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ABOUT COULOMB INFLUENCE ON ANTIPROTON – NUCLEI ANNIHILATION CROSS – SECTIONS

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ABOUT COULOMB INFLUENCE ON ANTIPROTON-NUCLEI ANNIHILATION CROSS-SECTION$:Preprint ITEP 93-13/

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The Coulomb corrections to the Glauber formula for the absorption cross-sections of hadrons by nuclei are calculated. The influence of the real part of the nuclear potential is taken into account. Calculations are made in the frame of High energy approach. A comparison with data of annihilation cross-sections of antiprotons on nuclei is made.

Fig. - 3, ref. - 6

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INTRODUCTION

In the work [1] the unusual behavior of the antiproton - nuclei cross-sections was noticed, namely too strong dependence on atomic weights A in comparison with expected dependence about $\approx A^{2/3}$. It was supposed that such behavior may be described by the influence of Coulomb potential. In this article the attempt to evaluate Coulomb influence on the antiproton-nuclei annihilation in the frame of High Energy approach [2] is made.

CALCULATION

We proceed from well known formula for divergence of flow in quantum mechanics:

$$\mathrm{div} j = \frac{2}{\hbar} \cdot \mathrm{Im}\, V \cdot |\psi|^2$$

here V is a potential and ψ is a wave function.

Using this formula one can easy obtain the expression for the absorption cross-section in the potential V :

$$\sigma_{abs} = -\frac{2}{\hbar v} \int (\mathrm{Im}\, V \cdot |\psi|^2)\, d^3r$$

To obtain the value of cross-section by means of this formula it is necessary to know the wave function in the region where the

imaginary part of potential is not zero. To find a wave function we will use the Lippmann-Shwinger integral equation:

$$\psi(r) = e^{ikr} - \frac{2m}{4\pi\hbar^2} \int \frac{e^{ik|r-r'|}}{|r-r'|} V(r') \psi(r') d^3r'$$

In the frame of High Energy approach one finds the solution of this equation in the form:

$$\psi(r) = e^{ikr} \cdot \phi(r)$$

Substituting this wave function into Lippmann-Shwinger equation after a number of usual transformations, one can get the integral equation for function $\phi(r)$:

$$\phi(x,y,z) = 1 - \frac{i}{\hbar v} \int_{-\infty}^{z} (1 - e^{2ik(z-\xi)}) V(x,y,\xi) \phi(x,y,\xi) d\xi$$

Here, as usual, the Z-axis direction is the direction of the initial particles momentum. In the standard way one can get from this an equivalent differential equation for $\phi(r)$:

$$\frac{d^2\phi}{dz^2} - 2ik \frac{d\phi}{dz} + \frac{2k}{\hbar v} V \cdot \phi = 0$$

We find the solution of this equation in the form:

$$\phi(x,y,z) = exp \left\{ \int_{-\infty}^{z} f(x,y,\xi) d\xi \right\}$$

After not difficult transformations one can get the equation for f :

$$f' + f^2 - 2ikf + \frac{2k}{\hbar v} V = 0$$

In the frame of the high energy approach we can neglect the derivative of f. After that, we obtain a usual quadratic equation

for f. Solutions of this equation are :

$$f_{1/2} = ik \pm \sqrt{-k^2 - \frac{2k}{\hbar v} V}$$

To choose the right sine we may compare this solution with Glauber model or with optical model, in which the wave function has the form:

$$\psi \approx exp\left\{ -\frac{i}{\hbar v} \int_{-\infty}^{z} V(\xi) d\xi \right\}$$

Expanding the square root, one can find that minus sine must be chosen. Thus we get the wave function in the form:

$$\psi(x,y,z) = e^{ikr} \cdot exp\left\{ ik \int_{-\infty}^{z} \left(1 - \sqrt{1 + \frac{2}{pv} V} \right) d\xi \right\}$$

Substituting this expression into the formula for the absorption cross-section, we finally get :

$$\sigma_{abs} = -\frac{2}{\hbar v} \int d^2b \, dz \, \mathrm{Im}\, V \cdot exp\left\{ -ik \int_{-\infty}^{z} \left[\sqrt{1 + \frac{2}{pv} \cdot V} - \sqrt{1 + \frac{2}{pv} \cdot V^*} \right] d\xi \right\}$$

It is our final formula, and we are going to investigate the influence of the Coulomb potential on the antiproton-nuclei annihilation cross-section by means of this expression. But as a first step we consider the relation between this formula and that of standard Glauber approach [3].

It is well known that in Glauber model the absorption cross section doesn't depend on the real part of the potential [4], and, as a consequence, on the Coulomb potential. Let us consider how it can be derived from our expression. To do it, one can take the expansion of the square root in the integral :

$$\sqrt{1+\frac{2}{pv}\cdot V} = 1 + \frac{1}{pv}\cdot V + O(V^2)$$

and keep only first two members. In this case the expression in the square brackets takes the form:

$$\sqrt{1+\frac{2}{pv}\cdot V} - \sqrt{1+\frac{2}{pv}\cdot V^*} \approx \frac{2i}{pv}\cdot \mathrm{Im}\, V$$

and the expression for the absorption cross section becomes:

$$\sigma_{abs} = -\frac{2}{\hbar v}\int d^2b\, dz\, \mathrm{Im}\, V\cdot exp\left\{\frac{2}{\hbar v}\int_{-\infty}^{z}\mathrm{Im}\, V d\xi\right\}$$

There is no dependence on the real part of the potential in this formula. From this derivation it is clear that this dependence disappears when we neglect the high powers of the potential in the square root expansion. If we don't do it, we can take into account the real part of nuclear potential and Coulomb potential and evaluate their influence on the absorption cross section. But before that we consider the transformation of our formula into the standard expression for the absorption cross section in Glauber approach.

After integration on z one gets :

$$\sigma_{abs} = \int d^2b\, (1 - exp\left\{\frac{2}{\hbar v}\int_{-\infty}^{\infty}\mathrm{Im}\, V d\xi\right\}$$

In optical model in the frame of Glauber approach the optical nuclear potential is expressed through the amplitude of the elastic scattering on zero angle $f(0)$ in the form:

$$-\frac{1}{\hbar v}\cdot V_{opt} = \frac{2\pi N}{k}\,\rho(r)\cdot f(0)$$

Here N is the number of nucleons in the nuclei; $\rho(r)$ is the

normalized nuclear density and k is the wave number of initial particles. The amplitude $f(0)$ may be expressed by means of Optical theorem through the total particle-nucleon cross section σ_{tot}. After this transformation we have:

$$V_{opt} = -\frac{\hbar v}{2} N\rho(r)\sigma_{tot}(\varepsilon + i)$$

In this formula ε is the ratio of the real part the amplitude to the imaginary one. Substituting this expression into the formula for the absorption cross-section, we get standard expression for this cross section in Glauber approach [3] :

$$\sigma_{abs} = \int d^2b\{1 - exp(-N\sigma_{tot}\int_{-\infty}^{\infty}\rho(b,z)dz)\}$$

Return now to our formula for the absorption cross section :

$$\sigma_{abs} = -\frac{2}{\hbar v}\int d^2b\, dz\, \mathrm{Im}\, V\cdot exp\left\{-ik\int_{-\infty}^{z}[\sqrt{1+\frac{2}{pv}\cdot V} - \sqrt{1+\frac{2}{pv}\cdot V^*}]dz\right\}$$

It's clear from this that both Coulomb potential and the real part of the optical nuclear potential have influence on the absorption cross section. In the case of the antiproton-nuclei interactions almost all absorption cross section is annihilation cross section and it seems to be easy to compare experimental results and calculations by means of this formula. But the true nuclear optical potential is not known. Moreover, exact Coulomb potential is not known too, because it depends on the exact charge distribution in nuclei, which is unknown. But, if we want only evaluate these influences, we may use any appropriate model. In this work we consider the optical potential in Glauber approach

(may be it seems to be strange make corrections to the formula by means of this formula). In this case, unknown factor is the ratio of real to imaginary parts of the amplitude. As the nuclear density we take Woods-Sakson distribution:

$$\rho(r) = \frac{\rho_0}{1 + \exp(\frac{R - r}{\tau})}$$

Here $R = r_0 \cdot A^{1/3}$ is the average nuclei radius, τ is a falloff parameter and $\int \rho(r) \cdot 4\pi \cdot r^2 dr = 1$. Parameters r_0 and τ are taken from [5] and equal $r_0 = 1.15 fm$, $\tau = 0.45\ fm$

As a result, the expression for the absorption cross section becomes:

$$\sigma_{abs} = -\frac{2}{\hbar v} \int d^2b\, dz\, \mathrm{Im}\, V_t \cdot \exp\left\{ -ik\int_{-\infty}^{z} [\sqrt{1 + \frac{2}{pv} \cdot V_t} - \sqrt{1 + \frac{2}{pv} \cdot V_t^*}\,] d\xi \right\}$$

Here V_t is the sum of nuclear and Coulomb potentials

$$V_t = V_n + V_c \quad ,$$

the nuclear potential has the form :

$$V_n = -\frac{\hbar v}{2} N\rho(r)\sigma_{tot}(\varepsilon + i)$$

and Coulomb potential is :

$$V_c(r) = -4\pi\, Ze^2 \left(\frac{1}{r} \int_0^r \rho(r')r'^2 dr' + \int_r^\infty \rho(r')r' dr' \right)$$

As far as the amount of the ratio of real to imaginary parts of the amplitude ε is not known, we have made calculations for two cases: one with $\varepsilon = 0.0$ and another with $\varepsilon = 0.5$. The amounts of σ_{tot} have been taken from [6]. The results of the calculations are in the table N 1. The comparison of the experimental results and the calculations for four energies is shown in the fig.1.

It's clear from our results that, at least in the frame of

our approach, neither Coulomb potential nor the real part of the nuclear potential can not describe the increase of A-dependence of the annihilation cross sections at low momenta observed in [1]. However, calculated corrections change amounts of annihilation cross sections in true direction. Maybe more exact nuclear optical potential and charge distributions in nuclei can give better approach to the experimental results (which, from the other hand, need to be more precise).

It must be mentioned that at low momenta corrections grow. From this point of view, it is interesting to calculate corrections to the usual Glauber formula at the low momenta. These corrections have been calculated for the momentum range from 0.30 GeV/c to 0.70 Gev/c. The possibility of application of high energy approach and Glauber model in this range of momentum has been considered in works [4]. In calculating of cross sections by our formula the same parameters of nuclei density were used. Total antiproton - nucleon cross sections were taken from [6]. Results are collected in the Table 2.

To see more precisely the dependence of corrections on the initial particle momentum, let's consider a relative corrections of the absorption cross sections, i.e. the ratio of the corrections to Glauber cross sections. Fig.2 shows the relative corrections at the change of the real to imaginary ratio ε from 0.0 to 0.5 in depending on the initial antiproton momentum. Y-axis represents the value: $S = \frac{\sigma(\varepsilon=0.5) - \sigma(\varepsilon=0.0)}{\sigma(\varepsilon=0)}$ and X-axis represents the initial antiproton momentum p. Fig.2-a represents data calculated without Coulomb potential and Fig.2-b represents data with Coulomb potential. These figurers show us that the

influence of the real part of nuclear potential rapidly increases, when the antiproton momentum decreases. The amounts of corrections can achieve tens of percents. Besides that one can see that the change of S doesn't depend on the atomic weight of the target nucleus A.

Fig.3 shows the relative corrections with and without Coulomb potential depending on the initial antiproton momentum. Y-axis represents the value: $S = \frac{\sigma(cul) - \sigma(no\ cul)}{\sigma(no\ cul)}$, and X-axis represents the initial antiproton momentum p. On the fig.3-a the data calculated without real part of the nuclear potential (i.e $\varepsilon = 0$) are represented, and the fig.3-b represents data calculated with this part (i.e. $\varepsilon=0.5$). From these figures one can see that the contribution of Coulomb potential to the annihilation cross section essentially depends on the atomic weight of the target nucleus or, more accurately, on it's charge Z. The growth of corrections at low momenta is rather considerable, at least, for heavy nuclei.

CONCLUSIONS

From results presented here, one can make conclusion that anomalous A-dependence of annihilation cross sections observed in [1] at the moment 0.70 GeV/c, hardly may be accounted for the influence of Coulomb potential, at least in the frame of high energy approach. Although Coulomb potential gives necessary type corrections to the annihilation cross sections, i.e. increasing their A-dependence, the amounts of corrections are not enough to explain the effect. Perhaps, it is necessary to have more detail and more exact consideration or more adequate model.

From the other hand, demonstrated calculations show that the

annihilation cross sections depend on both Coulomb potential and the real part of the nuclear potential. Corresponding corrections rapidly increase, as the initial momentum decrease, and their dependencies on atomic weight are very different: while the corrections due to the real part of the nuclear potential don't depend on the atomic weight, the corrections due to Coulomb potential strongly depend on it.

From all mentioned above, one can conclude that further investigation of the antiproton nuclei annihilation cross sections at low momenta is the interesting and important problem from theoretical and experimental points of view.

SUPPLEMENT

Table 1. Calculating cross-sections (in fm^2)

A	Z	p GeV/c	σ(hadr)	σ(ε=0) cul	σ(ε=0) no cul	σ(ε=0.5) cul	σ(ε=0.5) no cul
9	4	0.70	13.80	38.2510	37.5155	38.6392	38.5334
9	4	0.95	12.60	36.4215	36.3344	37.0963	37.0403
9	4	1.26	10.78	34.4308	34.3715	34.8599	34.8370
9	4	1.53	9.97	33.4388	33.3872	33.7434	33.7522
9	4	1.76	9.55	32.8798	32.8440	33.1438	33.1594
9	4	2.50	8.20	30.9675	30.9127	31.1124	31.1321
12	6	0.70	13.80	44.3205	44.1056	45.5880	45.4076
12	6	0.95	12.60	42.9701	42.8293	43.8402	43.7375
12	6	1.26	10.78	40.8235	40.7249	41.3864	41.3335
12	6	1.53	9.97	39.7504	39.6708	40.1788	40.1496
12	6	1.76	9.55	39.1655	39.0876	39.5179	39.5002
12	6	2.50	8.20	37.6405	37.0130	37.8340	37.2997
27	13	0.70	13.80	69.9383	69.3089	72.3130	71.7439
27	13	0.95	12.60	68.1125	67.6990	69.7754	69.4228
27	13	1.26	10.78	65.4438	65.1234	66.5502	66.3068
27	13	1.53	9.97	64.1115	63.8396	64.9663	64.7816
27	13	1.76	9.55	63.3780	63.1303	64.0928	63.9462
27	13	2.50	8.20	60.8300	60.6144	61.2566	61.1849
56	26	0.70	13.80	105.8737	104.3023	109.8888	108.3694
56	26	0.95	12.60	103.3402	102.2835	106.1294	105.1864
56	26	1.26	10.78	99.8676	99.1533	101.7629	101.1784
56	26	1.53	9.97	98.1766	97.6001	99.6519	99.2349
56	26	1.76	9.55	96.2544	96.7578	98.4936	98.1661
56	26	2.50	8.20	94.1265	93.7521	94.9000	94.7483
64	29	0.70	13.80	114.2423	112.4769	118.6052	116.9350
64	29	0.95	12.60	111.6546	110.3737	114.7140	113.5817
64	29	1.26	10.78	108.0130	107.1281	110.0969	109.3550
64	29	1.53	9.97	106.2469	105.5252	107.8962	107.3098
64	29	1.76	9.55	105.2924	104.6291	106.6608	106.1799
64	29	2.50	8.20	102.0342	101.5880	102.8754	102.6820

Table 1.(continue)

A	Z	p GeV/c	σ(hadr)	σ(ε=0) cul	σ(ε=0) no cul	σ(ε=0.5) cul	σ(ε=0.5) no cul
112	48	0.70	13.80	158.4574	154.9359	164.7546	161.4246
112	48	0.95	12.60	154.6179	152.3856	159.0951	157.0395
112	48	1.26	10.78	150.0972	148.5550	153.1731	151.8344
112	48	1.53	9.97	147.9015	146.6704	150.3369	149.3118
112	48	1.76	9.55	146.7260	145.6556	148.7666	147.9491
112	48	2.50	8.20	142.8483	142.0495	144.1450	143.6715
207	82	0.70	13.80	228.3856	221.1527	237.7829	230.8657
207	82	0.95	12.60	222.5782	217.9946	229.2823	224.9809
207	82	1.26	10.78	216.5513	213.4240	221.2131	218.3798
207	82	1.53	9.97	213.6723	211.1776	217.3165	215.1800
207	82	1.76	9.55	212.0944	209.9510	214.8765	213.4408
207	82	2.50	8.20	207.6596	206.0452	209.6686	208.5740

Table 2. Calculating cross-sections (in fm^2)

A	Z	p GeV/c	σ(hadr)	σ(ε=0) cul	σ(ε=0) no cul	σ(ε=0.5) cul	σ(ε=0.5) no cul
9	4	0.30	23.85	45.8788	45.0305	49.1308	48.2805
9	4	0.36	21.74	44.1996	43.7722	46.8621	46.4718
9	4	0.40	19.97	42.8500	42.5076	45.1097	44.8002
9	4	0.45	18.46	41.6877	41.4009	43.6432	43.3862
9	4	0.50	17.45	40.8247	40.5928	42.4946	42.2919
9	4	0.55	15.65	39.3636	39.1697	41.1264	41.2641
9	4	0.60	13.88	38.3920	38.2383	39.8374	38.7831
9	4	0.70	13.80	38.2510	37.5155	38.6392	38.5334
12	6	0.30	23.85	53.5889	52.3476	57.5592	56.3288
12	6	0.36	21.74	51.6219	50.9190	54.9133	54.2600
12	6	0.40	19.97	50.0919	49.5263	52.9030	52.3807
12	6	0.45	18.46	48.7938	48.3187	51.2353	50.8020
12	6	0.50	17.45	47.8142	47.4325	49.9111	49.5653
12	6	0.55	15.65	46.2178	45.8954	47.3844	47.6984
12	6	0.60	13.88	44.4961	44.2304	45.2552	45.7098

Table 2.(continue)

A	Z	p GeV/c	σ(hadr)	σ(ε=0) cul	σ(ε=0) no cul	σ(ε=0.5) cul	σ(ε=0.5) no cul
12	6	0.70	13.80	44.3206	44.1056	45.5880	45.4076
27	13	0.30	23.85	82.9304	79.9475	89.9590	87.1099
27	13	0.36	21.74	80.0545	78.0905	85.8071	83.9469
27	13	0.40	19.97	77.8349	76.2469	82.7981	81.3007
27	13	0.45	18.46	76.0182	74.6778	80.3819	79.1245
27	13	0.50	17.45	74.5865	73.5048	78.3639	77.3567
27	13	0.55	15.65	72.4920	71.5745	75.7350	74.8854
27	13	0.60	13.88	70.2777	69.5048	72.8769	72.2845
27	13	0.70	13.80	69.9383	69.3089	72.3130	71.7439
56	26	0.30	23.85	125.1912	118.2745	136.1514	129.3877
56	26	0.36	21.74	120.3830	115.5439	129.5788	124.9485
56	26	0.40	19.97	117.0886	113.1446	125.1985	121.4376
56	26	0.45	18.46	114.4559	111.1210	121.6294	118.4583
56	26	0.50	17.45	112.2709	109.5790	118.5165	115.9675
56	26	0.55	15.65	109.4612	107.1623	114.8948	112.6965
56	26	0.60	13.88	106.5079	104.5829	110.9631	109.2949
56	26	0.70	13.80	105.8737	104.3023	109.8888	108.3694
64	29	0.30	23.85	135.2592	127.2949	146.9779	139.3742
64	29	0.36	21.74	130.0257	124.4057	140.1082	134.4688
64	29	0.40	19.97	126.3908	121.8259	135.3980	130.8474
64	29	0.45	18.46	123.8634	119.8156	131.2415	127.5584
64	29	0.50	17.45	121.1623	118.4219	127.8613	124.9014
64	29	0.55	15.65	118.1551	115.4927	123.9670	121.4485
64	29	0.60	13.88	114.9923	112.7668	119.8103	117.8506
64	29	0.70	13.80	114.2423	112.4769	118.6052	116.9350
112	48	0.30	23.85	188.1622	173.1678	205.0650	190.4128
112	48	0.36	21.74	180.3315	169.4693	194.5709	184.0335
112	48	0.40	19.97	175.1171	166.2358	187.4363	178.9841
112	48	0.45	18.46	171.1469	163.6247	182.0968	174.8630
112	48	0.50	17.45	167.5689	161.5877	177.3256	171.3961
112	48	0.55	15.65	163.7914	158.5739	172.1830	167.2083
112	48	0.60	13.88	159.7455	155.3624	166.7339	162.7345

Table 2.(continue)

A	Z	p GeV/c	σ(hadr)	σ(ε=0) cul	σ(ε=0) no cul	σ(ε=0.5) cul	σ(ε=0.5) no cul
112	48	0.70	13.80	158.4574	154.9359	164.7546	161.4246
207	82	0.30	23.85	274.3561	244.4991	299.0828	269.9330
207	82	0.36	21.74	261.5813	239.5006	282.4788	261.1280
207	82	0.40	19.97	253.3968	235.3717	271.6685	254.2507
207	82	0.45	18.46	247.3634	232.0393	263.6485	248.8526
207	82	0.50	17.45	241.8426	229.4055	256.0888	244.1158
207	82	0.55	15.65	236.3450	225.6907	248.8188	238.5729
207	82	0.60	13.88	230.7472	221.7565	241.2056	232.8026
207	82	0.70	13.80	228.3856	221.1527	237.7829	230.8657

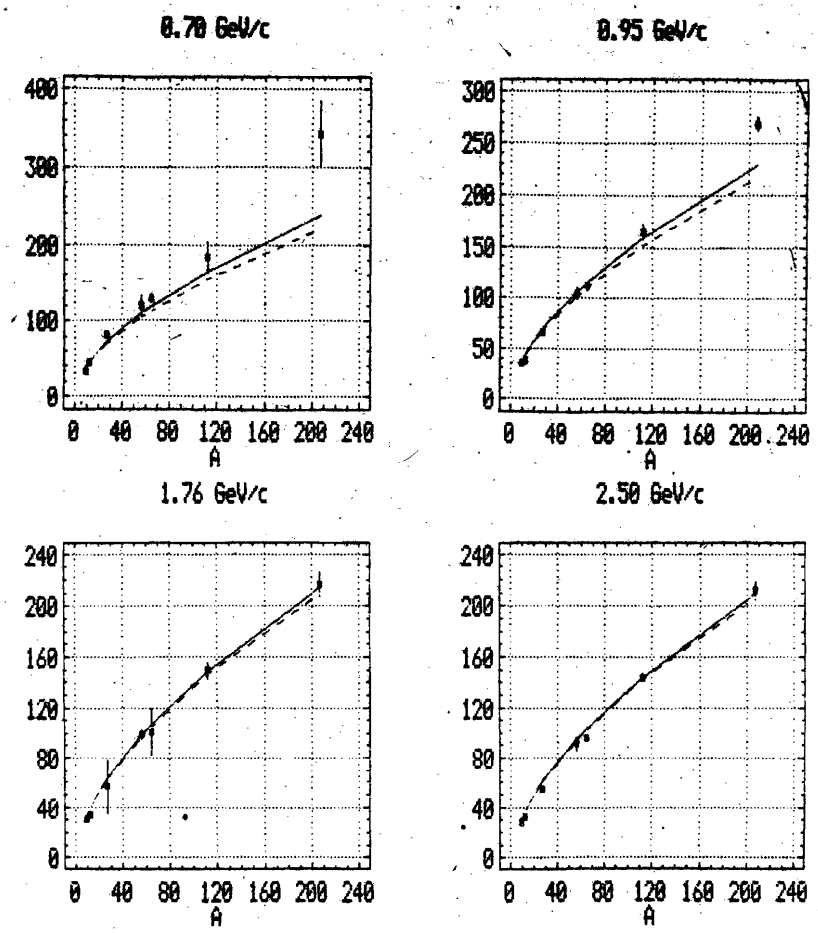

Fig.1 The comparison with the experiment at four initial momenta: 0.70, 0.95, 1.76 and 2.50 GeV/c.(in fm^2)
Solid curves - calculations with Coulomb and with ε = 0.5,
dashed curves - without Coulomb and with ε = 0.0;

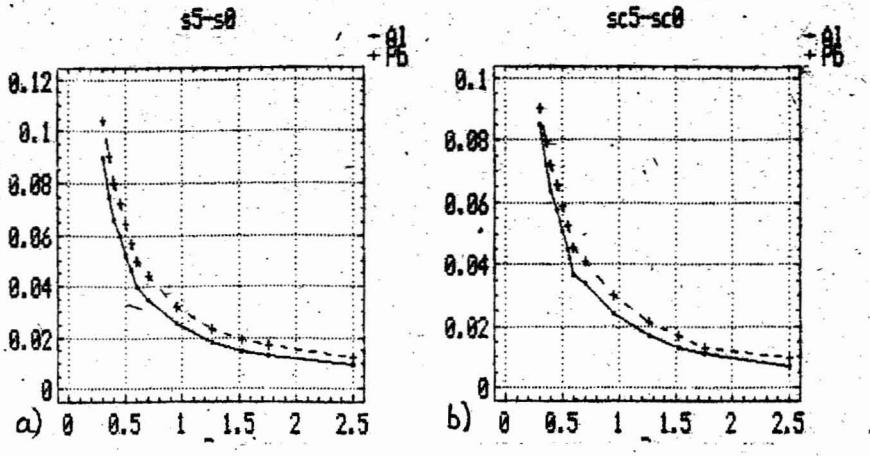

Fig.2 The relative corrections at the change ε from 0.0 to 0.5 vs momentum for two targets: Al and Pb. Y-axis represents the value $S = \frac{\sigma(\varepsilon=0.5) - \sigma(\varepsilon=0.0)}{\sigma(\varepsilon=0)}$.

fig.2-a: without Coulomb, fig.2-b: with Coulomb;

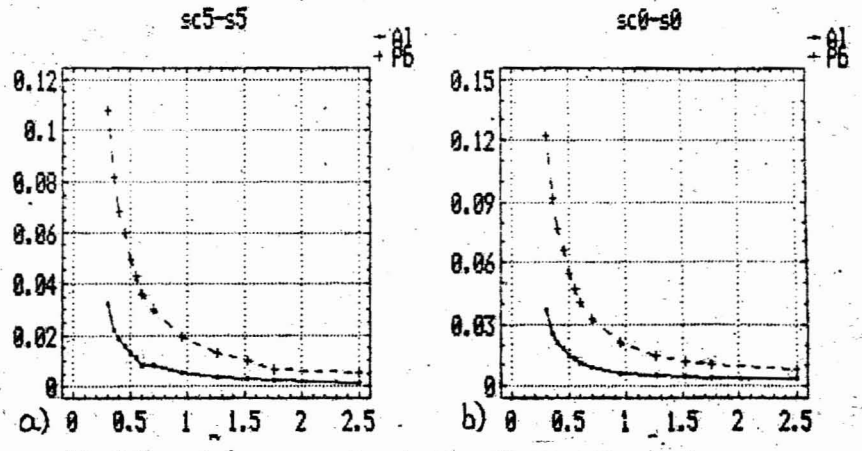

Fig.3 The relative corrections with and without Coulomb potential vs momentum for two targets: Al and Pb. Y-axis represents the value $S = \frac{\sigma(\text{cul}) - \sigma(\text{no cul})}{\sigma(\text{no cul})}$.

fig.3-a: at ε = 0.0, fig.3-b: at ε =0.5;

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