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E.P.Shabalin

CP VIOLATING DECAY $\eta \to 2\pi$ IN DIFFERENT THEORIES OF BREAKDOWN OF CP SYMMETRY

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CP VIOLATING DECAY $\eta \to 2\pi$ IN DIFFERENT THEORIES OF BREAKDOWN OF CP SYMMETRY[1] :Preprint ITEP 4-96/

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In the Standard Model (SM) of electroweak interaction with CP invariant strong interaction, the CP violating effects in flavour-conserving processes are expected to be very tiny and look beyond the experimental reach in the near decade(s).

If, however, both the Kobayashi-Maskawa mechanism and mechanism of spontaneous CP violation work, such a theory allows to have a picture of CP violation consistent with existing limits on parameters ϵ' and d_n and gives for the upper limit of the $\eta \to 2\pi$ rate a value one order larger than the upper limit for this rate originated by θ-term of QCD lagrangian.

Ref. - 20.

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1 Introduction

In order to clarify the nature of CP violation it is very important that in addition to studying flavour-changing processes also to look for CP effects in flavour-conserving transitions, such as $\eta \to 2\pi$ decay or interaction of baryons with electromagnetic field through their electric dipole moment. The reason is that predicting more or less the same picture of CP violation in strangeness-changing decays of $\{K^0, \bar{K}^0\}$ system, the different theories predict very different values for flavor-conserving processes, in particular, for CP violating $\eta \to 2\pi$ decay. The table 1 illustrates the situation.

Table 1.
Expected values of CP violating effects in flavour-changing and flavour-conserving processes originated by different sources of breakdown of CP symmetry.

Strangeness-changing processes	Flavour-conserving processes
Standard Model with CP invariant strong interaction [1]	
$\eta_{\pm} = \varepsilon + \varepsilon'$ $\eta_{00} = \varepsilon - 2\varepsilon'$ $\lvert\varepsilon'/\varepsilon\rvert^{th} \approx (3 \div 6)10^{-4}$ [3]	$d_n \leq 10^{-32} e$ cm [2] $R_{\eta 2\pi} = \frac{\Gamma(\eta \to 2\pi)}{\Gamma(\eta \to \mathrm{all})} \leq 2 \cdot 10^{-27}$ [4]
Superweak interaction [5]	
$\eta_{\pm} = \eta_{00}$ $\varphi_{\pm} = \varphi_{00} = \varphi_{SW} \simeq 43.7^0$	Amplitudes $\sim g_{SW} G_F^2 \sin^2\theta_C$ In particular, $R_{\eta 2\pi} \approx 3 \cdot 10^{-49}$
Spontaneous CP violation in theory with natural flavour conservation [6]	
$\lvert\varepsilon'/\varepsilon\rvert^{th} \sim 5 \cdot 10^{-2}$ [7] versus $\lvert\varepsilon'/\varepsilon\rvert^{exp} = (1.5 \pm 0.8)10^{-3}$ [9]	$d_n^{th} \approx 10^{-24} e$ cm [8] versus $d_n^{exp} \leq 1.1 \cdot 10^{-25} e$ cm [9] $R_{\eta 2\pi} \gg R_{\eta 2\pi}$ (SM)
Strong CP violation due to θ-term in QCD [10]	
	$d_n \approx 2 \cdot 10^{-16} \lvert\theta\rvert e$ cm [11] $R_{\eta 2\pi} \approx 350 \cdot \theta^2 \leq 1.2 \cdot 10^{-16}$ [12] at $\theta \leq 6 \cdot 10^{-10}$ from d_n^{exp}

The Table 1 leads to conclusion that $\eta \to 2\pi$ decay will have a chance to be observed in future only in the case of breakdown of CP symmetry in strong interaction with magnitude of θ not far from the existent upper limit on this quantity. The other source of $R_{\eta 2\pi} \gg R_{\eta 2\pi}(SM)$ – the spontaneous CP violation in Higgs sector, seems to be excluded by experimental data. But the last conclusion relates to the situation when the spontaneous CP violation is the only source of breakdown of CP symmetry. If, instead, this mechanism works simultaneously with Kobayashi-Maskawa mechanism [13] of CP violation then the picture is changed in a considerable way. In such a theory, it turns to be possible to have [14]

$$1) \qquad |\varepsilon'/\varepsilon| = (\text{anyvalue}) \leq (1.5 \pm 0.8)10^{-3}$$

$$2) \qquad d_n^{th} \leq 0.1 d_n^{ex} \tag{1}$$

3) $\quad R_{\eta 2\pi} \leq 1.8 \cdot 10^{-15}$

The theory with two sources of CP violation could be especially useful, if the future experiments will confirm that $\varepsilon'_{exp} > \varepsilon'^{th}(SM)$.

Let's now turn to foundation of the results concerning $\eta \to 2\pi$ decay in different models of CP violation.

2 $\eta \to 2\pi$ in SM with CP invariant strong interaction

In SM, a violation of CP invariance occurs due to imaginary part of coupling constants. But Hermicity of the $\eta\pi\pi$ interaction does not allow a complex coupling constant for such a vertex if η meson and $(\pi\pi)$ system are the self-conjugate states. Therefore, $\eta \to 2\pi$ transition could take place only if η or $(\pi\pi)$ states have a small admixture of some non-self-conjugate states, for instance, $K^0(\bar{K}^0)$ or $(K\pi)$ respectively. In other words, $\eta \to 2\pi$ transition must be the two-step transition [2]

$$\eta \to \{K^0, \bar{K}^0\} \to 2\pi \ , \ \text{ or } \ \eta \to \{K\pi, \bar{K}\pi\} \to 2\pi \tag{2}$$

and $\eta \to 2\pi$ transition becomes allowed if the phase Δ and δ of first-step and second-step transition, defined as

$$< K^0|\eta >= | < K^0|\eta > |e^{i\Delta} \ \text{ and } \ < 2\pi|K^0 >= | < 2\pi|K^0 > |e^{-i\delta} \tag{3}$$

are different, that is, $\Delta \neq \delta$.

A calculation of the matrix elements in eqs. (3) can be performed using the effective non-leptonic lagrangian for $\Delta S = \pm 1$ transition given in ref. [15].

$$L(|\Delta S| = 1) = \sqrt{2} G_F \sin\theta_C \cos\theta_C \sum_{j=1}^{\hat{c}} (c_j O_j + c_j^* O_j^\dagger) \tag{4}$$

where

$$\begin{aligned}
O_1 &= (\bar{s}_L \gamma_\mu d_L)(\bar{u}_L \gamma_\mu u_L) - (\bar{s}_L \gamma_\mu u_L)(\bar{u}_L \gamma_\mu d_L), \ (\{8_f\}, \ \Delta I = 1/2), \\
O_2 &= 2(\bar{s}_L \gamma_\mu d_L) \sum_{q=u,d,s} (\bar{q}_L \gamma_\mu q_L) - O_1, \ (\{8_f\}, \ \Delta I = 1/2), \\
O_3 &= O_2 - 5(\bar{s}_L \gamma_\mu d_L)(\bar{s}_L \gamma_\mu s_L), \ (\{27\}, \ \Delta I = 1/2), \\
O_4 &= (\bar{s}_L \gamma_\mu d_L)(\bar{u}_L \gamma_\mu d_L)(\bar{u}_L \gamma_\mu u_L) + (\bar{s}_L \gamma_\mu U_L)(\bar{u}_L \gamma_\mu d_L) -
\end{aligned} \tag{5}$$

[2] The intermediate states with non-zero charm, or beauty give considerably smaller contribution to $\eta \to 2\pi$ transition.

$$
\begin{aligned}
& - \ (\bar{s}_L\gamma_\mu d_L)(\bar{d}_L\gamma_\mu d_L), \ \ (\{27\}, \ \ \Delta I = 3/2), \\
O_5 &= (\bar{s}_L\gamma_\mu\lambda^a d_L)(\sum_{q=u,d,s}\bar{q}_R\gamma_\mu\lambda^a q_R)(\{8\}, \ \ \Delta I = 1/2), \\
O_6 &= (\bar{s}_L\gamma_\mu d_L)(\sum_{q=u,d,s}\bar{q}_R\gamma_\mu q_R)(\{8\}, \ \ \Delta I = 1/2).
\end{aligned}
$$

The above set of operators is not sufficient for calculation of the ratio ε'/ε in $K_L \to 2\pi$ decay [3, 16] but, as it will be seen later, is quite sufficient for our purpose.

The diquark operators can be expressed through the physical mesonic field using the relations [17]

$$\bar{q}^j_L\gamma_\mu q^i_L = \frac{if_\pi^2}{4}\{(\partial_\mu V)V^+ - V(\partial_\mu V^+) - \frac{r}{\Lambda^2}[m(\partial_\mu V^+ - \partial_\mu V)]\}_{ij} \tag{6}$$

$$\bar{q}^j_R q^k_L = -\frac{1}{4}f_\pi^2 r[V - \frac{1}{\Lambda^2}\partial^2 V]_{kj} \tag{7}$$

where

$$V = exp(i\sqrt{2}\hat{\pi}/f_\pi)\ ,\ r = \frac{2m_\pi^2}{m_u + m_d}\ ,\ \Lambda \sim 1GeV\ ,\ f_\pi \approx 93MeV \tag{8}$$

and

$$\hat{\pi} = \begin{pmatrix} \frac{\pi_0}{\sqrt{3}} + \frac{\pi_8}{\sqrt{6}} + \frac{\pi_3}{\sqrt{3}} & \pi^+ & K^+ \\ \pi^- & \frac{\pi_0}{\sqrt{3}} + \frac{\pi_8}{\sqrt{6}} - \frac{\pi_3}{\sqrt{2}} & K^0 \\ K^- & \bar{K}^0 & \frac{\pi_0}{\sqrt{3}} - 2\frac{\pi_8}{\sqrt{6}} \end{pmatrix} \tag{9}$$

The result for $\eta \to 2\pi$ matrix element is

$$< \pi^+\pi^-|\eta >= 6.02 \cdot 10^{-15}\beta Rec_5\frac{Imc_5}{Rec_5}[-\sqrt{3}\sin\theta_P(c_1 + c_2) +$$

$$+ \frac{10\cos\theta_P + 2\sqrt{2}\sin\theta_P}{\sqrt{6}}c_3 + (\frac{2\sin\theta_P}{\sqrt{3}} + \frac{\cos\theta_P}{\sqrt{6}})c_4 e^{i(\delta_2 - \delta_0)}]GeV \tag{10}$$

Using then the numerical values of c_i obtained in ref. [18] and [19] and estimate [20] giving $|Imc_5/Rec_5| = (1\pm0.4)|s_2 s_3 \sin\delta|$ and taking $\theta_P = -19,47^0$ we come to result

$$R_{+-} = \frac{\Gamma(\eta \to \pi^+\pi^-)}{\Gamma(\eta \to \text{all})} \leq (0.66^{+0.63}_{-0.32})10^{-27}\sin^2\delta \tag{11}$$

where δ is the Kobayashi-Maskawa phase. The rate of $\eta \to \pi^0\pi^0$ is half as big.

As it follows from eq. (10), the amplitude of $\eta \to 2\pi$ decay is determined mainly by $\Delta I = 1/2$ part of intermediate $K \to 2\pi$ amplitude and for this

reason is not practically sensitive to corrections introducting by the operators corresponding to so-called electroweak penguin and box diagrams.

The result (11) does not leave a hope to observe this decay if the phase δ is the only source of CP violation. But importance of this result consists in possibility to affirm that the observation of such decay would be the evidence for other sources of CP violation, such as θ-term in QCD lagrangian, or spontaneous CP violation in Higgs sector of the theory.

3 $\eta \to 2\pi$ decay caused by CP violation in strong interaction

A complicated topological structure of QCD vacuum state originates CP violation described by θ-term

$$L^{\theta}_{QCD} = -\theta \frac{\alpha_s}{8\pi} G^a_{\mu\nu} \tilde{G}^a_{\mu\nu} \quad [10] \tag{12}$$

In chiral theory it performs into

$$L^{\theta}(\eta \to 2\pi) = -\theta \frac{m_\pi^2}{2\sqrt{2} f_\pi} \cdot \frac{\Delta m_P^2}{\Delta m_P^2 + \frac{3}{2} m_\pi^2} (\pi^+\pi^- + \frac{\pi^{0^2}}{2})(\sqrt{2}\pi_0 + \pi_8) \tag{13}$$

demonstrating the direct dependence of the result on solution of $U(1)$ problem because Δm_P^2 is a difference between magnitudes of $(m^2_{\pi^0})^{phys}$ and $m^2_{\pi^0}$ following from Gell-Mann-Okubo mass relation. In eq. (13), $\pi^0 = \pi_3 \neq \pi_0$. At $\theta \leq 6 \cdot 10^{-10}$ (from the data on d_n) one gets

$$R^{\theta}_{\eta 2\pi} \leq 1.2 \cdot 10^{-16} \tag{14}$$

4 $\eta \to 2\pi$ decay in theory with Kobayashi-Maskawa and Weinberg mechanism of CP violation [6]

If the SM is complicated by extension of the Higgs sector up to three Higgs doublet, the additional source of CP violation appears – the spontaneous breakdown of CP symmetry in Higgs sector [6]. Then, in addition to diagrams in Fig. 1 contributing in Kobayashi-Maskawa scenario to CP violating parameter ε and ε' in $K \to 2\pi$ decay, the new set of diagrams shown in Figs. 2 and 3 give

a contribution to these parameters originated by Weinberg mechanism of CP breakdown.

If this last source is considered as the only one, such a mechanism gives too large values for ε'/ε [7] and d_n [8] in comparison with experimental data. The cardinal solution of the problem consists in assumption that the KM mechanism works also and gives the main contribution to ε. Then the Weinberg mechanism can contribute considerably to ε' and can originate a large enough magnitude of $\eta \to 2\pi$ decay. But to estimate $R_{\eta 2\pi}$ one needs first to extract a value of unknown parameter ImA of the Weinberg theory from the data on ε'. The important new result in the analysis of the problem concerning ε' in Weinberg model consists in the observation [14] that the diagrams in Fig. 3 give by 5 times larger contribution into ε' than the diagram in Fig. 2b considered previously in literature [7]. The same set of diagram as in Fig. 3 originates the $\eta \to 2\pi$ transition and this allows to express $g_W^{eff}(\eta\pi\pi)$ through $\varepsilon'_W = \varepsilon'_{exp} - (\varepsilon')^{th}_{KM}$.

From the existing data $|\varepsilon'/\varepsilon|^{exp} = (1.5 \pm 0.8)10^{-3}$ and theoretical prediction $|\varepsilon'_{KM}/\varepsilon|^{th} \sim 3{\cdot}10^{-4}$ [3] one can conclude that $|\varepsilon'_W/\varepsilon| \leq 2{\cdot}10^{-3}$. Then calculating a contribution of diagrams in Fig. 2 and 3 into ε'_W and comparing it with the experimental limit on ε'_W one can extract the value of ImA. Inserting this value into diagrams like the ones in fig. 3 but conserving the flavour one can obtain for the coupling constant of effective interaction

$$L^{eff}(\eta \to 2\pi) = g_W \eta\pi\pi$$

the result

$$|g_W| \leq 3.6 \cdot 10^{-10} \cdot (\frac{-\sin\theta_P + \sqrt{2}\cos\theta_P}{\sqrt{3}}) GeV \tag{15}$$

At $\theta_P = -19.47^0$

$$R_{\eta 2\pi} \leq 1.8 \cdot 10^{-15} \tag{16}$$

5 Electric dipole moment of the neutron in theory with KM and Weinberg mechanisms of CP violation

Our estimate of ImA is

$$ImA = 7.1 \cdot 10^{-8}[(\ln\frac{m_H^2}{m_c^2} - 1)(1 + x\frac{V_{td}V_{st}}{V_{cd}V_{sc}})]^{-1} \tag{17}$$

where

$$x = \frac{V_{ts}}{V_{cs}} \cdot \frac{m_t^2}{m_c^2} \cdot \frac{m_H^2}{m_t^2 - m_H^2}(1 - \frac{m_H^2}{m_t^2 - m_H^2}\ln\frac{m_t^2}{m_H^2}).$$

$$\cdot (\ln \frac{m_H^2}{m_c^2} - 1)^{-1} \tag{18}$$

Inserting this ImA into the formulae of ref. [8] one obtains

$$|D_n| \leq 0.11 \cdot 10^{-25} \left(\frac{1 + x \frac{V_{td}^2}{V_{cs}^2}}{1 + x \frac{V_{td}}{V_{cd}} \frac{V_{st}}{V_{dt}}} \right) ecm \tag{19}$$

In the above formulae, V_{ij} are the elements of K-M matrix taken at $\delta = 0$.

6 Conclusions

1. The minuteness of $R_{\eta 2\pi}$ in SM with CP invariant strong interaction presents a golden opportunity for manifestation of the other sources of CP violation.
2. Observation $R_{\eta 2\pi}$ at the level 10^{-16} would not be obligatory the evidence of CP violation due to θ-term in QCD lagrangian. The spontaneous CP violation in the Higgs sector could give also such a rate when it is accompanied by the Kobayashi-Maskawa mechanism of CP violation.
3. Observation $R_{\eta 2\pi}$ with a magnitude few times larger than $R^{\theta}_{\eta 2\pi}$ would be the evidence of CP violation in Higgs sector.
4. In $K_L \to 2\pi$ decays, it is possible, in principle, that the part ε'_{KM} is cancelled by ε'_W. Therefore, $\varepsilon' \to 0$ would not be inevitable the evidence of superweak CP violation.
5. Observation (at DAΦNE) $\varepsilon' > (\varepsilon')^{th}_{SM}$ would be the evidence of CP violation in Higgs sector!

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References

[1] S.L.Glashow, Nucl. Phys. **22** (1961) 579
S.Weinberg, Phys. Rev. Lett. **19** (1967) 1264
A.Salam, in Elementary Particle Theory, Ed. N.Svartholm (Almqvist and Wiksell, 1968).

[2] E.Shabalin, Usp. Fiz. Nauk **139** (1983) 561 (Sov. Phys. Usp. **26(4)** (1983) 297).

[3] G.Buchalla, A.J.Buras and M.K.Harlander, Nucl. Phys. **B337** (1990) 313
M.Ciuchini, E.Franko, G.Martinelli and L.Reina, Phys. Lett. **B301** (1993) 263.

[4] C.Jarlskog and E.Shabalin, Preprint Lund-MPh-95/01.

[5] L.Wolfenstein, Phys. Rev. Lett. **13** (1964) 569.

[6] S.Weinberg, Phys. Rev. Lett. **37** (1976) 657.

[7] N.G.Deshpande, Phys. Rev. **D23** (1981) 654
A.I.Sanda, Phys. Rev. **D23** (1981) 2647.

[8] A.R.Zhitnitskii and I.B.Khriplovich, Yad. Fiz. **34** (1981) 167 [Sov. J.Nucl. Phys. **34** (1981) 95].

[9] Particle Data Group, Phys. Rev. **D50** (1994) 1173.

[10] G.t'Hooft, Phys. Rev. Lett. **37** (1976) 8.

[11] V.Baluni, Phys. Rev. **D19** (1979) 2227
R.J.Crewther *et al.*, Phys. Lett. **B88** (1979) 123
E.P.Shabalin, Yad, Fiz. **36** (1982) 981 [Sov. J. Nucl. Phys. **36** (1982) 575]
H.J.Schnitzer, Phys. Lett. **139B** (1984) 217.
L.J.Dixon *et al.*, Preprint SLAC-PUB-5344, 1990.

[12] R.J.Crewther *et al.*, Phys. Lett. **88B** (1979) 123
M.A.Shifman, A.I.Vainshtein and V.I.Zakharov, Nucl. Phys. **B166** (1980) 494
C.Jarlskog and E.Shabalin, this Report.

[13] M.Kobayashi and T.Maskawa, Prog. Theor. Phys. **49** (1973) 652.

[14] C.Jarlskog and E.Shabalin, Preprint Lund-MPh-95/05.

[15] A.I.Vainshtein, V.I.Zakharov and M.A.Shifman, JETP **72** (1977) 1275.

[16] J.M.Flynn, L.Randall, Phys. Lett. **B224** (1989) 221
E.A.Paschos and Y.L.Wu, Mod. Phys. Lett. **A6** (1991) 93.

[17] W.A.Bardeen, A.J.Buras and J.-M-Gerard, Nucl. Phys. **B293** (1987) 787.

[18] L.B.Okun, "Leptons and Quarks" (North-Holland Publ. Comp., 1982) p.p. 315-323.

[19] E.Shabalin, Nucl. Phys. **B409** (1993) 87.

[20] M.B.Voloshin, Preprint ITEP-22 (1981), unpublished.

Шабалин Е.П.

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