26

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A space charge field of the ion beam inside the cooling section may limit application of electron cooling to high current storage ring. The influence of this effect on the damping rates is investigated by use of the Focker-Planck equation, which describes evolution of particle distribution in invariant space. This equation is solved by Monte Carlo method for two models of electron-ion binary collisions: classical model of interaction with non-magnetized electrons and model of interaction with partially magnetized electrons. The Monte Carlo code is validated by comparison with solution of Langevin equation for averaged values of particle invariants. Algorithm for calculation of electron cooling is included in unified code, which takes into acccunt intra-beam scattering and interaction with charge exchange target. This code applied to analysis of beam accumulation in TWAC storage ring (ITEP, Moscow), synchrotron SIS and HAR - High Intensity Accumulation Ring (GSI, Darmstadt). Calculations have shown that equilibrium distribution of particles in invariant space could be very far from Gaussian one. It is found that for the coasting ion beam the ion space charge does not limit application of the electron cooling for accumulation of high current ion beams. For bunched beams the longitudinal cooling rate decreases with the current more sharply due to longitudinal modulation of the ion line density, which results to modulation of the electron longitudinal velocity.

Fig.- 14, ref. - 16 names.

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1. Introduction.

Accumulation in storage ring of ion beams with high current and high stored energy is very actual problem for applications to heavy ion fusion, plasma experiments and etc. Beam current and quality in such storage rings could be significantly improved by use of electron cooling system (ECS). The main question is space charge interaction between cooling electron beam and the ion beam, which can result in decrease of damping rates and to excitation of the ion coherent oscillations. A goal of the present paper is an investigation of influence of ion space charge fields on ECS damping rates.

It is well known that influence of electron space charge diminishes the damping rates due to two effects: 1) momentum shift of electrons relative to a center of the ion beam; 2) drift of the cooling electrons in crossed Coulomb transverse field and longitudinal magnetic field /1/. These effects are estimated in code "BETACOOL" /2/, which calculates evolution of one-particle invariants (usually it is assumed that initial particle invariants corresponds to values, averaged on the ensemble).

A more correct algorithm for analysis of the problem consists in a solution of the Langevin equations for averaged values of invariants, which are used in /3/ for analysis of damping rates with account of ion and electron space charge. This model includes the following simplifying assumptions: 1) the ion beam has Gaussian distribution on all degrees of freedom, which does not change during the process; 2)

Coulomb field is calculated by analytical expressions, derived for the ion beam density depends only on r (transverse distance from the beam axe) and, for bunched beam, on s (longitudinal distance from the beam center); 3) bunch length is much more than radius of vacuum chamber. For coasting beam such approach requires five dimensional integration (on two transverse invariants, two transverse phases and momentum deviation), and for bunched beam one requires six dimensional integration (on three transverse invariants and three phases).

In this paper we use the most correct method of self-consistent solution, based on a system of equations, which includes Focker-Planck equation for analysis of the ion beam evolution in phase space, equation connecting ion beam density with phase space distribution and Poisson equation: two dimensional one for coasting ion beam and (for saving of computer time) two and half dimensional one for bunched beam). For solution of Focker-Planck equation we use Monte-Carlo method, applied in papers /4,5/ to analysis of beam evolution in invariant space with account of intra-beam scattering and (or) interaction with charge exchange target. Cooling process is described by two models: 1) classical binary collisions model (CBCM) of the ions with non-magnetized electrons /6/; 2) "magnetized" binary collisions model (MBCM), which takes into account the electron magnetization /1,7/.

For the first model our approach allows to calculate damping rates for "smoothened" electron distribution (with longitudinal temperature small in compare with transverse one) without any simplified expressions for friction force. Moreover, Monte Carlo method permits to take into account a dependence of Coulomb logarithm on collision parameters. For the second model we use an approximate equations for cooling friction force in laboratory system. Validation of the first model is made by comparison with the Langevin model for the round Gaussian beam (in this case a number of necessary integrations can be reduced to four).

Algorithm for calculation of the cooling rate is included in unified Monte Carlo code, which can take into account electron cooling, intra-beam scattering and (or) interaction with charge-exchange target. The code is applied to numerical analysis of beam evolution in TWAC ring (ITEP, Moscow) and SIS synchrotron (GSI, Darmstadt).

2. Initial equations and models of friction force.

For numerical simulations of particle motion it is convenient to use laboratory system. Let us introduce vector $\vec{u}$, whose components are defined by:

$$u_1 = \frac{1}{\gamma}\frac{\Delta p}{p}, u_2 = x', u_3 = y' \qquad (1)$$

where Δp is electron momentum deviation, x and y are transverse deviations (correspondingly, horizontal and vertical), x' and y' are derivatives from coordinates on longitudinal variable, s is a distance from the bunch center (in a case of bunched beam).

In these variables equation of motion can be written as follows:

$$\frac{d\vec{u}_i}{dt} = -2AL_C(\theta)\frac{\vec{u}_i - \vec{u}_e}{\theta^{3/2}} \qquad (2)$$

where $\vec{u}_{i,e}$ corresponds to electron and ion velocity, $\theta = |\vec{u}_i - \vec{u}_e|$. In a presence of space charge effects $\vec{u}_e = \vec{u}_{sc} + \Delta\vec{u}_e$, where $\vec{u}_{sc}$ is a space charge component of the electron velocity, $\Delta\vec{u}_e$ is a random one,

$$A = f\frac{2\pi r_i r_e c n_e}{\beta^3\gamma^5} \qquad (3)$$

In Eq.(3) constant $f = l_{cool}/l_{ring}$ (l_{cool} is the cooler length, l_{ring} is the ring circumference), r_i, r_e are electron and ion classical radii, n_e is the electron density inside the cooler, c is light speed, β,γ are relativistic parameters.

The Coulomb logarithm is defined by

$$L_C(\theta) = \ln(\rho_{max}/\rho_{min}) \qquad (4)$$

where ρ_{max}, ρ_{min} are maximal and minimal values of collision parameter, which can be estimated by use of the following formula:

$$\begin{cases} \rho_{max} = \min(r_{sh}, r_{dr}, a_i) \\ \rho_{min} = Zr_e / \beta^2\theta^2 \end{cases} \tag{5}$$

These parameters have the following physical sense: the electrical shelding parameter r_{sh} is equal to relation of the mean velocity to the electron plasma frequency, r_{dr} is a mean displacement of the ion during a drift of the cooling system, a_i is a vertical half size of the ion beam (all parameters correspond to the rest frame). They are expressed through the beam parameters in the laboratory system by the following formulae:

$$r_{sh} = \sqrt{\frac{\theta\beta^2\gamma^3}{4\pi n_e r_e}}, r_{dr} = \frac{l_{cool}\theta}{\gamma} \tag{6}$$

After averaging on the electron random velocities we can write Eq.(2) in the following form:

$$\begin{cases} \frac{d\vec{u}}{dt} = 2A\vec{F}_{fr} \\ \vec{F}_{fr} = \int f(\Delta\vec{u}_e) L_c(|\vec{u} - \vec{u}_{sc} - \Delta\vec{u}_e|) \frac{\vec{u} - \vec{u}_{sc} - \Delta\vec{u}_e}{|\vec{u} - \vec{u}_{sc} - \Delta\vec{u}_e|^{3/2}} d(\Delta\vec{u}_e) \end{cases} \tag{7}$$

where $\vec{F}_{fr}$ is "dimensionless friction force", $f(\Delta\vec{u}_e)$ is the distribution of the electrons on transverse random velocities. Such integration is not a trivial program and could be made analitically only for constant value of Coulomb logarithm ($\langle L_C \rangle = L_C(\sqrt{\langle\theta^2\rangle})$, where $\langle\theta^2\rangle$ is found by the averaging on the ensemble) and the simplest case of Maxwellian distribution:

$$f(\Delta\vec{u}_e) = \frac{1}{(2\pi)^{3/2}\theta_e^{3/2}} \exp[-\frac{|\Delta\vec{u}_e|^2}{2\theta_e^2}] \tag{8}$$

After integration we obtain [1]:

$$\vec{F}_{fr} = \langle L_C \rangle \begin{cases} \frac{\vec{U}}{|\vec{U}|^3}\varphi(x) \\ \varphi(x) = erf(x) - \sqrt{\frac{2}{\pi}} x \exp(-x^2/2) \end{cases} \tag{9}$$

Here $x = |\vec{U}| / \theta_e, \vec{U} = \vec{u}_i - \vec{u}_{sc}$.

Really in ECS longitudinal temperature is much less than the transverse one; the corresponding "flattened"distribution is defined by:

$$f(\Delta\vec{u}_e) = \frac{1}{(2\pi)^{3/2}\theta_{eper}{}^2\theta_{lon}} \exp[-\frac{|\Delta u_{ex}|^2 + |\Delta u_{ey}|^2}{2\theta_{tr}{}^2} - \frac{|\Delta u_{lon}|^2}{2\theta_{lon}{}^2}] \tag{10}$$

At this case exact integration can not be made analitically.

Binary collisions model for magnetized electrons is based on separation of three regions in relative electron-ion velocities: 1) region 1, where $\Theta = |\vec{U}| > \theta_{tr}$; 2) region 2, where $\theta_{tr} > \Theta > \theta_{lo}$; 3) region 3, where $\Theta < \theta_{lon}$. All collisions, in dependence on region and impact parameter, are divided on two kinds: 1)"classical" collisions, when magnetic field does not influence on the process; 2) adiabatic collisions, for which electron-ion interaction has adiabatic character. Resulting formulae for friction force in laboratory system are given in /1,2/.

3. **Electron space charge velocities.**

Let us consider firstly a coasting beam with azimuthal symmetry $\rho = \rho(r)$, where $\vec{r}$ is transverse radius vector, $r = |\vec{r}|$. A difference of energies $\Delta W(r)$ between central electron with r=0 and electron with coordinate r is defined by:

$$\Delta W(r) = \int_0^r F_{tr}(r)dr = e\int_0^r [E_r(r) - \beta H_\varphi(r)]dr = \frac{e}{\gamma^2}[U(r) - U(0)] \tag{11}$$

Here $E_r(r), H_\varphi(r), U(r)$ are, correspondingly, radial component of electrical field, azimuthal component of magnetic field and potential, which appear due to the space charge effect. Taking into account, that $\frac{\Delta p}{p} = \frac{\Delta W}{E_0\beta^2\gamma}$ (E_0 is the electron rest energy), we obtain:

$$u_1{}^{sc} = \frac{1}{\gamma}(\frac{\Delta p}{p})^{sc} = \frac{e[U(r) - U(0)]}{\beta^3\gamma^4 E_0} \tag{12}$$

For the bunched beam with an arbitrary dependence of Coulomb field on $\vec{r}$

$$u_1^{sc} = \frac{1}{\gamma}\left(\frac{\Delta p}{p}\right)^{sc} = \frac{e[U(\vec{r},s) - U(0,0)]}{\beta^3\gamma^4 E_0} \qquad (13)$$

where s is a distance from the bunch center; in a presence of metallic vacuum chamber with radius b all the potentials should be calculated for boundary condition $U(b,s) = 0$. In these expression we assume that longitudinal velocity of the central particle of the cooling beam coincides with longitudinal velocity of the central particle of the ion beam (with r=0 for coasting beam and with r,s=0 for bunched beam). Let us underline that this assumption is not trivial one: in process of the beam evolution the potential of the central particle $U(0,0)$ is changed due to changes in the longitudinal density and the transverse size of the ion beam. Matching of the longitudinal velocities of the central particle can be made by use of the special feedback system, which can change frequency of the accelerating field or electron energy to support the synchronism. In the coasting beams effect is significantly less, and the compensation may be performed by variation electron energy or by change of the ion energy using special system of betatron acceleration.

For long bunches with account of the screening we can find potential U by use of two-dimensional Poisson equation:

$$\Delta_\perp U = -4\pi e[Z_i \rho_i(x,y,s) - \rho_e(r)] \qquad (14)$$

where Z_i is ion charge number, e is an electron charge, $\rho_i(x,y,s)$ and $\rho_e(r)$ are the transverse ion and electron density.

For round beam the azimuthal electron drift velocity in the crossed fields (transverse electromagnetic space charge field and guiding longitudinal magnetic field with a magnetic induction B_s) is defined by

$$V_d = c\frac{E_r - \beta H_\varphi}{B_s} = \frac{c}{\gamma^2}\frac{E_r}{B_s} \qquad (15)$$

For beam with arbitrary dependence on $\vec{r}$

$$x'_{sc} = x'_d = \frac{1}{\beta\gamma^2 B_s}\frac{dU}{dy}, y'_{sc} = y'_d = -\frac{1}{\beta\gamma^2 B_s}\frac{dU}{dx} \qquad (16)$$

Let us consider simple particular case of round beam with Gaussian density, when

$$\rho_i(r,s) = \rho_0 \exp[-(\frac{r}{a_\perp})^2]F(s) \tag{17}$$

Here $a_\perp$ is r.m.s. radial deviation of the ion beam, a_s is r.m.s. half lengths of bunch, $F(s) = \exp(-s^2/2a_s^2)$ for bunched beam, $F(s) = 1$ for coasting beam, density at the center of the beam $\rho_0 = N_b / \pi\sqrt{2\pi} a_s a_\perp^2$ for bunching beam, $\rho_0 = N / \pi a_\perp^2 L$ for coasting beam. Solving Poisson equation, we obtain:

$$\frac{1}{\gamma}(\frac{\Delta p}{p})_{sc}(t,s) = \alpha\{G_i[1 - F(s)] + \Phi(t)F(s) - \eta_0 \frac{t^2}{2}\} \tag{18}$$

$$v_d(t,s) = \alpha\Lambda[\Theta(t)F(s) - \eta_0 \cdot t] \tag{19}$$

with

$$\alpha = \frac{2I_i}{\beta^3\gamma^4 I_{Alf}}, \Lambda = \frac{\beta\gamma^2 E_0}{eB_s a_\perp} \tag{20}$$

$$\Phi(t) = \int_0^t \frac{1-\exp(-v^2)}{v} dv, \Theta(t) = \frac{d\widetilde{U}}{dr}(t) = \frac{1-\exp(-t^2)}{t} \tag{21}$$

In these equations I_i is the ion current (for bunched beam this current is calculated for the bunch center, I_{Alf} is electron Alfven current (I_{Alf}=1.7*10^4 A), $t = r/a_\perp$, and neutralization degree and longitudinal form-factors are defined by

$$\eta_0 = (I_e a_i^2)/(I_i a_e^2), G_{i,e} = 0.577 + \ln(b/a_{i,e}) \tag{22}$$

Let us underline that condition of synchronism between central particles of the electron cooling beam and the ion beam can be written as follows:

$$p_0^e = p_0^i(1 - \frac{2I_e G_e}{\beta^3\gamma^4 I_{Alf}} + \alpha\gamma G_i) \tag{23}$$

The third term in the brackets is varied due to change of form-factor (due to dependence of the ion beam size on time) and, for bunched beam, due to time dependence of longitudinal beam size on time). As it was mentioned above, for high current ion beams this effect should be compensated by special matching system.

Analytical expressions for round beam allow to estimate important physical parameters of the beam: 1) mean momentum shift between the electron and ion beam due to the space charge effect; 2) mean change of squared velocities due to the space charge effect (both parameters are averaged on transverse coordinates). Using Eq. (18), we obtain:

$$\left\langle \left(\frac{\Delta p}{p}\right)_{sc} \right\rangle = \alpha\gamma[G_i(1-k)+0.35k-0.5\eta_0] \tag{24}$$

In Eq. (24) $k=\sqrt{2}/2$ for bunched beam, k=1 for coasting beam. We see from Eq. (24) that mean momentum shift is equal to zero for $\eta_0^{\min}=2[G_i(1-k)+0.35k]$; for $\eta_0<\eta_0^{\min}$ the momentum shift is negative due to the electron space charge, for $\eta_0>\eta_0^{\min}$ the momentum shift becomes positive due to action of the ion space charge.

Averaged change of squared velocities due to the space charge is defined by

$$\left\langle \theta_{sc}^2 \right\rangle = \alpha^2[G_i^2C_1+2G_iC_2+k_1(0.19+0.29\Lambda^2)-k\eta_0(0.6+\Lambda^2)+\eta_0^2(0.5+\Lambda^2)] \tag{25}$$

Here $k_1=1/\sqrt{3}$ for bunched beam, k_1=1 for coasting beam, $C_1=1-2k+k_1, C_2=\eta_0(1-k)-0.69(k-k_1)$.

These expressions allow a scaling of the ion space charge effect relative to the ion energy. Let us consider, for simplification, only the ion space charge effect (η_0=0); then we obtain (in physical variables):

$$\left\langle \theta_{sc}^2 \right\rangle = \left(\frac{2I_i}{\beta^3\gamma^4 I_{A\psi}}\right)^2(G_i^2C_1+2C_2G_i+0.19)+0.29\left(\frac{2I_i}{\beta^2\gamma^2 I_{A\psi}}\right)^2\left(\frac{E_0}{eB_s a_\perp}\right)^2 \tag{26}$$

The first term appears due to the space charge momentum shift, the second one due to the drift velocity. Incoherent Coulomb tune shift limits the maximal ion current by $I_i \propto (\beta\gamma)^3$; substitution of this current in Eq. (26) shows, that the maximal value of the first term decreases with energy as $1/\gamma^2$, the second one increases as $(\beta\gamma)^2$.

4. Analytical model for calculation of damping rates in round Gaussian beam.

It is convenient to consider an evolution of particle parameters in a space of invariants (integrals of motion I_k), where

$$\begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix} = \begin{cases} H_{lon} \\ \gamma_x x_f^2 + 2\alpha_x x_f x_f' + \beta_x x_f'^2 \\ \gamma_y y^2 + 2\alpha_y y y' + \beta_y (y')^2 \end{cases} \tag{27}$$

Here $x_f = x - \Psi_x \frac{\Delta p}{p}$, $x_f' = x' - \Psi_x' \frac{\Delta p}{p}$, Ψ_x and Ψ_x' are dispersion function and its derivative; longitudinal invariant is defined by:

$$H_{lon} = \begin{cases} \frac{1}{\gamma^2}(\frac{\Delta p}{p})^2 \\ \frac{1}{\gamma^2}\{(\frac{\Delta p}{p})^2 + \frac{1}{\Omega_s^2}[\frac{d}{dt}(\frac{\Delta p}{p})]^2\} \end{cases} \tag{28}$$

In Eq. (28) upper line corresponds to coasting beam, lower line to bunched beam with constant parameters (for changing synchrotron frequency it is necessary to use adiabatic invariant instead of energy). Components of vector $\vec{U}$ are defined by:

$$U_1 = \frac{\Delta p}{p}, U_2 = x_f' + \Psi' \frac{\Delta p}{p}, U_3 = y_f' \tag{29}$$

If the friction force is known, the invariant changes due to the electron cooling are defined by:

$$\frac{d\vec{I}}{dt} = \begin{cases} 2U_1 \frac{dU_1}{dt} \\ 2(x_f D_1 + x_f' D_2)\frac{dU_1}{dt} + 2(\beta_x x_f' + \alpha_x x_f)\frac{dU_2}{dt} \\ 2U_3 \frac{dU_3}{dt} \end{cases} \tag{30}$$

where

$$D_1 = \gamma_x \Psi + \alpha_x \Psi', D_2 = \alpha_x \Psi + \beta_x \Psi' \tag{31}$$

Let us assume, that the ion distribution on betatron phases χ is uniform in an interval [0, 2π] (stationary beam) and, moreover, $F(\vec{I})$ (the ion distribution function on the invariants) is known; then we can find average damping times, which are defined by

$$\frac{1}{\tau_k} = -\frac{1}{\langle I_k \rangle}\left\langle \frac{dI_k}{dt} \right\rangle \tag{32}$$

Here sign $\langle\ \rangle$ means averaging on ensemble, which for coasting Gaussian beam can be made by integration on five variables: three components of vector $\vec{I}$ and betatron phases χ_x, χ_y (for the bunched beam it is necessary to add an integration on synchrotron phase χ_s). For the round coasting beam a number of integrations can be reduced up to four. If electron and ion beam have Maxwellian distribution with equal dispersion on all degrees of freedom, damping times can be found analytically /6/:

$$\frac{1}{\tau_{Max}} = 2A\langle L_C \rangle \frac{2\sqrt{2}}{3\sqrt{\pi}}(\theta_e^2 + \theta_i^2)^{-3/2} \tag{33}$$

For estimation of cooling time for arbitrary distributions electrons and ions on different degrees of freedom in a presence of the space charge, we can use the same formula, adding space charge term:

$$\frac{1}{\tau_{Max}} = 2A\langle L_C \rangle \frac{2\sqrt{2}}{3\sqrt{\pi}}(\theta_e^2 + \theta_i^2 + \frac{1}{3}\langle \theta_{sc}^2 \rangle)^{-3/2} \tag{34}$$

Here θ_e^2 and θ_i^2 are averaged energy of electrons and ions on one degree of freedom, factor 1/3 takes into account equipartioning of energy on three degrees of freedom. We see from this formula that for high ion currents cooling time decreases as I_i^{-3}(however, let us mark that this expression can strongly underestimate the space charge effect).

For more accurate account of the space charge we should make integration on the beam distribution. It is clear, that for the round beam the transverse decrements are equal. Then we obtain:

$$\frac{1}{\tau_{tr}} = A\frac{\langle L_C\rangle}{\theta_e{}^3 Y_{tr}{}^2}\left\langle\frac{[v^2 - vv_{sc}(t)\cos\theta]\varphi(x)}{x^{3/2}}\right\rangle \tag{35}$$

$$\frac{1}{\tau_{lon}} = 2A\frac{1}{\theta_e{}^3 Y_{lon}{}^2}\left\langle\frac{[y^2 - yy_{sc}(t)\cos\theta]\varphi(x)}{x^{3/2}}\right\rangle \tag{36}$$

where

$$x = [v^2 - 2vv_{sc}(t)\cos\theta] + v_{sc}(t)^2 + (y - y_{sc}(t))^2]^{1/2} \tag{37}$$

Sign $\langle F\rangle$ means integration on parameters y, v, t, θ (corresponding intervals $[-\infty,\infty],[0,\infty],[0,\infty],[0,\pi]$) function F, multiplied on distribution function

$$f(y,v,t,\theta) = \frac{2vt\exp(-\frac{y^2}{2} - \frac{v^2}{2} - t^2)}{\sqrt{2\pi}Y_{tr}{}^2 Y_{lon}} \tag{38}$$

5. Monte Carlo method.

The Monte Carlo method calculates an evolution of the beam distribution on the invariants in a presence of the cooler by use of iterative procedure, which includes the following steps:

1. Random choice of betatron phases and, for the bunched beam, synchrotron ones, and the following calculation of the particles coordinates and velocities at a moment of an interaction with the cooler.

2. Calculation of the transverse ion density in each longitudinal layer (for coasting beam it is assumed that all ions are located in one layer).

3. Solution of two-dimensional Poisson equation in each layer by use of FPS2H code from MSML Libraries (the code uses Hodie finite-difference scheme with uniform mesh).

4. Calculation of space charge electron velocities, corresponding particle coordinate by use of Eq. (13) and Eq. (16).

5. Calculation of change of the vector $\vec{u}$ due to an interaction with the cooler for time interval t. A change of the ion velocity $\delta\vec{u}$ is found by use of one of the following models: 1) classical binary collision model (CBCM); 2) magnetized binary collisions model (MBCM)

$$\delta\vec{u} = -A\tau\begin{cases} L_C(|\vec{u}_i - \vec{u}_{sc} - \Delta\vec{u}_e|)\dfrac{\vec{u}_i - \vec{u}_{sc} - \Delta\vec{u}_e}{|\vec{u}_i - \vec{u}_{sc} - \Delta\vec{u}_e|^{3/2}} \\ L_C(|\vec{u}_i - \vec{u}_{sc}|)\vec{F}_{fr}(\vec{u}_i - \vec{u}_{sc}) \end{cases} \tag{39}$$

The first line in Eq. (39) corresponds to CBCM, the second one to MBCM. For CBCM components of vector $\Delta\vec{u}_e$ are found as random variables, corresponding to Gaussian distributions with given dispersions θ_{tr} and $\theta_{lon}{}^{e}$.

For binary collisions model value of Coulomb logarithm corresponding to calculated value of $|\vec{u} - \vec{u}_{sc} - \Delta\vec{u}_e|$ is determined by use of Eq. (4-6); for model of magnetized electrons these logarithms are calculated in accordance with chosen equation for the friction force.

To diminish beam diffusion due to discrete algorithm we use a condition, that in each event $\left|\frac{\Delta u_k}{u}\right| > 1$.

6. Calculation of new invariants values by use of known values of $\delta\vec{u}$.

Scheme contains the following free calculation parameters: number of particles N_p, time interval between the interactions with the cooler τ, number of cells in transverse space for solving of Poisson equation $n_\perp$ and number of longitudinal layers n_s. Typical values of these parameters: $\tau = 0.01 \div 1\,\mathrm{sec}$, $N_p = 10^3 \div 10^6$, $n_\perp = 100 \div 1000$, $n_s = 10$.

The described above algorithm is included in developed earlier Monte Carlo code, which can take into account intra-beam scattering and interaction with charge exchange target. We plan to include in code the calculation of particles losses due to radiative electron capture of the ions in electron cooler and losses by scattering of ion beam on residual gas in vacuum chamber.

6. Validation and numerical applications.

Validation of Monte Carlo code for CBCM model is made by comparison with analytical model for round beam described above. Calculations are made for the parameters, corresponding to of storage scenarios for TWAC accelerator facility, which is under construction in ITEP /8/. Let us mark, that to use analytical model we consider "smoothened" structure with zero dispersion and alpha functions.

Moreover, developed Monte Carlo code is applied to analysis of beam dynamics in SIS synchrotron /9/ and to project of High-Current Accelerator Ring (HAR) /10/.

List of parameters is given at following Table:

	TWAC	SIS	HAR
Kind of ions	$^{27}Al^{13+}$	$^{238}U^{28+}$	$^{238}U^{28+}$
Ring circumference [m]	251.0	216.0	278.6
Average beta-function	4.3	---	---
r.m.s. transversal beam emittance [mm*mrad]	1.64	2.0	2.0
r.m.s. beam momentum spread [%]	0.08	0.1	0.1
Energy of ions [MeV/u]	738	11.4	195
Cooler section length [m]	1.0	2.8	2.8
Electron beam radius [cm]	2.0	2.16	2.16
Dispersion in cooler section [m]	-0.25	0.0	0.3
Electron current [A]	4.0	0.6	3.0
Magnetic field in cooler [Gs]	2000	600	2000
Transversal and longitudinal temperatures of electron beam [eV]	0.1; 0.1	0.1; 0.001	0.1; 0.001

Results of numerical calculations a dependence of transverse and longitudinal damping rates on beam current are given at Fig.1 and Fig.2. Calculations are made for Maxwellian distributions of electrons and ions (equal temperatures on all degrees of freedom), parameters correspond to TWAC storage ring, which is now under construction in ITEP (Moscow). Curve 1 is calculated by analytical code, curve 2 by Monte Carlo code, CBCM model of friction force (for both methods Coulomb logarithm is constant); curve 3 (CBCM) and 4 (MBCM) are calculated by Monte Carlo code with integrated Coulomb logarithm. Good coincidence between the first two curves confirms validity of Monte Carlo code. Let us underline, that for considered parameters maximal ion current (corresponding to incoherent tune shift 0.15) is about 5 A; we see that for this current space charge effect decreases the damping rates on 30-40 %.

At Fig.3 and Fig.4 results of the calculation are given for the same parameters, besides longitudinal energy of electrons, which is equal to 0.001 eV. The calculated damping rates exceed the damping rates for Maxwellian electron beam distribution on 10-30 %. MBCM gives more high damping rates, than CBCM; maximal difference is about 1.5, and it is diminished with increase of the ion current. Comparatively small difference between magnetized and non-magnetized models can be explained by high transversal and longitudinal temperatures of the ion beam, which enhances with ion current due to addition of space charge temperature.

For injection in SIS synchrotron we have investigated process of emittance evolution under action of intra-beam scattering and electron cooling for constant number of ions (coasting beam, $N_i = 10^{11}, I_i = 0.1$ A). Important feature of this case is connected with the small ion energy: for non-relativistic beams the ion current is usually less than the electron one, what results in more sophisticated physics due to possible neutralization effects. Calculated dependencies of $\varepsilon_{lon} = \left\langle (\frac{\Delta p}{p})^2 \right\rangle$ and r. m. s.

horizontal emittance ε_x on time for different values of electron current I_e are plotted, correspondingly, at Fig.5 and Fig.6. We see that with an increase of time these parameters go to equilibrium or "quasi- equilibrium" (slowly growing) values. This growth probably is connected with "scattering" on high angles, which results to appearance of non-Gaussian tails.

Typical equilibrium distributions on $\frac{\Delta p}{p}$ and I_x are plotted, correspondingly, at Fig. 7 and Fig.8. We see from Fig.7, that center of distribution is shifted relative to $\frac{\Delta p}{p}$=0 due to space charge shift of the electron longitudinal velocities. Moreover, for both distributions long non-Gaussian tails appear.

The investigation of the emittance evolution under action of intra-beam scattering and electron cooling were made as well for the HAR project, which is now under consideration in GSI /10/. We consider two options: 1) coasting beam; 2) bunched beam (at this case number of bunches is equal to 4).

For coasting beam calculated dependencies of averaged invariants on time have qualitatively the same form, as for SIS; time of transient process is about 5 s (see Fig.9). The dependence of equilibrium invariants ε_{lon}, ε_x and ε_y on number of ions N_i is plotted at Fig.10. We see from the picture that for high ion beam intensities equilibrium emittances are approximately proportional to N_i (as it is well-known, for small intensities this dependence is proportional to $N_i^{0.4}$).

For the bunched beam the shift of the momentum due to space charge does not take place. Calculated dependencies ε_{lon} and ε_x on time for different numbers of ions per bunch N_i are given, correspondingly, at Fig. 11 and 12. We see from the pictures that longitudinal cooling sharply decreases with N_i; this effect appears due to dependence longitudinal electron velocity on the ion current, which varies inside the bunch. Let us mark, that r. m. s. beam size $a_s = a_s^{\,0}\sqrt{\varepsilon_{lon}/\varepsilon_{lon}^{\,0}}$, where $a_s^{\,0} = 10\,\mathrm{m}$,

$\varepsilon_{Ion}{}^{0}=10^{-6}$. Fig. 13 and 14 show dependence of cooling rates on particles numbers for transversal emittances corresponding to space charge betatron tune shift 0.15.

7. Conclusion.

Developed Monte Carlo code seems to be a very useful method for detailed investigation physics of high current ion beams. It is clear, that due to complicated dependence of the electron cooling damping rates on particle invariants, which appears in a presence of space charge effects, stationary solution can be absent or to be much more sophisticated than standard Gaussian distribution. Application of the code has allowed to investigate new effects, for example, a displacement a center of distribution on $\frac{\Delta p}{p}=0$ due to the space charge effect, creation of long non Gaussian tails, particle losses and so on.

Comparison of e-cooling damping rates calculated by use of two binary collisions models (classical model and model of partially magnetized collisions) has shown, that for medium energy (500-1000 MeV/u) ion beams difference between these models is moderate (usually model of magnetized collisions gives damping rates, which exceed the classical ones on 10-50 %). A question about possible effects, which are out of scope of binary collisions model (for example, discussed in /11/ experimental dependence of damping rates on the ion charge as $Z_i^{\lambda}, \lambda < 2$) is open and requires additional investigation. In a frame of BCM we find, that for coasting beam decrease of damping rates (for reasonable values of the ion current and beam emittance) due to the ion space charge is not too high and, in principle, allows using EQS in high current storage rings.

For bunched beams the longitudinal cooling rate decreases with the current more sharply due to longitudinal modulation of the ion line density, which results to modulation of the electron longitudinal velocity. Let us mark, that application of EQS to high current ion beams could require dynamical matching of the electron longitudinal velocity with velocity of central particle of the beam.

Excitation of longitudinal and transverse instabilities due to electron-ion coherent interaction (see, for example, /12-15/) seems to be the serious problem, which needs additional investigations. Experiments in SIS [16] have shown that for coasting beams in non relativistic storage ring it is possible to reach currents, which is near to space charge limit with $\Delta\nu_{inc}$=0.1. This fact permits to hope that for medium energy rings such Coulomb shift parameters is achievable as well.

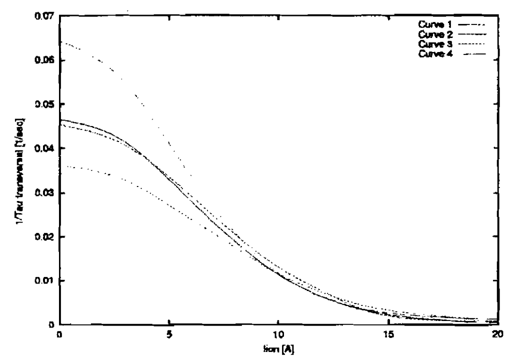

Figure 1. Dependence of transverse damping rate on ion current.

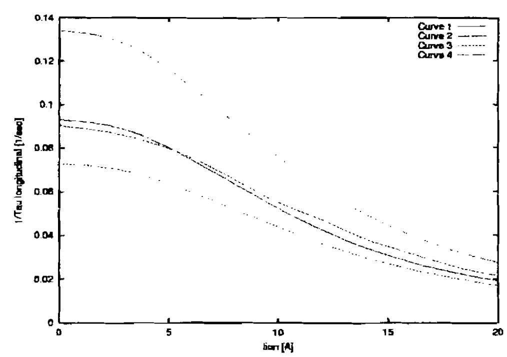

Figure 2. Dependence of longitudinal damping rate on ion current

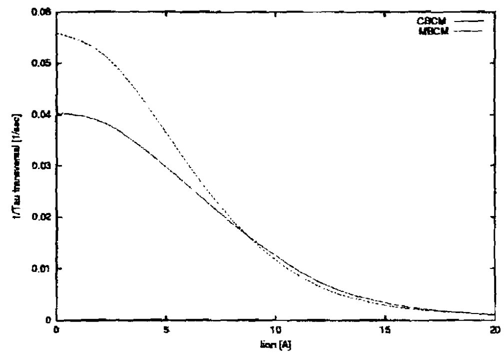

Figure 3. Dependence of transverse damping rate on ion current for TWAC project

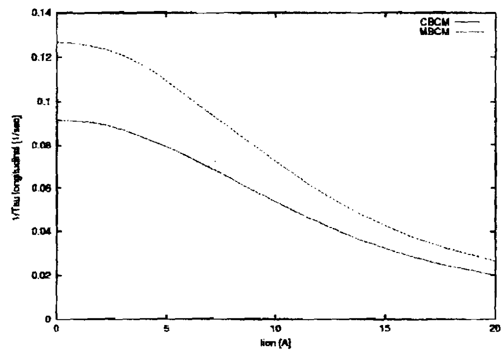

Figure 4. Dependence of longitudinal damping rate on ion current for TWAC project

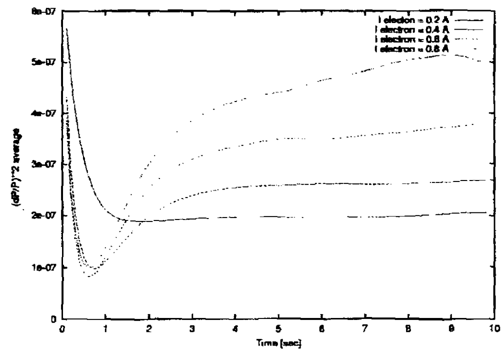

Figure 5. Dependence of momentum spread on time with electron current as parameter (SIS)

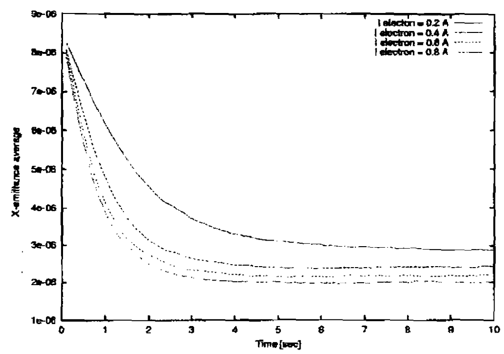

Figure 6. Dependence of X-emittance on time with electron current as parameter (SIS)

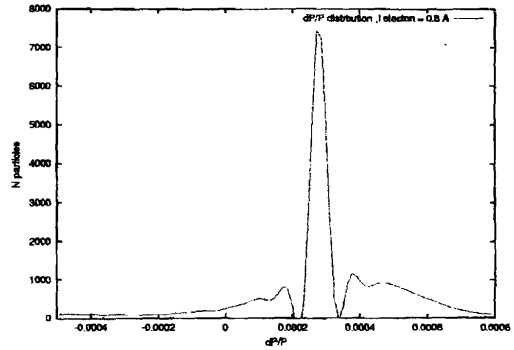

Figure 7. Final momentum spread distribution (SIS)

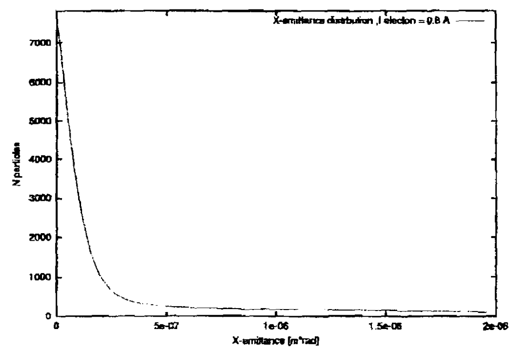

Figure 8. Final X-emittance distribution (SIS)

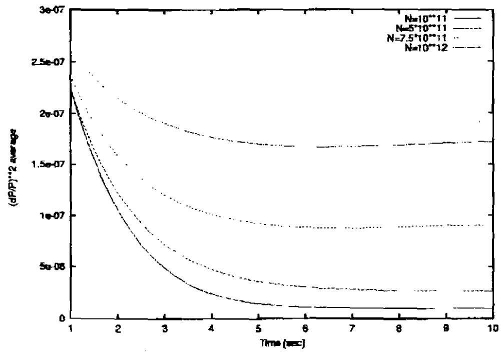

Figure 9. Dependence of momentum spread on time with particles number as parameter (HAR, coasting beam)

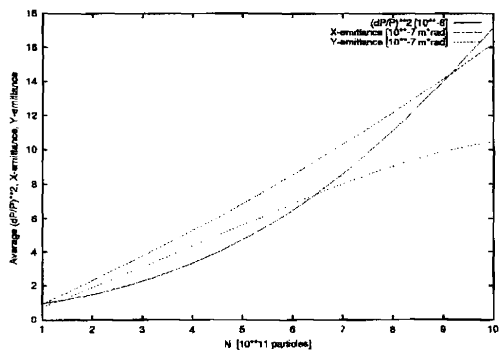

Figure 10. Dependence of equilibrium emittances on particles number (HAR, coasting beam)

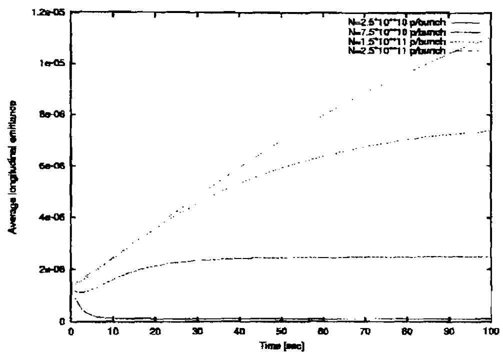

Figure 11. Dependence of longitudinal emittance on time with particles number as parameter (HAR, bunched beam)

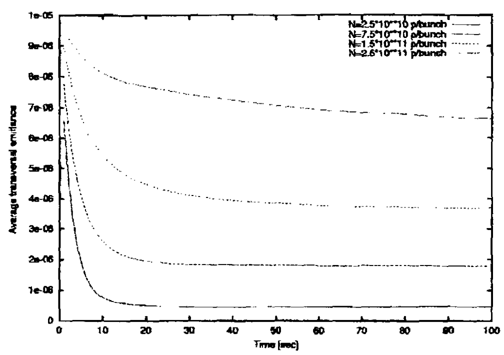

Figure 12. Dependence of transversal emittance on time with particles number as parameter (HAR, bunched beam)

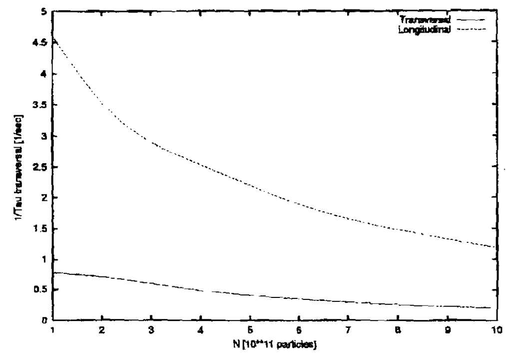

Figure 13. Dependence of damping rates on particles number for emittances corresponding space charge tune shift=0.15 (SIS, coasting beam)

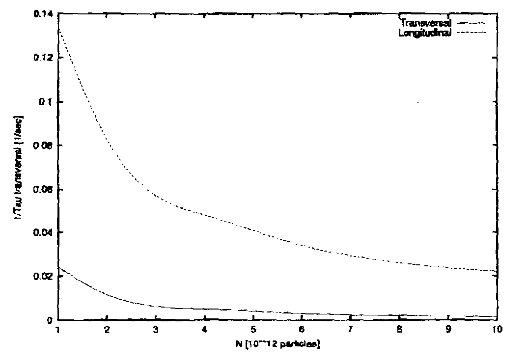

Figure 14. Dependence of damping rates on particles number for emittances corresponding space charge tune shift=0.15 (TWAC, coasting beam)

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