STATE RESEARCH CENTER OF RUSSIA
Institute of Theoretical
and Experimental Physics

FERMILAB
APR 8 1997
30-96
LIBRARY
Fermilab Library
0 1160 0065883 5
ITEP-30-96

E.P.Shabalin

ON THE ROLE OF QUARK-GLUON OPERATOR IN $K^0 \to 2\pi$ DECAY

Moscow 1996

UDK 530.1 M-16

ON THE ROLE OF QUARK-GLUON OPERATOR IN $K^0 \to 2\pi$ DECAY: Preprint ITEP 30-96/

E.P.Shabalin - M., 1996 - 8p.

The different approaches to calculation of the $K^0 \to 2\pi$ transition amplitude originated by quark-gluon operator are reanalyzed leading to conclusion that the magnitude of the corresponding matrix element is

$$g_s < \pi^+\pi^-|\bar{s}_R\sigma_{\mu\nu}\hat{G}_{\mu\nu}d_L|K^0 >= -(6-12)m_K^2 F_\pi/(m_s+m_u)$$

Fig. - 3, ref. - 16.

© Институт теоретической и экспериментальной физики, 1996

The role of the quark-gluon operator

$$T = \bar{s}_R \sigma_{\mu\nu} \hat{G}_{\mu\nu} d_L \tag{1}$$

where

$$\hat{G}_{\mu\nu} = \frac{1}{2} \lambda^a G^a_{\mu\nu} \tag{2}$$

in the decays $K^0 \to 2\pi$ was widely discussed in the literature. This operator appears at one loop level in the theories incorporating the right-handed currents in addition to usual left-handed currents and its role was discussed in connection with the $\Delta I = 1/2$ rule [1, 2].

In the Standard Model (SM), the operator T arises at one loop level in the order $G_F m_q^2 / M_W^2$ [3] and at two-loop level in the order G_F with small coefficient [4, 5].

It appears also in SM with extended Higgs sector incorporating two, or more Higgs doublets and was considered in connection with the question on a magnitude of the parameter ε' [5] -[8].

The numerical coefficient with which the operator T arises in the effective $|\Delta S| = 1$ Hamiltonian obviously depends of a sort of theory. But to get a conclusion on significance of this operator for $K \to 2\pi$ decay, it is not sufficient to know only the corresponding coefficient. The final conclusion depends also on the result of hadronization of T.

The question is not simple because of presence in T of the gluon field explicitly. This gluon can be absorbed by another quark [6, 8], or one more gluon can be emitted from external s and d quarks [8] giving rise in appearance of the colourless products $(\bar{s}q)(\bar{q}d)$, or $(\bar{s}q)(G^a_{\mu\nu})^2$ which can be easily interpreted in terms of the physical mesons. But we shall deal in these cases with the new operators, different of T, which are dependent on the large distances region.

As for the attempts of the direct hadronization of the operator T, in the literature, there are two opposite points of view on the result:

1) The operator T originates $K^0 \to 2\pi$ transition with a magnitude of order $F_\pi^{-3} m_0^2 < \bar{q}q >$ [3], where $m_0^2 \approx 0.8 GeV^2$.

2) It does not induce $K^0 \to 2\pi$ transition to leading order in expansion of the amplitude in squared momenta of external mesons [7, 5].

The first statement is not true because the technics used in ref. [3] did not distinguish the operators T and $\bar{s}_R d_L$, but the last one does not produce $K^0 \to 2\pi$ decay, because it can be taken away by the redefinition of the quark

fields. As it will be shown below, the non-zero result of ref. [3] appeared as a result of neglect of very rapid variation of the amplitude from the point $\{p_{\pi,\pi'} = 0; p_k = 0\}$ to point $\{p^2_{\pi,\pi'} = m^2_\pi; p^2_k = m^2_K\}$.

The second statement does not clear up the question on a magnitude of the effect because each of the operators of the effective $\Delta S = -1$ Hamiltonian produces $K^0 \to 2\pi$ transition just in the same order in $p^2_{\pi,K}$.

Let's now consider, why an application of the soft-pion technics gave a wrong result concerning the influence of T on $K^0 \to 2\pi$ transition.

The result of ref. [3] was obtained using the reduction formula, PCAC relations and usual current-algebra relations which are non-sensitive to the presence of gluonic field. Transferring $\pi^+(p_1)$ and $\pi^-(p_2)$ fields under the time-ordered product of the operators and using the first of the PCAC relations

$$\partial_\mu A^{(1-3)}_\mu = F_\pi m^2_\pi \pi^{(1-3)} \ , \ \partial_\mu A^{(4-7)}_\mu = F_K m^2_K \pi^{(4-7)} \tag{3}$$

one gets

$$< \pi^+(p_1)\pi^-(p_2)|T|K^0(k_1) >_{p_{1,2}\to 0} = -\frac{1}{2F^2_\pi} < 0|T|K^0(p_K = 0) > \tag{4}$$

Applying the same procedure to the rest matrix element one comes to the result

$$\begin{aligned} < 0|T|K^0(p_K = 0) > \ &= \ -\frac{i}{2\sqrt{2}F_K} < 0|\bar{d}\sigma_{\mu\nu}\hat{G}_{\mu\nu}d + \\ &+ \ \bar{s}\sigma_{\mu\nu}\hat{G}_{\mu\nu}s|0 > \end{aligned} \tag{5}$$

The last vacuum expectation value usually is represented in the form

$$< 0|\bar{q}\sigma_{\mu\nu}\hat{G}_{\mu\nu}q|0 >= m^2_0 < 0|\bar{q}q|0 > \tag{6}$$

with $m^2_0(1GeV) = (0.8 \pm 0.2)GeV^2$ [9].

Let us stress now that replacing the operator T by the operator $\bar{s}_R d_L$ one would come to the result

$$< \pi^+(p_1)\pi^-(p_2)|\bar{s}_R d_L|K^0(k_1) >= \frac{i}{2\sqrt{2}F^2_\pi F_K} < 0|\bar{d}d + \bar{s}s|o > \tag{7}$$

that differs from the results (4) - (6) by the absence of the numerical factor m^2_0 only.

The crucial points in such calculations is that both results were obtained in limit $p_{1,2} = 0$ and $p_K = 0$.

Using the similarity of the operators T and $\bar{s}_R d_L$ with respect to the applied technics of calculation, let us show why the amplitude (4) turns into zero on mass shell.

The operator $\bar{s}_R d_L$ can be represented in terms of unitary 3×3 matrix U containing the pseudoscalar meson fields:

$$\bar{s}_R d_L = -\frac{F_\pi^2 m_\pi^2}{2(m_u + m_d)}(U - \frac{1}{\Lambda^2}\partial^2 U)_{23} \quad ,[10] \tag{8}$$

where the general form of U is as follows [11]

$$\begin{aligned} U &= 1 + \frac{i\sqrt{2}\hat{\pi}}{F_\pi} + (\frac{i\hat{\pi}}{F_\pi})^2 + a_3(\frac{i\hat{\pi}}{\sqrt{2}F_\pi})^3 + \\ &+ 2(a_3 - 1)(\frac{i\hat{\pi}}{\sqrt{2}F_\pi})^4 + ... \end{aligned} \tag{9}$$

As it was established in ref. [11], a fulfillment of the PCAC relations (3) demands $a_3 = 0$. We adopt this condition, especially as the on-mass-shell result does not depend on a magnitude of a_3. Then the effective P-odd, $\Delta S = -1$ lagrangian is

$$L^{eff}_{P-odd}(\bar{s}_R d_L) = -\frac{iF_\pi m_\pi^2}{\sqrt{2}(m_u + m_d)}(\hat{\pi} - \frac{1}{\Lambda^2}\partial^2\hat{\pi})_{23} \tag{10}$$

Such an "interaction" is able to produce only the transition $K^0 \to$ vacuum:

$$< 0|\bar{s}_R d_L|K^0 >= \frac{-i}{2\sqrt{2}F_K} < 0|\bar{d}d + \bar{s}s|0 > \tag{11}$$

Comparing this expression with those given by eqs. (5) and (6) we see that

$$< 0|T|K^0 >= m_0^2 < 0|\bar{s}_R d_L|K^0 > \tag{12}$$

Turning to $K^0 \to 2\pi$ decay, we conclude that for the case $a_3 = 0$, the amplitude originated by the operator $\bar{s}_R d_L$ is described by the diagram in Fig. 1.

The matrix element corresponding to this diagram is

$$\begin{aligned} & < \pi^+(p_1)\pi^-(p_2)|\bar{s}_R d_L|K^0(k_1) >= \\ = & \lim_{k_2 \to 0}\{< \pi^+(p_1)\pi^-(p_2)|\mathcal{H}^{strong}|K^0(k_1)\bar{K}^0(k_2) > \cdot \\ \cdot & \frac{-i \cdot m_0^{-2}}{m_K^2 - k_2^2} < 0|T|\bar{K}^0(k_2) >\} \end{aligned} \tag{13}$$

where $\pi\pi \to K^0\bar{K}^0$ scattering amplitude to leading order in p_i^2 in a theory with PCAC conditions is as the follows: [12]

$$< \pi^+(p_1)\pi^-(p_2)|K^0(k_1)\bar{K}^0(k_2) >= \frac{1}{2F_\pi^2}[s + t - m_K^2 - m_\pi^2] \qquad (14)$$

where

$$s = (k_1 + k_2)^2 = (p_1 + p_2)^2 \ ; \ t = (k_2 - p_1)^2 = (k_1 - p_2)^2 \qquad (15)$$

At $k_2 = 0$ one has $s = m_K^2$, $t = m_\pi^2$ and the amplitudes (13) and (14) turn into zero.

Using the linear σ-model with broken $U(3)_R \otimes U(3)_L$ symmetry in the form [13]

$$L_\sigma = \frac{1}{2}Tr\{\partial_\mu U \partial_\mu U^+\} - cTr\{UU^+ - A^2\}^2 + \frac{F_\pi}{2\sqrt{2}}Tr\{M(U + U^+)\} \qquad (16)$$

where

$$U = \hat{\sigma} + i\hat{\pi} \qquad (17)$$

$$M \sim diag\{m, m, m_s\} \ , \ m = \frac{1}{2}(m_u + m_d) \qquad (18)$$

and σ_a are the scalar partners of π_a, it can be shown that the amplitude of $\pi\pi \to K^0\bar{K}^0$ transition turns into zero to <u>all order</u> in s and t, if $k_2 = 0$. Namely (see ref. [14]),

$$\begin{aligned} &< \pi^+(p_1)\pi^-(p_2)|K^0(k_1)\bar{K}^0(k_2) >= \\ &= \frac{\Lambda^2}{2F_\pi^2}[\frac{3 - 2R}{1 - (s - m_\pi^2)/\Lambda^2} + \frac{(2R - 1)^2}{(2R - 1) - (t - m_K^2)/\Lambda^2} - 2] \ , \end{aligned} \qquad (19)$$

where $R = F_K/F_\pi$, $\Lambda^2 = (m_K^2 - m_\pi^2)(R - 1)^{-1}(2R - 1)^{-1}$ and s and t are determined by eq. (15). At $k_2 = 0$ this amplitude is equal to zero.

We come to the conclusion that the operator $\bar{s}_R d_L$ does not contribute to $K^0 \to 2\pi$ transition, as it was expected from the general consideration [15]. In view of existence of this general consideration, our above exercise could seem as the superfluous one. But using the formulae (12), (13) and (14) we are able to understand the origin of the result (4) of ref. [3]. As it was noticed before, it was obtained in the limit p_1, p_2, k_1 (and consequently, k_2) turning into zero, and the used technics did not distinguish the dynamical properties

of T and $\bar{s}_R d_L$. In limit of zero momenta of all mesons the amplitude (14) is not equal to zero, and as the result

$$\lim_{p_{1,2}\to 0, k_{1,2}\to 0} \{< \pi^+(p_1)\pi^-(p_2)|T|K^0(k_1) >\} =$$
$$= -\frac{1}{2F_\pi^2}(1 + \frac{m_\pi^2}{m_K^2}) < 0|T|K^0 > \qquad (20)$$

Consequently, the result of ref. [3] arised because of neglect of very strong dependence of $\pi\pi \to K\bar{K}$ amplitude of the external momenta. It is an example of the situation, when the soft-pion technics leads to wrong result in a region of momenta $p_i^2 \leq m_{\bar{K}}$.

The main difference between the operators T and $\bar{s}_R d_L$ is the dynamical one and consists in the ability to induce the $sq \to qd$ scattering through T. The operator $\bar{s}_R d_L$ can not make this. Besides, the operator T can not be rotated away by redefinition of the quark fields. It is obvious, because, in particular, the diagonal terms $\bar{s}\sigma_{\mu\nu}\gamma_5 G^a_{\mu\nu} s$ and $\bar{d}\sigma_{\mu\nu}\gamma_5 G^a_{\mu\nu} d$ can not arise in CP-invariant theory.

And so, the effect of T can be estimated using the diagram in Fig. 2, as it was done for the first time in the papers of ref. [6]. The result of hadronization of such quark-quark amplitude was criticized in ref. [7] as though not corresponding to general requirement of proportionality to the squared momenta of the external mesons.

Really it is not so, because the quantity m_G^2 figurating in ref. [6] does not disappear at $m_q \to 0$ (see ref. [8]), but the total coefficient does not agree with the recent result [8]. Namely

$$< \pi^+\pi^-|T|K^0 >= \frac{2\sqrt{2}F_\pi g_s}{3} \cdot \frac{m_\pi^2}{m_d + m_u} \cdot$$
$$\cdot \frac{m_K^2}{m_G^2} \begin{cases} 1 + F_K/F_\pi + m_K^2 F_K/(m_\sigma^2 F_\pi) & , \text{ [6] (Deshpande)} \\ \frac{m_s - m_d}{m_s + m_d} \cdot \frac{m_K^2 - m_\pi^2}{\Lambda^2} & , \text{ [8]} \end{cases}$$

The prediction of ref. [8] is by 13 times smaller than obtained in ref. [6] (Deshpande). But, as it was pointed out in ref. [8], the diagrams in Fig. 3 give a contribution which is numerically by 5 times larger than the contribution from Fig. 2.

More careful analysis shows that the physically meaningful magnitude of the constant in eq. (12) of ref. [8] has to be multiplied by the factor

$(m_s + a)/(m_s + a + b)$ where the denominator is the constituent mass of s-quark, nominator is a part of this mass without $< G^a_{\mu\nu})^2 >_0$ contribution and a corresponds to contribution of $< \bar{q}q >_0$. This factor arises as a result of diagonalization of the mass part of the lagrangian describing the constituent quarks and must be of order 1 as the main contribution into the nucleon mass (and consequently, the mass of constituent quark) appears from $< \bar{q}q >_0$ [16, 9].

The result [8] of hadronization of T basing on the diagrams in Fig. 3 is

$$< \pi^+\pi^-|T|K^0 >= -\frac{32\pi^2\sqrt{2}}{27g_s} \cdot \frac{m_K^2}{m_s + m_u} \cdot I \cdot F_\pi \tag{21}$$

where the quantity I disappears in limit $m_q \to 0$ and numerically is estimated to be 1.54.

The different approach to estimation of the result of hadronization of T used in ref. [5] gave

$$< \pi^+\pi^-|T|K^0 >= -\frac{11}{2\sqrt{2}g_s} \cdot \frac{m_K^2}{m_s + m_u}(\frac{F_K}{F_\pi})^2\frac{m_\pi^2}{F_\pi} \tag{22}$$

Being absolutely unlike in the forms these estimates numerically are very close one to another. Namely, the estimate (21) is by two times larger than the estimate (22).

Such a small difference of the results (21) and (22) obtained applying the unlike ideas on the procedure of hadronization of T allows to hope that the eqs. (21) and (22) give the right order of a magnitude of $K^0 \to 2\pi$ amplitude originated by this operator.

The author is grateful to Professor S.Bertolini for many useful discussions of the problem. He also wish to thank the Department of Elementary Particle Physics of Scuola Internazionale Superiore di Studi Avanzati, Trieste for kind hospitality.

This research was supported in part by INTAS under the Grant No. 94-3986.

Figures

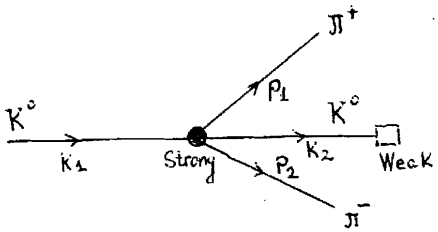

Fig. 1.

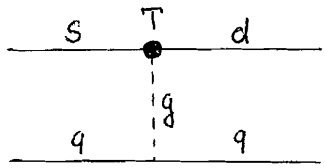

Fig. 2.

Fig. 3.

References

[1] H.Fritzsch and P.Minkowsky, Phys. Lett. **61B** (1976) 275.

[2] A.I.Vainshtein, V.I.Zakharov and M.A.Shifman, JETP Letters **23** (1976) 656.

[3] A.A.Penin and A.A.Pivovarov, Nuovo Cim. **106A** (1993) 19.

[4] A.I.Vainshtein, V.I.Zakharov and M.A.Shifman, Yad. Fiz. **72** (1977) 1275.

[5] S.Bertolini, N.Fabbrichesi and E.Gabrielli, Phys. Lett. **B327** (1994) 136.

[6] N.G.Deshpande, Phys. Rev. **D23** (1981) 2654;
A.I.Sanda, Phys. Rev. **D23** (1981) 2647.

[7] J.F.Donoghue and B.R.Holstein, Phys. Rev. **32** (1985) 1152.

[8] C.Jarlskog and E.Shabalin, Phys. Rev. **D52** (1995) 6327.

[9] V.M.Belyaev and B.L.Ioffe, Z.Eksp. Teor. Fiz. **83** 1982) 876.

[10] W.A.Bardeen, A.J.Buras and J.-M.Gerard, Nucl. Phys. **B293** (1987) 787;
R.S.Chivukula, J.M.Flynn and H.Georgi, Phys. Lett. **171B** (1986) 453.

[11] J.Cronin, Phys. Rev. **161** (1967) 1483.

[12] H.Osborn, Nucl. Phys. **B15** (1970) 501.

[13] C.Rosensweig, J.Schechter and G.Trahern, Phys. Rev. **D21** (1980) 3388

P. Di Vecchia and C.Veneziano, Nucl. Phys. **B171** (1980) 253.

[14] E.Shabalin, Nucl. Phys. **B409** (1993) 87 (Appendix A).

[15] G.Feinberg, P.K.Kabir and S.Weinberg, Phys. Rev. Lett. **3** (1959) 527;
S.Weinberg, Phys. Rev. Lett. **31** (1973) 494; Phys. Rev. **D8** (1973) 605.

[16] A.Smilga, Yad. Fiz. **35** (1982) 473 [Sov. J. Nucl. Phys. **35** (1982) 271].