations in accordance with the decomposition

$$N \otimes \bar{N} = singlet \oplus adjoint$$

for oppositely directed loops, and

$$N \otimes N = symmetric \oplus antisymmetric$$

for parallelly directed loops,so that the average of two loops can be (symbolically) presented as

$$< W(C_1)^{\alpha}_{\beta} W(C_2)^{\gamma}_{\delta} >=$$
$$(singlet)^{\alpha}_{\delta}(singlet)^{\gamma}_{\beta} + (N^2-1)(adjoint)^{\alpha}_{\delta}(adjoint)^{\gamma}_{\beta}$$

or

$$< W(C_1)^{\alpha}_{\beta} W(C_2)^{\gamma}_{\delta} >=$$
$$\frac{N(N+1)}{2}(sym)^{\alpha\gamma}(sym)_{\beta\delta} + \frac{N(N-1)}{2}(antisym)^{\alpha\gamma}(antisym)_{\beta\delta}$$

As a consequence, for large contours in quasiflat geometry the following expressions are obtained:

$$< SpW(C_1)SpW(C_2) >= \sum_r D^{(r)} exp\{-\sigma^{(f)}(S_1 - S_2) - \sigma^{(r)} S_2\},$$
$$S_1 > S_2; \quad S_1, S_2 \gg T_g^2$$

where $D^{(r)}$ is the dimension of the given irreducible representation $\{r\}$, $\sigma^{(r)}$ is the corresponding string tension, and $\sigma^{(f)}$ is the fundamental string tension entering the area law for a single loop. Therefore we connect the average of two Wilson loops (in fundamental representation) to the average of single Wilson loop in representations in accordance with the geometry of the contours. The expressions for the selfintersecting loop (with the geometries of Fig.2) are also obtained.

We perform the reduction of our resuts to the 1+1 gauge theory, and find the agreement with the calculations [8,9] based on the Migdal–Makeenko equations [10] and non-abelian Stokes theorem [11]. We note that recently in 1+1 QCD the average of arbitrary number of Wilson loops on an arbitrary two-dimensional manifold has been calculated and the relation to the string theory of maps was obtained [12].

Correlation of Wilson loops was also studied numerically in lattice simulations [13]. The case when one of the loops is small and serves as a probe

to study the field distribution around the string is a topic of a vivid interest for several years [14-16]. When the loops are large, both measurements and interpretation are much more difficult and accuracy is not yet enough.

The plan of the paper is as follows.

In Section 2 we introduce the cluster expansion on the example of a single Wilson loop, using the Fock–Schwinger gauge to simplify our derivation.

In Section 3 we derive expressions in terms of the cumulants for the average of two loops.

The explicit answers in terms of the corresponding string tensions and areas are given in Section 4 for the case of large contours embedded into the same plane.

Discussion and comparison with the two-dimensional $SU(N)$ theory is done in Section 5.

Two appendices A and B are devoted to the derivation of scalar clusterization condition and the relations between string tensions for 1+1 SU(N) theory respectively.

2 Cluster expansion method.

Before proceeding with the case of two loops we remind the essentials concerning one loop; the details may be found in [5]. The quantity to be considered here is

$$< W(C) >=< Sp\ P\ exp\ ig \int_C A_\mu dx_\mu >, \tag{1}$$

where brackets mean the average over all gluonic fields with the QCD weight. The expression (1) may be rewritten using the non-abelian Stokes theorem [5,11,17] in the explicitly gauge–invariant form

$$< W(C) >=< Sp\ P\ exp\ ig \int_S d\sigma_{\mu\nu}(x) F_{\mu\nu}(x, x_0) >, \tag{2}$$

where S is the minimal surface bounded by the contour C, and

$$F_{\mu\nu}(x, x_0) = \Phi(x_0, x) F_{\mu\nu}(x) \Phi(x, x_0), \tag{3}$$

$\Phi(x_0, x) = P\ exp\ ig \int_{x_0}^{x} A_\mu dz_\mu$ is the parallel transporter along the path connecting the points x and x_0.

We shall use at the intermediate stages the Fock–Schwinger gauge

$$A_\mu^a(x)(x - x_0)_\mu = 0, \tag{4}$$

$$A^a_\mu(x) = \int_0^1 \alpha d\alpha (x - x_0)_\nu F^a_{\nu\mu}((x - x_0)\alpha + x_0)$$

for which $\Phi(x_0, x)$ in (3) is equal to unity, and choose the point x_0 inside the contour, as it is shown at Fig.3; for such choice the integration in (1) takes place actually along the identically deformed contour of Fig.3, and (2) follows from (1) in the most straightforward way.

We present the expression (1) in the form

$$< W(C) >= N \sum_{n=0}^{\infty} (ig)^n I_n \ , \quad I_0 = 1, \tag{5}$$

$$I_n = \frac{1}{N} P \int dx_1 ... dx_n < Sp\ (A(x_1)...A(x_n)) > \tag{5^a}$$

with ommited Lorentz indices. For the contours without selfintersections the coordinates $\{x_i\}$ on the contour C can be parametrized in the unique way by ordering parameres $\{s_i\}, 0 < s_i < 1$, which are determined, in turn, by angular variables $\{\theta_i\}$ along the contour (see Fig.3). It is naturally to assume that for the ensemble of stochastic fields the angular clusterization condition holds true,

$$< (A(s_1)...A(s_m))^\alpha_\beta (A(s_{m+1})...A(s_{m+n}))^\rho_\sigma > \rightarrow \tag{6}$$
$$\rightarrow < (A(s_1)...A(s_m))^\alpha_\beta >< (A(s_{m+1})...A(s_{m+n}))^\rho_\sigma >$$

if the cluster of points $\{s_1...s_{m+n}\}$ is separated spatially into two clusters, $\{s_1...s_m\}$ and $\{s_{m+1}...s_{m+n}\}$, so that

$$|x_i - x_j| \gg T_g, \quad i\epsilon\{1...m\}, \quad j\epsilon\{m+1..m+n\},$$

where T_g is the gluonic correlation length.

Due to the condition (6) the correlators of a given order in eq.(5) contain the smaller order correlators as disconnected parts (the parts which do not vanish at large intercluster distances), and it is useful to represent $< W(C) >$ in terms of the connected correlators, the so-called cumulants [4]. It can be done by calculation of $ln < W(C) >$. Formally, the cumulants $\mathcal{K}^\alpha_{m\beta}(x_1...x_m)$ are defined from the equation [4]

$$< (W(C))^\alpha_\beta >= \sum_{n=0}^{\infty} (ig)^n (I_n)^\alpha_\beta = \tag{7}$$

$$\{exp \sum_{m=1}^{\infty} \frac{(ig)^m}{m!} \mathcal{K}_m\}^\alpha_\beta,$$

where

$$(\mathcal{K}_m)^\alpha_\beta = \int dx_1 ... dx_m K^\alpha_{m\beta}(x_1 ... x_m). \tag{8}$$

It was shown in [4] that if the clusterization condition (6) takes place, then the functions $K^\alpha_{m\beta}(\{x_m\})$ defined in accordance with eqs.(7), (8) vanish if the cluster of points $\{x_m\}$ is separated spatially into any number of clusters $\{x_{m_1}\}, \{x_{m_2}\} ...$. Equivalently, for large smooth surfaces the field strength cumulants (with the Lorentz indexes structure, which contains the part expressed only in terms of Kronecker delta functions) coming to eq.(8) from the expression (4) for Fock-Schwinger gauge lead to the following result for the corresponding integrals [5]

$$\frac{(ig)^m}{m!}(\mathcal{K}_m)^\alpha_\beta = (\sigma_m)^\alpha_\beta S + (\mu_m)^\alpha_\beta p \tag{9}$$

where S/p is the area/perimeter of the surface, and σ, μ may be (for arbitrary stochastic ensemble) non-unity matrices. Large and smooth means that both the size of the contour and the averaged size of contour irregularities are much larger than the correlation length T_g.

The equation (9), however, is not sufficient in general for the "scalar" area law

$$< Sp\ W(C) > \rightarrow exp(-\sigma S)$$

we are to have in QCD. The latter is achieved if there is no fixed colour direction in the vacuum, i.e., in addition to eq.(6), the condition

$$< (A(s_1) ... A(s_n))^\alpha_\beta > = \delta^\alpha_\beta \mathcal{A}(s_1 ... s_n), \tag{10}$$

$$\mathcal{A}(s_1 ... s_n) = \frac{1}{N} < Sp(A(s_1) ... A(s_n)) > .$$

is imposed. Then the "matrix" area law (9) is obviously reduced to the "scalar" one,

$$(\mathcal{K}_m)^\alpha_\beta = \delta^\alpha_\beta \mathcal{K}_m,$$

$$(\sigma_m)^\alpha_\beta = \delta^\alpha_\beta \sigma_m\ , \quad (\mu_m)^\alpha_\beta = \delta^\alpha_\beta \mu_m,$$

and for large smooth contours one has [5], up to shape corrections,

$$< Sp\ W(C) > = N exp(-\sigma S - \mu p), \tag{11}$$

where

$$\sigma = \sum_{m=2}^{\infty} \sigma_m\ , \quad \mu = \sum_{m=2}^{\infty} \mu_m\ . \tag{12}$$

(It is supposed that seria (12) for σ and μ converge).

The conditions (6) and (10) can be combined into the "scalar" clusterization condition for the functions $\mathcal{A}$

$$\frac{1}{N} < Sp(A(s_1)...A(s_{m+n})) > \rightarrow \tag{13}$$

$$\rightarrow \frac{1}{N} < Sp(A(s_1)...A(s_m)) > \cdot \frac{1}{N} < Sp(A(s_{m+1})...A(s_{m+n})) >$$

if the cluster $\{s_1...s_{m+n}\}$ is separated into two clusters $\{s_1...s_m\}$ and $\{s_{m+1}...s_{m+n}\}$.

This condition enables one to reduce the matrix exponentiation (7) to the scalar one which will be necessary for two loops case analysis. To this end we are to tranform the ordered in $\{s\}$ integration $s_{i+1} > s_i$, into the integration over the whole region $0 < s_j < 1$, so that I_n in (5) can be rewritten as

$$I_n = P \int ds_1...ds_n \mathcal{A}(s_1...s_n) = \tag{14}$$

$$\frac{1}{n!} \int (ds_i)^n \mathcal{A}(s_1...s_n) \tag{14^a}$$

where the ordering of colour matrices in $\mathcal{A}(s_1...s_n)$ in (14^a) follows the ordering of variables $\{s_i\}$. In such representation the application of ordinary scalar exponentiation [4] to eq. (5) gives the definition of scalar cumulants K_m, vanishing at large distances due to eq. (13):

$$\frac{1}{2!} K_2(x_1 x_2) = \mathcal{A}(x_1 x_2)$$

$$\frac{1}{3!} K_3(x_1 x_2 x_3) = \mathcal{A}(x_1 x_2 x_3) \tag{15}$$

$$\frac{1}{4!} K_4(x_1 x_2) = \mathcal{A}(x_1 x_2 x_3 x_4) - \mathcal{A}(x_1 x_2)\mathcal{A}(x_3 x_4) - \mathcal{A}(x_1 x_3)\mathcal{A}(x_2 x_4) - \mathcal{A}(x_1 x_4)\mathcal{A}(x_2 x_3)$$

and so on. Therefore, we demonstrate that the condition (13) leads to scalar area law (11).

3 Two Wilson loops average

We start with the evaluation of

$$< (W(C_1))^\alpha_\beta (W(C_2))^\gamma_\delta > \tag{16}$$

for two contours in the same plane inserted one into another. We have deformed the contours identically by making tentacles from the open ends of the contours to the point x_0 inside the both loops (Fig.4). This point participates in the definition (3) of the $F_{\mu\nu}(x,x_0)$ and in the Fock–Schwinger gauge (4), which is used in what follows. The straightforward application of the exponentiation procedure (7) to the expression (16) leads only to the "matrix" area law, as it is clear already from the analysis of the one loop case. Moreover, in contrast to the one loop case, the condition (10) does not lead, as we shall see, automatically to the area law.

To develop a regular method we are to express the average (16) in terms of the proper scalar short range correlators. Colour structure of the expression (16) which can be interpreted as the circulation of two fundamental colour charges, is decomposed into the pairs corresponding to the circulation of charges in irreducible representation accordingly to the formulae

$$N \otimes \bar{N} = singlet \oplus adjoint \tag{17}$$

for the oppositely directed loops, and

$$N \otimes N = symmetric \oplus antisymmetric \tag{18}$$

for the parallelly directed loops.

We prove that scalar correlators corresponding to each term of the decomposition of $< SpW(C_1)SpW(C_2) >$ in accordance with (17),(18) satisfy the scalar clusterization condition similar to (13). Consequently, the application of scalar exponentiation separately to each part of this decomposition gives the definition of short range scalar cumulants and generalized area law.

Instead of eq. (6) for two loop case we consider the average of the type

$$< (A(x_1^{(1)})...A(x_m^{(1)}))^{\alpha}_{\beta}(A(x_1^2)...A(x_n^{(2)}))^{\gamma}_{\delta} > \tag{19}$$

where points $\{x_i^{(1)}\}$ belong to the contour C_1 and points $\{x_k^{(2)}\}$ belong to the contour C_2. First we generalize the clusterization condition (6) for our two loops case, and prove that it leads with necessity to the decomposition (17), (18) for the definition of short range scalar cumulants.

Due to the interaction between the loops the average (19) is not reduced to the product of averages if the points on both contours are correlated in angles defined with respect to a halfline, starting at x_0. Therefore only if the cluster of points $\{s_1^{(1)}...s_{m+n}^{(1)}, s_1^{(2)}...s_{k+l}^{(2)}\}$ is separated in angles into two

clusters $\{s_1^{(1)}...s_m^{(1)}, s_1^{(2)}...s_k^{(2)}\}$ and $\{s_{m+1}^{(1)}...s_{m+n}^{(1)}, s_{k+1}^{(2)}...s_{k+l}^{(2)}\}$ (the superscripts (1) and (2) stand to label the points from the first and the second loop), the generalized clusterization condition holds true for the ensemble of stochastic fields:

$$
\begin{aligned}
&< (A(s_1^{(1)})...A(s_m^{(1)})^\alpha_\beta (A(s_{m+1}^{(1)})...A(s_{m+n}^{(1)})^\gamma_\delta \cdot \\
&\cdot (A(s_{k+l}^{(2)})...A(s_{k+1}^{(2)}))^\rho_\sigma (A(s_k^{(2)})...A(s_1^{(2)}))^\mu_\nu > \rightarrow \\
&\rightarrow < (A(s_1^{(1)})...A(s_m^{(1)}))^\alpha_\beta \cdot (A(s_k^{(2)})...A(s_1^{(2)}))^\mu_\nu > \cdot \\
&\cdot < (A(s_{m+1}^{(1)})...A(s_{m+n}^{(1)})^\gamma_\delta \cdot (A(s_{k+i}^{(2)})...A(s_{k+1}^{(2)}))^\rho_\sigma >
\end{aligned}
\tag{20}
$$

for opposite ordering of loops, and

$$
\begin{aligned}
&< (A(s_1^{(1)})...A(s_m^{(1)})^\alpha_\beta (A(s_{m+1}^{(1)})...A(s_{m+n}^{(1)})^\gamma_\delta > \cdot \\
&\cdot (A(s_1^{(2)})...A(s_k^{(2)}))^\mu_\nu (A(s_{k+1}^{(2)})...A(s_{k+l}^{(2)}))^\rho_\delta > \rightarrow \\
&\rightarrow < (A(s_1^{(1)})...A(s_m^{(1)}))^\alpha_\beta (A(s_1^{(2)})...A(s_k^{(2)}))^\mu_\nu > \cdot \\
&\cdot < (A(s_{m+1}^{(1)})...A(s_{m+n}^{(1)})^\gamma_\delta \cdot (A(s_{k+1}^{(2)})...A(s_{k+l}^{(2)}))^\rho_\sigma >
\end{aligned}
\tag{20'}
$$

for parallel ordering. Again, as for the case of one loop, in the Fock– Schwinger gauge with the point x_0 inside the loops the clusterization condition (20) is actually the angular one.

In what follows the shorthand notations are used:

$$
< (A(s_1^{(1)})...A(s_i^{(1)})^\alpha_\beta (A(s_j^{(2)})...A(s_1^{(2)}))^\gamma_\delta > = < (a_i)^\alpha_\beta (b_j)^\gamma_\delta > \tag{21}
$$

and

$$
< (A(s_1^{(1)})...A(s_i^{(1)})^\alpha_\beta (A(s_1^{(2)})...A(s_j^{(2)}))^\gamma_\delta > = < (a'_i)^\alpha_\beta (b'_j)^\gamma_\delta > . \tag{21'}
$$

To reduce, as in one loop case, the SU(N) matrix problem to the effective scalar one we first note, that the natural generalization of eq.(10) is the following expression for the four-point averages (21) and (21′) in terms of two scalar functions $\mathcal{A}, \mathcal{B}$

$$
< (a_i)^\alpha_\beta (b_j)^\gamma_\delta > = \mathcal{A}(i,j)\delta^\alpha_\beta \delta^\gamma_\delta + \mathcal{B}(i,j)\delta^\alpha_\delta \delta^\gamma_\beta \tag{22}
$$

$$
< (a'_i)^\alpha_\beta (b'_j)^\gamma_\delta > = \mathcal{A}'(i,j)\delta^\alpha_\beta \delta^\gamma_\delta + \mathcal{B}'(i,j)\delta^\alpha_\delta \delta^\gamma_\beta \tag{23}
$$

instead of one function $\mathcal{A}$ of eq.(10). It is demonstrated in Appendix A that the use of the decomposition (17) leads to the following scalar correlators, satisfying the scalar clusterization condition

$$\frac{1}{N} < Sp(a_{i1+i2}b_{j2+j1}) > \rightarrow \tag{24}$$

$$\rightarrow \frac{1}{N} < Sp(a_{i1}b_{j1}) > \cdot \frac{1}{N} < Sp(a_{i2}b_{j2}) >$$

and

$$\frac{2}{N^2-1} < Sp(a_{i1+i2}\lambda_a b_{j2+j1}\lambda_a) > \rightarrow \tag{25}$$

$$\rightarrow \frac{2}{N^2-1} < Sp(a_{i1}\lambda_b b_{j1}\lambda_b) > \cdot \frac{2}{N^2-1} < Sp(a_{i2}\lambda_c b_{j2}\lambda_c) >$$

with

$$\frac{1}{N} < Sp(a_i b_j) >= \mathcal{A}(i,j) + N\mathcal{B}(i,j) = J_{ij}^{(0)} \tag{26}$$

and

$$\frac{2}{N^2-1} < Sp(a_i\lambda_a b_j\lambda_a) >= A(i,j) = J_{ij}^{(adj)}. \tag{27}$$

Similarly, from the decomposition (18) we have

$$\frac{1}{N(N\pm 1)} < \{Sp(a'_{i1+i2})Sp(b'_{j1+j2}) \pm Sp(a'_{i1+i2}b'_{j1+j2})\} > \rightarrow \tag{28}$$

$$\frac{1}{N(N\pm 1)} < \{Sp(a'_{i1})Sp(b'_{j1}) \pm Sp(a'_{i1}b'_{j1})\} > \cdot$$

$$\cdot \frac{1}{N(N\pm 1)} < \{Sp(a'_{i2})Sp(b'_{j2}) \pm Sp(a'_{i2}b'_{j2})\} >$$

with

$$\frac{1}{N(N\pm 1)} < \{Sp(a'_i)Sp(b'_j) \pm Sp(a'_i b'_j)\} >= \tag{29}$$

$$= \mathcal{A}'(i,j) \pm \mathcal{B}'(i,j) = J_{ij}^{(s,a)}$$

To take advantage of these scalar correlators we decompose the pairs of colour indeces in $< W(C_1)^\alpha_\beta W(C_2)^\gamma_\delta >$ in accordance with the decomposition (17),(18), that corresponds to the following:

$$< SpW(C_1)SpW(C_2) >= \tag{30}$$

$$\frac{1}{N} < Sp(W(C_1)W(C_2)) > +2 < Sp(W(C_1)\lambda_a W(C_2)\lambda_a >$$

with

$$< Sp(W(C_1)W(C_2) >= N \sum_n (ig)^n \; I_n^{(0)} \tag{31}$$

$$< Sp(W(C_1)\lambda_a W(C_2)\lambda_a >= \frac{N^2-1}{2} \sum_n (ig)^n I_n^{(adj)}, \tag{32}$$

for the case of opposite direction in the loops, and to

$$< Sp(\overset{*}{W}(C_1))Sp(W(C_2)) >= \tag{33}$$

$$\frac{1}{2}\{< SpW(C_1)SpW(C_2) + < Sp(W(C_1)W(C_2)) >\} +$$

$$+\frac{1}{2}\{< SpW(C_1)SpW(C_2) > - < Sp(W(C_1)W(C_2)) >\}$$

with

$$< SpW(C_1)SpW(C_2) \pm SpW(C_1)W(C_2)) >= (N \pm 1) \sum_n (ig)^n I_n^{(s,a)} \tag{34}$$

for the case of parallel directions. The quantities $I_n^{(r)}$ ((r) stand for singlet (0), adjoint (adj), symmetric (s) and antisymmetric (a) representations) are

$$I_n^{(r)} = \sum_{i+j=n} I_{n,ij}^{(r)} \; , \quad I_0^{(r)} = 1 \; ,$$

$$I_{n,ij}^{(r)} = P \int ds_1^{(1)} ... ds_i^{(1)} ds_1^{(2)} ... ds_j^{(2)} J_n^{(r)}(i,j) \tag{35}$$

and $J_n^{(r)}(i,j)$ are defined in (26),(27) and (29).

In order to perform the final reduction to the scalar case, as it was done for one loop (see eq.(14^a)), one is to represent the integration in (35) over the ordered region as the integration over the whole region:

$$I_n^{(r)} = \sum_{i+j=n} I_{n,ij}^{(r)} = \sum_{i+j=n} \frac{1}{i!}\frac{1}{j!} \int (ds^{(1)})^i (ds^{(2)})^j J_n^{(r)}(i,j) = \tag{36}$$

$$= \sum_{i+j=n} \frac{1}{n!} \int (ds)^{i+j} J_n^{(r)}(i+j)$$

where the integration is performed over the whole contour $C_1 + C_2$, and the ordering of the fields (and λ matrices – insertions in eq. (27)) follows the ordering in $\{s\}$.Due to the eqs. (24)-(29) the averages $J_n^{(r)}(i+j)$ satisfy the clusterization condition for angularly separated clusters. Therefore the scalar exponentiation procedure applied separately to each term in eqs.(30),(33) leads

to the cumulants which vanish [4] if the points on loops from any clusters are separated in angles.

The result of such exponentiation is

$$< Sp(W(C_1)SpW(C_2)) >= \tag{37}$$

$$exp \sum_{m=2}^{\infty} \frac{(ig)^m}{m!} \mathcal{K}_m^{(0)} + +(N^2-1) exp \sum_{m=2}^{\infty} \frac{(ig)^m}{m!} \mathcal{K}_m^{(adj)}$$

for oppositely directed loops, and

$$< Sp(W(C_1)Sp(W(C_2)) >= \tag{38}$$

$$\frac{N(N+1)}{2} exp \sum_{m=2}^{\infty} \frac{(ig)^m}{m!} \mathcal{K}_m^{(s)} + \frac{N(N-1)}{2} exp \sum_{m=2}^{\infty} \frac{(ig)^m}{m!} \mathcal{K}_m^{(a)}$$

We have introduced the integrated cumulants $\mathcal{K}_m^{(r)}$

$$\mathcal{K}_m^{(r)} = \sum_{i+j=m} \mathcal{K}_{m,ij}^{(r)} \tag{39}$$

$$\mathcal{K}_{m,ij}^{(r)} = \int dx_1^{(1)} ... dx_i^{(1)} dx_1^{(2)} ... dx_j^{(2)} K_{m,ij}^{(r)} (x_1^{(1)} ... x_i^{(1)} x_1^{(2)} ... x_j^{(2)}) \tag{40}$$

which are related to $J_m^{(r)}$ of eq. (36) by the recurrent procedure similar to (7),(15).

4 Area law for the average of two Wilson loops

Let's analyse the expressions (37),(38) for the large contours C_1 and C_2 of Fig.4, and obtain the generalization of the area law (using the Fock–Schwinger gauge (4)). In what follows we neglect the perimeter type contributions, which always come as in one loop case (11). The integrals (40) contain, in addition to "angular" integration over $\{s_i\}$, the integration over the radial variables $(xd\alpha)_i$, see eq.(4); this double integration can be rewritten as the summation over the corresponding sectors. Due to the existence of short range cumulants for a given angle only the overlaps of the sectors beloning to different loops contribute to the area law (see Fig.3). Therefore, only the $\mathcal{K}_{m,m0}^{(r)}$ are proportional (in the large area limit) to $S_1 (S_1 > S_2)$, and the $\mathcal{K}_{m,ij}^{(r)}$, with $j \neq 0$ are proportional to S_2. We stress again that we assume that the cumulants $\ll F_{\mu\nu} F_{\rho\sigma} \gg$ contain the part expressed only in terms of Kronecker delta functions (see [5] for the details). It is this part which originates the long range interaction between quarks leading to the area law.

First we note that due to the condition (10) we have for $\mathcal{K}^{(r)}_{m,m0}(S_1 > S_2)$

$$\frac{(ig)^m}{m!}\mathcal{K}^{(r)}_{m,m0} = \sigma^{(f)}_m S_1, \tag{41}$$

where $\sigma^{(f)}_m = \sigma_m$ is the contribution to the fundamental string tension of eq.(12) which arises from the cumulant of m-th order. Second, due to the finite gluonic correlation length all the $\mathcal{K}^{(r)}_{m,ij}$ with $j \neq 0$ are proportional to S_2, and therefore one can reduce the integration over C_1+C_2 to the integration over $C_2' + C_2$, where C_2' coincides with C_2 and directed as C_1. Consequently,

$$\frac{(ig)^m}{m!}\sum_{i+j=m, j\neq 0}\mathcal{K}^{(r)}_{m,ij} = -\sigma^{(f)}S_2 + \frac{(ig)^m}{m!}\sum_{i+j=m}\bar{\mathcal{K}}^{(r)}_{m,ij},$$

where $\bar{\mathcal{K}}^{(r)}_{m,ij}$ is defined by integral (40) over $C_2' + C_2$.

Observing that

$$D^{(r)}exp(\sum_m \frac{(ig)^m}{m!}\sum_{i+j=m}\bar{\mathcal{K}}^{(r)}_{m,ij}) =< Sp\ P\ exp\ ig\int_{C_2}(A^a_\mu \Lambda^{(r)}_a)dx_\mu > \tag{42}$$

where $\Lambda^{(r)}$ and $D^{(r)}$ are the generators and dimensions of the corresponding irreducible representations r, we finally get the following expression:

$$\sum_m \frac{(ig)^m}{m!}\mathcal{K}^{(r)}_m = \sigma^{(f)}(S_1 - S_2) + \sigma^{(r)}S_2 \tag{43}$$

(The string tension $\sigma^{(0)}$ in the singlet representation is obviously equal to zero).

Now we arrive to the main result of the paper. The area law that follows from the equations (37), (38) reads

$$\begin{aligned} < Sp(W(C_1))Sp(W(C_2)) >= exp(-\sigma^{(f)}(S_1 - S_2)) + \\ +(N^2-1)exp(-\sigma^{(f)}(S_1 - S_2) - \sigma^{(adj)}S_2), S_1 > S_2 \end{aligned} \tag{44}$$

for oppositely directed loops, and

$$\begin{aligned} < Sp\ (W(C_1))Sp\ (W(C_2)) >= \frac{N(N-1)}{2}exp(-\sigma^{(f)}(S_1 - S_2)) - \sigma^{(a)}S_2) + \\ +\frac{N(N+1)}{2}exp(-\sigma^{(f)}(S_1 - S_2)) - \sigma^{(s)}S_2) \end{aligned} \tag{45}$$

for parallel loops, with $\sigma^{(adj)}$, $\sigma^{(a)}$ and $\sigma^{(s)}$ and $\sigma^{(s)}$ given by (43). We stress here that due to eq. (42), (43) we express the average of two Wilson loops (in fundamental representation) in terms of the proper combinations of single Wilson loop averages in representations corresponding to the geometry of the contours.

If the main contribution to the string tension arises from the lowest cumulant K_2, then, as it is shown in the Appendix B, the string tensions $\sigma^{(r)}$ are related to each other by the formulae

$$\sigma^{(adj)} = \frac{2N^2}{N^2-1}\sigma^{(f)} , \tag{46}$$

$$\sigma^{(a)} = \frac{2(N-2)}{N-1}\sigma^{(f)} , \tag{46'}$$

$$\sigma^{(s)} = \frac{2(N+2)}{N+1}\sigma^{(f)} . \tag{46''}$$

that follow from the relations between quadratic Casimir operators. Lattice calculations [18] demonstrate that the relation (46) between $\sigma^{(f)}$ and $\sigma^{(adj)}$ holds with a good accuracy. This fact allows to hope that the series (12) for string tension is saturated to a large extent by the second order contribution.

One can also write out the expressions for the average of a loop with selfintersections. For a contour with one selfintersection the usual area law

$$< Sp(W(C_1))W(C_2) >= N exp(-\sigma(S_1+S_2))$$

takes place, as it is clear from the given above considerations, only if the areas inside both loops do not overlap (Fig.4^a). In the case of Fig 4^b the average can be calculated similarly to the case (33) of parallelly directed loops:

$$< Sp(W(C_1)W(C_2)) >= \frac{1}{2}\{< SpW(C_1)SpW(C_2) > + < Sp(W(C_1)W(C_2)) >\}-$$
$$-\frac{1}{2}\{< SpW(C_1)SpW(C_2) > - < Sp(W(C_1)W(C_2)) >\}$$

with the result

$$< Sp(W(C_1)W(C_2)) >= \frac{N(N+1)}{2}exp(-\sigma^{(f)}(S_1-S_2)-\sigma^{(s)}S_2)- \tag{47}$$
$$-\frac{N(N-1)}{2}exp(-\sigma^{(f)}(S_1-S_2)-\sigma^{(a)}S_2).$$

We note that eq.(47) is less then zero when

$$\frac{N+1}{N-1} < exp(+(\sigma^{(s)} - \sigma^{(a)})S_2)$$

which is achieved for $\sigma^{(r)}$ determined by eqs (46) if S_2 is sufficiently large.

Let us briefly discuss what result is to be expected if the contours are not in the same plane. If the average distance between the contours is less than the correlation length T_g, in the limit of large areas one obtains the same expressions (44)-(45). If the distance between the planes is larger then T_g, the interaction between the loops, in contrast to the flat geometry case, becomes small. To extract the factor corresponding to interaction we first gather the terms $K^{(r)}_{m,m0}$ into the $exp(-\sigma S_1)$ and the terms $K^{(r)}_{m,0m}$ into the $exp(-\sigma S_2)$ separating in this way the part corresponding to noninteracting Wilson loops. The remains of expression (40) are responsible for the interaction, so that the explicit dependence of $K^{(r)}_{m,ij}, i,j \neq 0$ on the distance between the loops enters now the answers. One is to conclude therefore that the latter cannot be expressed in terms of areas and string tensions, but more detailed knowledge of the dynamics of field strength correlators is required. When the distance between loops is much more then T_g (and their average sizes) the asymptotics of the interaction is described usually [19] by the exchange of the glueball with lightest mass.

5 Discussion

Our analysis was performed for the SU(N) gauge theory with arbitrary number of dimensions. In the case of 1+1 gauge theory our expressions are simplified, because in two dimensions the SU(N) theory can be represented [20] as the abelian one choosing the axial gauge

$$A_1^{(a)}(x_1, x_2) = 0.$$

As the result, only the quadratic cumulant exists, and one arrrives to the generalized area laws (44), (45), (47) with string tensions $\sigma^{(r)}$ in various representations related to the fundamental string tension $\sigma^{(f)}$ by the eqs. (46).

For the U(N) theory in two dimensions the generalized area law for several Wilson loops or for the Wilson loop with selfintersections was obtained in [11] using the non-abelian Stokes theorem. Also on the basis of Migdal-Makeenko equations the Wilson loops average was calculated in 1+1 both for U(N),

$N \to \infty$ in [8], and for U(N) and SU(N) theory in [9]. Our expressions (44), (45), (47) reduced to the two-dimensional case coincide with the U(N) results of [8], [11] for $N \to \infty$ and are equivalent to the SU(N) area laws obtained in [9]. In particular, for a Wilson loop with selfintersection in the geometry of fig.2^b the formula of [8]

$$< Sp(W(C_1)W(C_2)) > \to Nexp(-\sigma^{(f)}(S_1 - S_2))(1 - 2\sigma^{(f)}S_2)$$

$$N \to \infty \ , \quad \sigma^{(f)} \sim g^2 N \ ,$$

can be easily obtained from eq.(47).

Let us summarize the results. We have expressed the average of two Wilson loops in terms of irreducible field strength correlators. For large contours the generalized area law asymptotics is established, so that the average of two Wilson loops (in fundamental representation) is related to the averages of a single Wilson loop in the representations corresponding to the geometry of the contours.

The physical interpretation of the expressions (44),(45) can be obtained if one considers the problem of nonperturbative field distribution between colour charges(in overall colour singlet state). The area law for a single Wilson loop corresponds to the formation of the string between $q\bar{q}$ in the meson. In our two loops case the generalized area law reflects the formation of stringlike gluonic field configurations in four quarks system. We note that in the leading in $1/N$ approximation (when the interaction between colourless systems vanishes [21]) $< W(C_1)W(C_2) > \to < W(C_1) >< W(C_2) >$, so that one gets linear superposition of two mesonic strings.

Our analysis based on the cluster expansion method can be easily generalized to evaluate the average of several Wilson loops or of a loop with more than one selfintersection. To this end one is to consider the quantity

$$< (W(C_1))^{\alpha_1}_{\beta_1} ... (W(C_n))^{\alpha_n}_{\beta_n} > \tag{48}$$

and decompose the pairs of colour indeces in accordance with formulae generalizing expressions (17) or (18) depending on the geometry of the loops. The expression (48) can be rewritten then as a sum over combinations corresponding to all proper irreducible representations of colour group with the coefficients determined by the dimensions of these representations. With the assumption of the existence of a finite correlation length the cluster expansion method allows to demonstrate that each term in this sum exhibits the area

law behaviour and enables to calculate the string tensions entering this area law.

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Appendix A

Here we prove that only the linear combinations (26), (27) and (29) obey the "scalar" clusterization condition. To this end we treat the problem as an eigenvalue problem, looking for the numbers μ and ν for which

$$\mu\mathcal{A}(i_1+i_2,j_2+j_1)+\nu\mathcal{B}(i_1+i_2,j_2+j_1)\to \tag{A.1}$$
$$\to\{\mu\mathcal{A}(i_1,j_1)+\nu\mathcal{B}(i_2,j_2)\}\{\mu\mathcal{A}(i_1,j_1)+\nu\mathcal{B}(i_2,j_2)\}$$

and

$$\mu\mathcal{A}'(i_1+i_2,j_1+j_2)+\nu\mathcal{B}'(i_1+i_2,j_1+j_2)\to \tag{A.2}$$
$$\to\{\mu\mathcal{A}'(i_1,j_1)+\nu\mathcal{B}'(i_2,j_2)\}\{\mu\mathcal{A}'(i_1,j_1)+\nu\mathcal{B}'(i_2,j_2)\}$$

takes place at large intercluster distances.

For the oppositely directed loops one gets from (20)

$$<(a(i_1+i_2))^\alpha_\beta(b(j_2+j_1))^\gamma_\delta>=$$
$$<(a(i_1))^\alpha_\rho(a(i_2))^\rho_\beta(b(j_2))^\gamma_\sigma(b(j_1))^\sigma_\delta>\to$$
$$\to<(a(i_1))^\alpha_\rho(b(j_1))^\sigma_\delta>\cdot<(a(i_2))^\rho_\beta(b(j_2))^\gamma_\sigma>,$$

or, using eq.(22),

$$\mathcal{A}(i_1+i_2,j_2+j_1)\to\mathcal{A}(i_1,j_1)\mathcal{A}(i_2,j_2) \tag{A.3}$$

$$\mathcal{B}(i_1+i_2,j_2+j_1)\to\mathcal{A}(i_1,j_1)\mathcal{B}(i_2,j_2)+\mathcal{A}(i_2,j_2)\mathcal{B}(i_1,j_1)+ \tag{A.4}$$
$$N\,\mathcal{B}(i_1,j_1)\mathcal{B}(i_2,j_2)$$

Substituting (A.3) and (A.4) into (A.1) we have

$$\mu\mathcal{A}(i_1,j_1)\mathcal{A}(i_2,j_2)+$$
$$+\nu\{\mathcal{A}(i_1,j_1)\mathcal{B}(i_2,j_2)+\mathcal{A}(i_2,j_2)\mathcal{B}(i_1j_1)+N\mathcal{B}(i_1,j_1)\mathcal{B}(i_2,j_2)\}$$
$$=\{\mu\mathcal{A}(i_1,j_1)+\nu\mathcal{B}(i_1,j_1)\}\{\mu\mathcal{A}(i_2,j_2)+\nu\mathcal{B}(i_2,j_2)\},$$

that leads to the system of equations

$$\begin{cases}\mu=\mu^2\\ \nu N=\nu^2\\ \mu\nu=\nu\end{cases} \tag{A.5}$$

which has two non-trivial solutions $\mu_1=1,\ \ \nu_1=N$ and $\mu_2=1,\ \ \nu_2=0$; first solution corresponds to the eigenfunction (26) and the second solution

corresponds to the eigenfunction (27) (the latter is evident, if the identity $(\lambda_a)^\alpha_\beta(\lambda_a)^\gamma_\delta = \frac{1}{2}\delta^\alpha_\delta\delta^\gamma_\beta - \frac{1}{2N}\delta^\alpha_\beta\delta^\gamma_\delta$ is used).

Similarly, for parallel loops we write the (20') as

$$< (a'(i_1+i_2))^\alpha_\beta(b'(j_1+j_2))^\gamma_\delta >=$$
$$< (a'(i_1))^\alpha_\rho(a'(i_2))^\rho_\beta(b'(j_1))^\gamma_\sigma(b'(j_2))^\sigma_\delta >\rightarrow$$
$$\rightarrow< (a'(i_1))^\alpha_\rho(b'(j_1))^\gamma_\sigma > \cdot < (a'(i_2))^\rho_\beta(b'(j_2))^\sigma_\delta >,$$

or, using eq. (23),

$$\mathcal{A}'(i_1+i_2, j_1+j_2) \rightarrow \mathcal{A}'(i_1,j_1)\mathcal{A}'(i_2,j_2) + \mathcal{B}'(i_1,j_1)\mathcal{B}'(i_2,j_2) \tag{A.6}$$

$$\mathcal{B}'(i_1+i_2, j_1+j_2) \rightarrow \mathcal{A}'(i_1,j_1)\mathcal{B}'(i_2,j_2) + \mathcal{A}'(i_2,j_2)\mathcal{B}'(i_1,j_1) \tag{A.7}$$

Substituting (A.5) and (A.6) into (A.2) we have

$$\mu\{\mathcal{A}'(i_1,j_1)\mathcal{A}'(i_2,j_2) + \mathcal{B}'(i_1,j_1)\mathcal{B}'(i_2,j_2)\}+$$
$$+\nu\{\mathcal{A}'(i_1,j_1)\mathcal{B}'(i_2,j_2) + \mathcal{A}'(i_2,j_2)\mathcal{B}'(i_1,j_1)\} =$$
$$= \{\mu\mathcal{A}'(i_1,j_1) + \nu\mathcal{B}'(i_1,j_1)\}\{+\mu\mathcal{A}'(i_2,j_2) + \nu'\mathcal{B}(i_2,j_2)\},$$

that leads to the system of equations

$$\begin{cases} \mu = \mu^2 \\ \mu = \nu^2 \\ \nu = \mu\nu \end{cases} \tag{A.8}$$

with non-trivial solutions $\mu_1 = 1, \nu_1 = +1$ and $\mu_2 = 1, \nu_2 = -1$, corresponding to the eigenfunctions (29).

Appendix B

In the case when only the second order cumulants contribute to the string tensions, the expressions (44), (45) and (47) are simplified considerably, because in this case all the cumulants $K_2^{(r)}$ can be expressed in terms of a quadratic correlator of the fields Φ. As a result all string tensions can be related to the fundamental one. Introducing convenient notations

$$\delta_{mn}\Phi(z_1^{(i)}, z_2^{(j)}) =< A^{(m)}(z_1^{(i)})A^{(n)}(z_2^{(j)}) >, \quad i,j = 1,2, \tag{B.1}$$

where m, n are the colour octet indeces, we note first that one gets from (22),(23)

$$\mathcal{A}(2,0) = \mathcal{A}'(2,0) = \frac{N^2-1}{2N}\Phi(x_1^{(1)}, x_2^{(1)}) \tag{B.2}$$

$$\mathcal{A}(0,2) = \mathcal{A}'(0,2) = \frac{N^2-1}{2N}\Phi(x_1^{(2)}, x_2^{(2)}) \tag{B.3}$$

$$\mathcal{B}(2,0) = \mathcal{B}(0,2) = \mathcal{B}'(2,0) = \mathcal{B}'(0,2) = 0 \tag{B.4}$$

$$\mathcal{A}(1,1) = \mathcal{A}'(1,1) = -\frac{1}{2N}\Phi(x_1^{(1)}, x_1^{(2)}) \tag{B.5}$$

$$\mathcal{B}(1,1) = \mathcal{B}'(1,1) = \frac{1}{2}\Phi(x_1^{(1)}, x_1^{(2)}) \tag{B.6}$$

Therefore one can see from (35),(39) that all the $\mathcal{K}_2^{(r)}$ are related to the integrals

$$\Phi_{ij} = \frac{1}{2}\int_{S_i}\int_{S_j} d\sigma(z_1{}^{(i)})d\sigma(z_2^{(j)})\Phi(z_1^{(i)}, z_2^{(j)}) \tag{B.7}$$

in the following way

$$\frac{1}{2}\mathcal{K}_2^{(r)}(2,0) = \frac{N^2-1}{2N}\Phi_{11}, \tag{B.8}$$

$$\frac{1}{2}\mathcal{K}_2^{(r)}(0,2) = \frac{N^2-1}{2N}\Phi_{22}, \tag{B.9}$$

while

$$\frac{1}{2}\mathcal{K}_2^{(0)}(1,1) = \frac{N^2-1}{N}\Phi_{12}, \tag{B.10}$$

$$\frac{1}{2}\mathcal{K}_2^{(adj)}(1,1) = \frac{1}{N}\Phi_{12}, \tag{B.11}$$

$$\frac{1}{2}\mathcal{K}_2^{(s,a)}(1,1) = \pm\frac{N\mp 1}{N}\Phi_{12}. \tag{B.12}$$

For large contours embedded into the same plane we have

$$\Phi_{11} = \tilde{\sigma} S_1, \quad \Phi_{22} = \tilde{\sigma} S_2, \quad \Phi_{12} = \mp \tilde{\sigma} S_2, \quad S_1 > S_2, \tag{B.13}$$

where $-(+)$ stands for the opposite (parallel) directions in the loops, and

$$\tilde{\sigma} = \frac{2N}{N^2 - 1} \sigma^{(f)}.$$

Substituting (B.8)-(B.13) into the eq. (43) we obtain the relations (46) between string tensions in different representations.

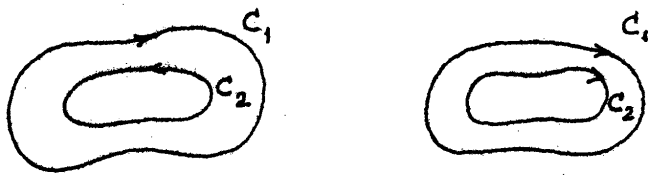

Fig.1. Geometries of Wilson loops under consideration.

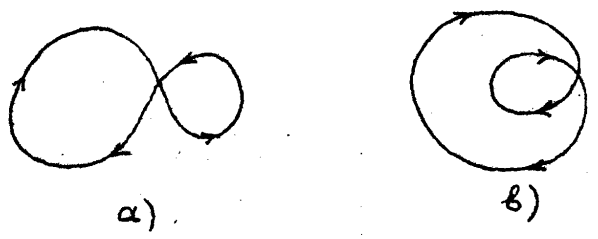

Fig.2. Geometries of Wilson loop with one selfintersection.

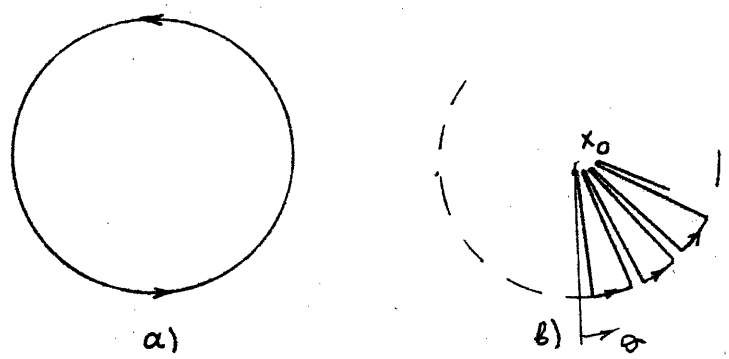

Fig.3. Transformation of the initial contour a) to the deformed one b) for the application of non-abelian Stokes theorem in the Fock–Schwinger gauge (4) for the case of a single Wilson loop.

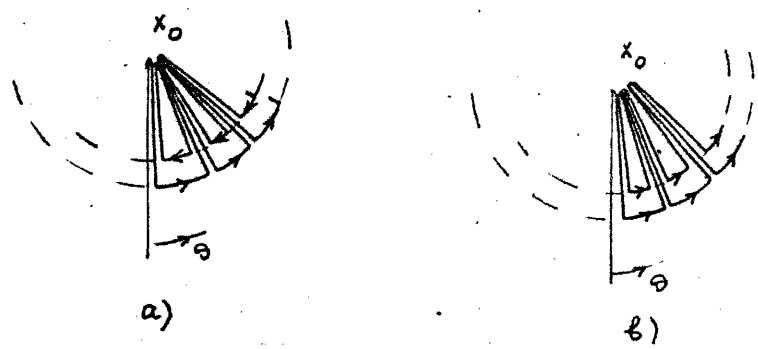

Fig.4. Deformed contours for the case of two Wilson loops (16), with angular correlations taken into account.

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