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V.G.Ksenzov

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ON THE VACUUM ENERGY DENSITY AND NON-PERTURBATIVE EFFECTS IN SIGMA MODELS

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ON THE VACUUM ENERGY DENSITY AND NON-PERTURBATIVE EFFECTS IN SIGMA MODELS: Preprint IREP 93-28/

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The vacuum energy density is obtained for $O(N)$ nonlinear sigma models. It was shown that non-perturbative contribution is connected with Casimir operator of the group $O(N)$.

Fig. - , ref. - 6

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After the discovery of nonperturbative fluctuations the question about their function in generating of physical amplitude is of particular concern in the field theory. The most effort goes into investigating these effects in the Yang-Mills theory, in which we are still far from a complete understanding because it is a notoriously difficult task. Therefore there is sense to study these problems in a simple theory to get a new insight into the problem. Such simple analogy of Yang-Mills theory is two-dimensional sigma models with fields transform according to the vector representation of the group $O(N)$ [1]. These models provide a possibility to perform the analysis conserving hope that obtained results are interesting in actual physics.

For a long time it has become clear that the dynamics is in some way related to a complicated vacuum structure, therefore investigation of the vacuum structure turns out to be the key to understanding actual dynamics.

In this paper, we will discuss the influence of non-perturbative fluctuations on the vacuum energy density ε_{vac} in the framework of the two-dimensional sigma model. We shall demonstrate this by considering the large N limit. In these models, non-perturbative effects take place and do not depend on the existence of instantons for $N > 3$.

Remind what physicists know about nonperturbative effects and ε_{vac} in sigma models. Lorentz invariance implies that the vacuum energy density may be defined as

$$g_{\mu\nu}\varepsilon_{vac} = < 0 \mid \theta_{\mu\nu} \mid 0 > ,$$

where $\theta_{\mu\nu}$ stands for the energy-momentum tensor. Taking into account the conformal anomaly we arrive at

$$\varepsilon_{vac} = \frac{1}{2} < 0 \mid \theta_{\mu\mu} \mid 0 > .$$

Then the vacuum energy may be written as a sum of two terms in sigma models at large N

$$\varepsilon_{vac} = \frac{N}{8\pi}(-M^2 + \frac{<\alpha>}{\sqrt{N}}),$$

M^2 is regulator mass, α is Lagrange multiplier and $<\alpha>$ is constant denoted by $m^2\sqrt{N}$. The first term is connected with the perturbative fluctuation in the vacuum, while the second one is due to non-perturbative fluctuations. These results were obtained in ref. [2]. Let us note that $<\alpha>$ is a fundamental value of the theory also in the case of phase transitions [3].

To demonstrate how the non-perturbative effects enter into the value $<\alpha> \neq 0$ one employs the Lagrangian

$$L = \frac{1}{2}\{(\partial_\mu \sigma^a)^2 + \frac{\alpha}{\sqrt{N}}(\sigma^a\sigma^a - \frac{N}{f})\} \ . \tag{1}$$

We want to decompose $\sigma^a(x)$ and $\alpha(x)$ as

$$\begin{aligned} \sigma^a(x) &= c^a(x) + q^a(x) \\ \alpha(x) &= \alpha_c(x) + \alpha_{qu}(x) \ , \end{aligned} \tag{2}$$

where $c^a(x)$ and $\alpha_c(x)$ are slowly varying fields, while $q^a(x)$ and $\alpha_{qu}(x)$ describe fast varying fields.

The equations of motion for slowly varying fields are

$$\partial^2 c^a = \frac{\alpha c^a}{\sqrt{N}} \tag{3}$$

$$c^a(x)c^a(x) = \frac{N}{f} \tag{4}$$

$$\frac{\alpha_c(x)\sqrt{N}}{f} = -(\partial_\mu c^a)^2 \ . \tag{5}$$

The equation (5) may be rewritten in a linear form for $(\partial_\mu c^a)$. Using the current symmetry J_μ^{ab} and the constraint (4) we get

$$(J_\mu^{ab})^2 = \frac{N}{f}(\partial_\mu c^a)^2 \ , \tag{6}$$

where $J_\mu^{ab} = c^a\partial_\mu c^b - \partial_\mu c^a \, c^b$, and

$$-\frac{\alpha_c(x)}{\sqrt{N}} = (\frac{f}{N}J_\mu^{ab})^2 \ . \tag{7}$$

The $\alpha_c(x)$ turns out to be Casimir operator of the group $O(N)$.

Varying the equation (5) with respect to $\partial_\mu c^a$ and taking account of (7) we can obtain a new equation

$$\partial_\mu c^a = -\frac{2f}{N} J_\mu^{ab} c^b(x) \,. \tag{8}$$

Deriving Eq.(8) we followed the procedure which may be applied only for a special form of the equation.

This equation asserts identically for all $c^a(x)$ which are subject to the constraint (4). These fields have a special property: their transformation under translations can be compensated by global isotopic transformations and they were first discussed in [4].

The classical energy-momentum tensor can be found from (8) and we arrive at

$$\theta_{\mu\nu} = \frac{2f}{N}(J_\mu^a J_\nu^a - \frac{g_{\mu\nu}}{2}(J_\mu^a)^2) \tag{9}$$

with apparent properties

$$\partial_\mu \theta_{\mu\nu} = 0 \,, \quad \theta_{\mu\mu} = 0 \,.$$

The energy-momentum tensor $\theta_{\mu\nu}$ turns out to be proportional to the product of conserved currents in these models. Similar facts are well known in the field theory [5] and in quantum mechanics [6].

To find the vacuum energy density ε_{vac}, we defined it as

$$\varepsilon_{vac} = \frac{1}{2} < 0 \mid \theta_{\mu\mu} \mid 0 > = \lim < 0 \mid M^2 q^a(x-\Delta) q^a(x+\Delta) \mid 0 > \,. \tag{10}$$

The limit of the right-hand-side of Eq. (10) is calculated at $M^2 \to \infty$ and $\Delta^2 \to 0$ and under a special condition $\Delta^2 M^2 = Const$

In equation (10) fast varying fluctuations $q^a(x)$ are not of arbitrary value and must satisfy the constraint [2]

$$2c^a q^a + q^a q^a = 0 \,. \tag{11}$$

Differentiating (11) and implying $q^a \partial_\mu q^a = 0$ (which is the first step approximation) we obtain

$$\partial_\mu q^a = -\frac{2f}{N} J_\mu^{ab} q^b \tag{12}$$

We have to consider (12) as an exact equation. Indeed, using (12) we can see that $q^a \partial_\mu q^a = 0$. If there exist fields q^a with property $q^a \partial_\mu q^a \neq 0$, we can get

$$\partial_\mu q^a = -\frac{2f}{N}(J_\mu^{ab} q^b + c^a q^b \partial_\mu q^b)$$

This equation can be rewritten as

$$q^a \partial_\mu q^a = -\frac{f}{N} q^b c^b q^a \partial_\mu q^a .$$

Using $c^a q^a = -\frac{1}{2} q^a q^a$ we have

$$q^a \partial_\mu q^a (1 - \frac{f}{2N} q^b q^b) = 0 .$$

Solving this equation, we find $q^a q^a = \frac{2N}{f}$, but this solution contradicts to the basic assumption that q^a describes small fluctuations and the condition $q^a \partial_\mu q^a \neq 0$.

Equation (12) connects quantum field fluctuations and non-perturbative fluctuations.

Our goal is to calculate (10) and we can do it knowing only equation (12). Indeed, we get

$$< 0 \mid \theta_{\mu\mu} \mid 0 >= lim \frac{1}{2} (0 <\mid M^2 q^a q^a \mid 0 > - \tag{13}$$

$$-\Delta_\mu \Delta_\nu M^2 < 0 \mid q^a q^k \mid 0 >< 0 \mid \left(\frac{2f}{N}\right)^2 J^{ab}_\mu J^{bk}_\nu \mid 0 > + \ldots)$$

Using the orthonormality condition we can conclude

$$< 0 \mid (\frac{f}{N})^2 J^{ab}_\mu J^{bk}_\nu \mid 0 >= -g_{\mu\nu} \delta^{ak} < 0 \mid J^2_\lambda \mid 0 >= +m^2 g_{\mu\nu} \delta^{ak}$$

$$< q^a q^k >= -\frac{\delta^{ak}}{4\pi} .$$

Remind that the stationary value of $\alpha(x)$ is denoted by $m^2\sqrt{N}$.

We assumed that the vacuum is an eigen state of Casimir operator of the group $O(N)$, but this statement can be proved. However, we will not do it as this is beyond our short report.

In this paper, we have concentrated on the non-perturbative contribution to the vacuum energy density. It was shown that non-perturbative fluctuations with a special property determine non-perturbative contribution into ϵ_{vac}. As a mater of fact, vacuum condensate of $< \alpha >$ is vacuum condensate of Casimir operator of the group $O(N)$.

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Ксензов В.Г.

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