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# Coherent dipole instability in storage ring due to interaction of circulating beam with beam of the electron cooler

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COHERENT DIPOLE INSTABILITY IN STORAGE RING DUE TO INTERACTION OF CIRCULATING BEAM WITH BEAM OF THE ELECTRON COOLER / Preprint ITEP 26 – 01/

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In a presence of electron cooling system stability of the hadron beam dipole oscillations in the storage ring is determined by eugen values of the ring transfer matrix, including the cooler transfer matrix, which is calculated with account of the coherent electron-ion interaction. *MathCad* code calculating these eugen values is applied to two rings: antiproton storage ring "recycler" (FNAL, USA) and ion storage ring ACR (RIKEN, Japan).

КОГЕРЕНТНАЯ ДИПОЛЬНАЯ НЕУСТОЙЧИВОСТЬ В НАКОПИТЕЛЬНОМ КОЛЬЦЕ ИЗ-ЗА ВЗАИМОДЕЙСТВИЯ ЦИРКУЛИРУЮЩЕГО ПУЧКА С ПУЧКОМ СИСТЕМЫ ЭЛЕКТРОННОГО ОХЛАЖДЕНИЯ

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Стабильность дипольных колебаний адронного пучка в накопительном кольце в присутствии системы электронного охлаждения определяется собственными числами матрицы передачи кольца, включающей матрицу передачи этой системы, рассчитанную с учетом когерентного электрон-ионного взаимодействия. Программа для вычисления собственных чисел этой матрицы, написанная на языке *MathCad*, применена к двум накопительным кольцам: антипротонному кольцу "ресайклер" (FNAL, США) и ионному накопительному кольцу ACR (RIKEN, Япония).

Fig. – 2, ref. – 4 names.

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Influence of the electron cooling system on stability of the transverse coherent oscillations of the ion beam in the storage ring is widely discussed at recent years /1-4/. In our previous paper /1/ the transverse coherent motion of the intense ion beam has been investigated with account of the electron drift in crossed longitudinal magnetic field of the cooler and transverse electrical field of the ion beam. This analysis has shown that the coherent interaction of the ion beam with the cooler (for long wave oscillations) can be described by four-dimensional transfer matrix $M_C$. This matrix was derived neglecting the influence of longitudinal magnetic field on ion beam motion (for zero ion Larmor frequency and without account of fringed field of the cooler solenoid). For oscillations stability the following necessary stability condition should take place: $\det M_C \leq 1$. It was shown that this condition is valid for typical cooler parameters.

However, this stability condition is not sufficient, and in common case it is necessary to investigate the eugen values of the transfer matrix of the ring; for stability moduluses of all eugen values should be less than unity /2,3/. Such analysis has been made in paper /3/ for a case, when Twiss parameters of unperturbed transfer matrix of the ring (calculated in absence of the cooler and compensating solenoids) are symmetrical (beta functions and phase advances are equal, alpha functions are equal to zero). For such "degenerated" case the complex transfer matrice of the second range describes the ion coherent motion in the ring. Analysis

of this case has shown that one of two modes is always unstable; the expression for the increment has been derived and investigated.

A goal of present paper is analysis of the beam stability for non-generated case; moreover, the increments in designed ion and antiproton rings are calculated with account of effects, which were omitted in previous work.

Dipole motion of the electron and ion beams in the cooler (for the round beams) is described by the following equations /1/:

$$\left.\begin{aligned} \frac{d^2 v_i}{ds^2} + i\Omega_L{}^i \frac{dv_i}{ds} + \Omega_{ie}{}^2 v_i &= \Omega_{ie}{}^2 v_e \\ \frac{d^2 v_e}{ds^2} - i\Omega_L{}^e \frac{dv_e}{ds} + \Omega_{ei}{}^2 v_e &= \Omega_{ie}{}^2 v_i \end{aligned}\right\} \tag{1}$$

Here $v_{i,e} = x_{i,e} + iy_{i,e}$, $x,y$ are transverse variables, $s$ is longitudinal variable; frequencies are defined by

$$\Omega_{i,e}{}^2 = \frac{2\pi Z n_e r_p}{A\beta^2\gamma^3},\ \Omega_{ei}{}^2 = \frac{2\pi Z n_i r_e}{\beta^2\gamma^3},\ \Omega_L{}^i = \frac{Z}{A\beta\gamma}\frac{Be}{E_p},\ \Omega_L{}^e = \frac{Be}{\beta\gamma E_e} \tag{2}$$

where $Z$, $A$ are charge and atomic numbers of the ion, $\beta,\gamma$ are relativistic parameters, $r_p, r_e$ are classical radii proton and electron, $E_p, E_e$ are corresponding rest energies, $B$ is magnetic induction of longitudinal magnetic field of the cooler. In drift approximation (valid if $\Omega_L{}^e >> \Omega_{ei}$) the second equation can be written as follows:

$$\frac{dv_e}{ds} + i\Omega v_e = \Omega v_i \tag{3}$$

Here the drift frequency

$$\Omega = \frac{\Omega_{ei}{}^2}{\Omega_L{}^e} = \frac{2\pi Z n_i r_e E_e{}^0}{\beta^2\gamma^2 eB} \tag{4}$$

Eigen frequencies of the third order uniform system of linear equations (Eq.1,3) are

$$\omega_{1,2} = \frac{\Omega + \Omega_L}{2} \pm \sqrt{\left(\frac{\Omega - \Omega_L}{2}\right)^2 + \Omega_i{}^2},\ \omega_3 = 0 \tag{5}$$

Eq. (5) is written for positively charged ions; for negatively charged ions (for example, antiprotons) it is necessary to change here (and in all further formulae) $\Omega_i^2$ on $-\Omega_i^2$. Let us mark, that at this case eugen frequencies can become imaginary.

Ion motion (for zero initial condition for electrons) is defined by:

$$\begin{pmatrix} v \\ v' \end{pmatrix} = (C)\begin{pmatrix} v_0 \\ v_0' \end{pmatrix} \tag{6}$$

where elements of $C$ (complex matrix of the second range) are given in Appendix.

In real variables the following second range transfer matrix describes the motion:

$$M_C = \begin{pmatrix} A & B \\ -B & A \end{pmatrix} \tag{7}$$

In Eq. (8) the second range transfer matrices $A$ and $B$ are defined by $A = \mathrm{Re}\,C, B = -\mathrm{Im}\,C$. Elements of the real transfer matrix $M_C$ are given in /1/.

Determinant of real matrix $M_C$ is connected with determinant of the complex matrix $D$ by

$$\det M_C = |\det D|^2 \tag{8}$$

Usually for typical cooler parameters all phases $\omega L << 1$ ($\omega = \Omega, \Omega_i, \Omega_L$). Expanding $C_{i,j}$ (Eq.7) on these small parameters (see Appendix) we can find the following approximate expression for determinant:

$$\det M_C \approx 1 - \frac{L^4}{12}\Omega_i^2\Omega(2\Omega - \Omega_L) \tag{9}$$

For negatively charged hadrons (for example, p-bars) it is necessary to change the sign before subtracting term on plus.

Dependence of $\det M_C$ on $L$ is plotted at Fig. 1 for ACR parameters (see Table1); solid curve corresponds to exact calculations, dotted curve to approximate Eq. (9). We see from the figure that for small $L$ both curves coincide.

Stability is determined by eigen values of one-turn matrix:

$$M_T = M_1 \cdot M_C \tag{10}$$

where $M_1$ is the transfer matrix from the beginning of cooler to its end.

Let $M_0$ is an unperturbed transfer matrix for the central point of the cooler. As a rule ring in calculation of Twiss parameters are all solenoids are considered as drift spaces. If the cooler is located in the beam crossover, unperturbed one-turn matrix $M_0$ is

$$M_0 = \begin{pmatrix} M_x & 0 \\ 0 & M_y \end{pmatrix}, M_{x,y} = \begin{pmatrix} \cos\nu_{x,y}L & \beta_{x,y}^{\ 0}\sin\nu_{x,y}L \\ -\dfrac{\sin\nu_{x,y}L}{\beta_{x,y}^{\ 0}} & \cos\nu_{x,y}L \end{pmatrix} \tag{11}$$

Then one turn matrix can be written as follows:

$$MT = ML^{-1}M_0ML^{-1}MC_{eff} \tag{12}$$

Here $ML$ is the matrix of drift space with length $L/2$, $ML^{-1}$ is the corresponding inverse matrix, matrix $M_C^{eff} = F \cdot M_C \cdot F^{-1}$, $F$ is a matrix of edge field kick at the solenoid entrance. For oscillations stability moduluses of all eigen-values of matrix $M_T$ should not succeed unity. These eigen-values were calculated numerically using *MathCad* program.

Let us mark that by use of the complex second range matrices for partial case of symmetrical focusing system ($\beta_1 = \beta_2 = \beta, \varphi_1 = \varphi_2 = \varphi$) it is possible to find the following analytical expression for the increment $\delta$:

$$\delta = |\lambda_{\max}| - 1 \approx \frac{1}{4}\beta_0\Omega\Omega_i^{\ 2}L^2 = \frac{1}{4}\frac{\beta_0\Omega_i^{\ 2}\Omega_e^{\ 2}L^2}{\Omega_L^{\ e}} \tag{13}$$

For insight in the physics of the effect let us rewrite Eq.(13) through the incoherent tune shifts of the ion beam: tune shift due to ion-ion Coulomb interaction $\Delta Q_{sc}^{\ ii}$ and tune shift due to Coulomb interaction of the ions with the cooling electron beam $\Delta Q_{sc}^{\ ie}$. Then we obtain:

$$\delta = \frac{\pi^2 r_e}{2r_p}\frac{Z}{A}\frac{\Delta Q_{sc}^{\ ii}\Delta Q_{sc}^{\ ie}}{\beta_0\Omega_L^{\ e}}f_C \tag{14}$$

Here $f_C = L/L_{ring}$, $L_{ring}$ is the ring circumference.

For non-equal beta functions and (or) phase advances we should use four dimensional formalism and values of increment can be found numerically. The results of calculations for ACR ring (parameters are given at Table 1) are plotted at Fig.1 and Fig.2:

Table 1. Main ring parameters and results of the calculations.

| Ring name | ACR | FNAL |
|---|---|---|
| Kind of ions | $^{27}Al^{+13}$ | p-bars |
| Ion energy (GeV/u) | 0.4 | 8 |
| Number of ions | $2\cdot10^{12}$ | $2.5\cdot10^{12}$ |
| Nominal tunes | 2.62/2.81 | 26.22/26.22 |
| Circumference (m) | 130 | 3000 |
| Cooler length (m) | 3.6 | 20 |
| Electron current (A) | 4 | 1 |
| Electron beam radius (cm) | 2.5 | 0.6 |
| Ion beam radius (cm) | 1.1 | 0.4 |
| Magnetic field in the cooler (Gs) | 2000 | 100 |
| $\beta$ function in the cooler (m) | 10 | 30 |
| Electron drift frequency $\Omega$ ($m^{-1}$) | 0.0627 | $1.32\cdot10^{-3}$ |
| Ion Larmor frequency $\Omega_L$ ($m^{-1}$) | 0.0287 | $3.4\cdot10^{-4}$ |
| Ion-electron frequency $\Omega_i$ ($m^{-1}$) | 0.0137 | $1.57\cdot10^{-3}$ |
| Increment per turn for nominal tunes | $3.09\cdot10^{-5}$ | $9.27\cdot10^{-6}$ |
| Increment per turn for equal tunes | $3.73\cdot10^{-4}$ | |

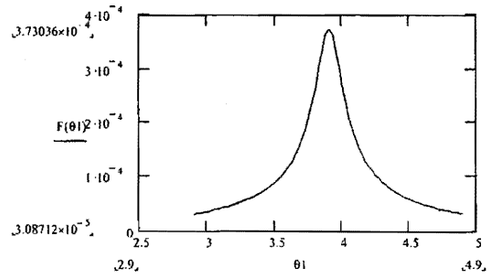


Fig.1. Dependence of the increment on horizontal phase advance (vertical phase advance is equal to 3.9).

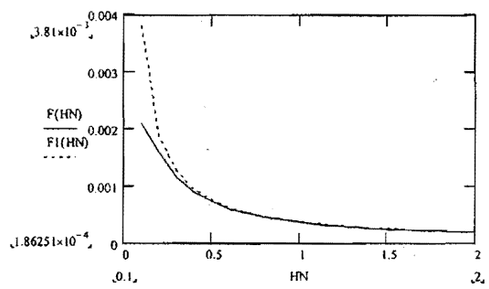


Fig.2. Dependence of the increment on normalized magnetic field ($HN = H / H^0$, where $H$ is magnetic field, $H^0$ is nominal magnetic field). Solid curve is result of exact calculations; dashed curve is proportional to $1/H$.

We see from the Fig.1 that increment has resonant character and sharply decreases with enhancement of the phase advance difference $|\varphi_1 - \varphi_2|$. Fig.2 shows that increase of magnetic field (and, correspondingly, increase of Larmor electron and ion frequencies and decrease of the electron drift frequency) results to decrease of the maximal increment.

Estimations have shown that for existing rings incoherent tune shifts and, correspondingly, increments, are small. Therefore we have considered more ambitious numbers corresponding to designing electron cooling systems: e-cool system of ACR ring, which is now under design in RIKEN (Japan) , electron cooling system of antiproton recycler (FNAL,USA). Main ring parameters and the results of calculations are given at Table1. We see from the table that maximal increments are very small and can be suppressed by tune difference or by increase of the cooler magnetic field, as well as by standard methods (Landau damping or feedback).

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**Appendix.**

Elements of the matrix $C$ are defined by:

$$\left.\begin{aligned} C_{11} &= 1+\frac{1}{\Delta}\left[\frac{\exp(-i\omega_1\tau)-1}{\omega_1}-\frac{1}{\Delta}\frac{\exp(-i\omega_2\tau)-1}{\omega_2}\right] \\ C_{12} &= \frac{i}{\Delta}\left[\frac{\exp(-i\omega_1\tau)-1}{\omega_1(\Omega-\omega_2)}-\frac{\exp(-i\omega_2\tau)-1}{\omega_2(\Omega-\omega_1)}\right] \\ C_{21} &= \frac{i}{\Delta}[\exp(-i\omega_2\tau-\exp(-i\omega_1\tau)] \\ C_{22} &= \frac{1}{\Delta}\left[\frac{\exp(-i\omega_1\tau)}{(\Omega-\omega_2)}-\frac{\exp(-\omega_2\tau)}{(\Omega-\omega_1)}\right] \end{aligned}\right\} \tag{A1}$$

Here $\Delta = \frac{1}{\Omega-\omega_1}-\frac{1}{\Omega-\omega_2}$.

Usually for typical cooler parameters phases $\omega L << 1$ ($\omega = \omega_{1,2}$). Expanding $C_{ij}$ (Eq. A1) on these small parameters we obtain:

$$\begin{aligned} C_{11} &= 1-\frac{\Omega_i^2L^2}{2}+i\frac{\Omega_i^2(\Omega+\Omega_L)L^3}{6}+\frac{\Omega_i^2[\Omega^2+\Omega_L(\Omega+\Omega_L)+\Omega_i^2)]L^4}{24} \\ C_{12} &= L(1-i\frac{\Omega_L L}{2}-\frac{\Omega_L^2L^2}{6}) \\ C_{21} &= \frac{1}{L}\{\Omega_i^2L^2-i\frac{\Omega_i^2(\Omega+\Omega_L)L^3}{2}-\frac{\Omega_i^2[\Omega^2+\Omega_L(\Omega+\Omega_L)+\Omega_i^2)]L^4}{6}\} \\ C_{22} &= 1-i\Omega_L L-\frac{\Omega_L^2L^2}{2}+i\frac{[\Omega_L^3-\Omega_i^2(2\Omega_L+\Omega)]}{6}+\frac{[\Omega_L^4-\Omega_i^2(3\Omega_L^2+2\Omega\Omega_L+\Omega^2-\Omega_i^2)]}{24} \end{aligned} \tag{A2}$$

In complex form elements of the solenoid matrix is defined by

$$S = \begin{pmatrix} 1 & \frac{1-\exp(-i\Omega_L s)}{i\Omega_L} \\ 0 & \exp(-i\Omega_L s) \end{pmatrix} \tag{A3}$$

Edge fringe field matrix

$$F = \begin{pmatrix} 1 & 0 \\ 0.5i\Omega_L & 1 \end{pmatrix} \tag{A4}$$

Real matrices can be found from equations (A1- A4).