IST-DF-6-92
IST-DF-6-1992
Fermilab Library
0 1160 0021840 8

Instituto Superior Técnico
DEPARTAMENTO DE FÍSICA
Av. Rovisco Pais, 1096 Lisboa Codex, Portugal

IST-DF-6/92
November, 1992

AN INFLATIONARY BIANCHI-V MODEL WITH SHEAR AND ROTATION

ALFREDO B. HENRIQUES and LUÍS E. MENDES[1]

Departamento de Física, Instituto Superior Técnico
Av. Rovisco Pais,
1096 Lisboa Codex, Portugal

Abstract

A Bianchi-V model, in the presence of a fluid with shear and rotation, is investigated. Asymptotic solutions are worked out, showing that shear and rotation may give rise to a conflict with present day observations, a problem that can be solved with the help of an inflationary solution.

[1]Supported by grant BM/253/90-RM from JNICT.

1 Introduction

Most of the modern cosmological models are based on the Friedmann-Lemaître-Robertson-Walker (FLRW) metric. This is due in part to the simplicity of the resulting Einstein equations, but mainly to the fact that the astronomical observations seem to indicate that we live in a universe which is isotropic and homogeneous on the largest scales; indeed, measurements of the cosmic microwave background radiation show that the universe was already isotropic at the time of decoupling. There are, nevertheless, puzzles not explained by the standard FLRW model, the most quoted being the homogeneity, the isotropy and the flatness of the observed universe. It was to solve them in a natural way, without having to assume special initial conditions, that the idea of an early inflationary phase was introduced by Alan Guth in the context of Grand Unification Theories [1]

Several versions of the inflationary scenario exist [2], but in most of them the implementation of the model is made in the framework provided by the FLRW metric, that is within a model already homogeneous and isotropic. While in such models the flatness problem is well understood and solved, the situation is not so clear concerning isotropy and homogeneity. In fact, what these models do is to solve the horizon problem, a necessary, but not sufficient, condition to the solution of those two problems. To claim that we have a solution, we would need to work in a general inhomogeneous model. Due to the obvious difficulties in dealing with non-homogeneous metrics, we shall confine ourselves to the isotropy problem, assuming an homogeneous isotropic space. In fact, no compelling reasons exist, apart from simplicity, to assume that the universe started in an isotropic condition, or that it emerged from the Planck era in an isotropic state. It is thus a matter of great interest to investigate alternative models, with a richer structure than the standard one, in particular models with both shear and rotation.

Many general results have been obtained and numerical calculations performed, using the different Bianchi and Kantowsky-Sachs metrics [3-10]. In particular, the influence of the anisotropy and shear on the inflation of the universe has been explored, sometimes with surprising conclusions, namely that in many circumstances shear may help, rather than hinder, inflation [6].

Some of these investigations have been generalized to include rotation

[11-18]. Although present astronomical limits on the rotation of the universe obtained from the anisotropy of the microwave background are rather severe [19,20] (for a Bianchi-V model we have $(\omega/H)_0 < 10^{-11}$), this might have necessarily not been the case near the singularity, and it is an important topic of reserch to inquire about the influence of rotation during the very early stages of the expansion, in particular to study its influence on the onset of inflation. We must also verify that inflation can damp out rotation, and bring it down to values in agreement with the observations. In recent years, important results concerning rotation have been derived by G. F. R. Ellis, Madsen, Matzner and Rothman among others [11-16]; in particular the influence of inflation on the rotation problem has been analysed by J. Ellis and K. Olive [21].

In this paper we investigate a rotating Bianchi-V model with, for simplicity, only one non-diagonal term in the metric. The energy content of the model is given by a perfect fluid plus a self-interacting scalar field, playing the role of the inflaton. Approximate asymptotic solutions near the singularity, and for large values of t, are worked out as well as estimates concerning the values of the rotation and compared with numerical simulations. The main results of our calculations are summarized at the end.

From now on we use the following conventions with respect to the indices: lower case latin indices refer to space indices and run from 1 to 3 and greek indices denote spacetime indices and run from 0 to 3. We use units such that $8\pi G = c = 1$.

2 The model

In our model the universe is filled with a perfect fluid described by the energy-momentum tensor

$$T_{\mu\nu}^{Matter} = (\rho + p)u_\mu u_\nu + g_{\mu\nu}p \tag{1}$$

where u_μ is the fluid four-velocity satisfying the normalization condition $u^\mu u_\mu = -1$, ρ its energy density and p its pressure. To consider a rotating fluid at least one of the u_i components must be different from zero, i.e. u_μ is not orthogonal to the three-surfaces of homogeneity. We shall use an

equation of state of the general form

$$p = (\lambda - 1)\rho \ , \quad 1 \leq \lambda \leq 2 \, . \tag{2}$$

We write the metric as

$$ds^2 = -dt^2 + g_{ij}\omega^i\omega^j \tag{3}$$

where g_{ij} is a symmetric 3×3 matrix and the ω^i satisfy the following relations

$$d\omega^i = \frac{1}{2} C^i_{jk} d\omega^j \wedge d\omega^k \ ,$$

the C^i_{jk} being the structure constants of the Bianchi-V isometry group

$$C^2_{21} = -C^2_{12} = 1$$
$$C^3_{31} = -C^3_{13} = 1$$

all the other being zero.

We choose g_{12} as the only non-diagonal element in the metric $g_{\mu\nu}$ and call $g_{11} = a^2(t)$, $g_{22} = b^2(t)$, $g_{33} = c^2(t)$ and $g_{12} = d^2(t)$.

To avoid an overdetermined system, the four-velocity u^μ must at least have two of the space components different from zero, which makes this case considerably more complicated than the case of a Bianchi-*IX* model with rotation: $u^\mu = (A, B, C, 0)$, where from the normalization condition of the four-velocity, the following relation applies:

$$A^2 = 1 + a^2B^2 + b^2C^2 + 2\, d^2 B\, C \, . \tag{4}$$

To get inflation we introduce a spatially homogeneous scalar field $\phi = \phi(t)$, the inflaton with an energy-momentum tensor given by

$$T^\phi_{\mu\nu} = \partial_\mu\phi\partial_\nu\phi - \frac{1}{2} g_{\mu\nu}\partial_\rho\phi\partial^\rho\phi - g_{\mu\nu}V(\phi), \tag{5}$$

where the potential is of the simple form

$$V(\phi) = \frac{1}{2} m^2\phi^2 \tag{6}$$

used in the inflationary chaotic models. From the Einstein equations and the local conservation of the energy-momentum tensor, $T^{\mu\nu}{}_{;\nu} = 0$, we find after some manipulations the following equations:

$$a^2\left[\frac{\ddot{a}}{a}+\left(\frac{\dot{a}}{a}\right)^2+\frac{\dot{a}}{a}\left(\frac{\dot{c}}{c}-\frac{1}{2}\frac{\dot{g}}{g}\right)+\frac{2}{g}\left(ab\dot{a}\dot{b}-d^2\dot{d}^2\right)-\frac{2b^2}{g}\right]=$$
$$\frac{a^2}{2}(2-\lambda)\rho+\lambda\rho\left(a^2B+d^2C\right)^2+a^2V(\phi) \qquad (7)$$

$$b^2\left[\frac{\ddot{b}}{b}+\left(\frac{\dot{b}}{b}\right)^2+\frac{\dot{b}}{b}\left(\frac{\dot{c}}{c}-\frac{1}{2}\frac{\dot{g}}{g}\right)+\frac{2}{g}\left(ab\dot{a}\dot{b}-d^2\dot{d}^2\right)-\frac{2b^2}{g}\right]=$$
$$\frac{b^2}{2}(2-\lambda)\rho+\lambda\rho\left(b^2C+d^2B\right)^2+b^2V(\phi) \qquad (8)$$

$$\frac{\ddot{c}}{c}+\frac{1}{2}\frac{\dot{c}}{c}\frac{\dot{g}}{g}-\frac{2b^2}{g}=$$
$$\frac{1}{2}(2-\lambda)\rho+V(\phi) \qquad (9)$$

$$d^2\left[\frac{\ddot{d}}{d}+\left(\frac{\dot{d}}{d}\right)^2+\frac{\dot{d}}{d}\left(\frac{\dot{c}}{c}-\frac{1}{2}\frac{\dot{g}}{g}\right)+\frac{2}{g}\left(ab\dot{a}\dot{b}-d^2\dot{d}^2\right)-\frac{2b^2}{g}\right]=$$
$$\frac{d^2}{2}(2-\lambda)\rho+\lambda\rho(a^2B+d^2C)(d^2B+b^2C)+d^2V(\phi) \qquad (10)$$

$$\frac{\dot{\rho}}{\rho}=\frac{\lambda}{(\lambda-1)(1-A^2)+A^2}(a\dot{a}B^2+b\dot{b}C^2+2d\dot{d}BC-A^2\tilde{\theta}-2AB) \qquad (11)$$

$$\frac{\dot{B}}{B}=\frac{2}{g}\left(\frac{C}{B}\left(b\dot{b}d^2-b^2d\dot{d}\right)+\left(d^3\dot{d}-a\dot{a}b^2\right)\right)-\frac{\lambda-1}{\lambda}\frac{\dot{\rho}}{\rho}+$$
$$\frac{1}{ABg}\left(b^2C-d^2B\right)^2 \qquad (12)$$

$$\frac{\dot{C}}{C}=\frac{2}{g}\left(\frac{B}{C}\left(a\dot{a}d^2-a^2d\dot{d}\right)+\left(d^3\dot{d}-a^2b\dot{b}\right)\right)-\frac{\lambda-1}{\lambda}\frac{\dot{\rho}}{\rho}-$$
$$\frac{1}{ACg}\left(a^2B+d^2C\right)\left(d^2B+b^2C\right) \qquad (13)$$

where we used the first derivative of equation (4) to eliminate $\dot{A}$ and

$$g\equiv a^2b^2-d^4=\frac{\det g_{ij}}{c^2}$$

$$\tilde{\theta} = n^{\alpha}{}_{;\alpha} = \frac{1}{2}\frac{\dot{g}}{g} + \frac{\dot{c}}{c}$$

with $n^{\alpha} = (1,0,0,0)$ being the normal to the three-surfaces of homogeneity ($\tilde{\theta}$ is the expansion scalar in the frame of the normals to the hipersurfaces of homogeneity, that is, in the frame were the velocity appears tilted). The Klein-Gordon equation for the inflaton is:

$$\ddot{\phi} + \tilde{\theta}\dot{\phi} + \frac{dV}{d\phi} = 0 \tag{14}$$

To these equations we must add the constraint equations coming from the (00), (01) and (02) components of the Einstein equations

$$\frac{1}{g}(ab\dot{a}\dot{b} - d^2\dot{d}^2) + \frac{1}{2}\frac{\dot{c}}{c}\frac{\dot{g}}{g} - \frac{3b^2}{g} = A^2\lambda\rho - (\lambda-1)\rho + V(\phi) + \frac{1}{2}\dot{\phi}^2 \tag{15}$$

$$\frac{a^2b^2}{g}\left(2\frac{\dot{a}}{a} - \frac{\dot{b}}{b}\right) - \frac{\dot{c}}{c} - \frac{d^4}{g}\frac{\dot{d}}{d} = -A(a^2B + d^2C)\lambda\rho \tag{16}$$

$$3\frac{d^2b^2}{g}\left(\frac{\dot{d}}{d} - \frac{\dot{b}}{b}\right) = -A(d^2B + b^2C)\lambda\rho. \tag{17}$$

Equations (15) to (17) correspond to constraints on the initial data, and the Bianchi identities make sure that, once these equations are satisfied by the initial conditions, they will always be verified.

A straightforward but very tedious computation gives the following complicated expressions for the expansion, acceleration, rotation and shear scalars:

$$\theta = \dot{A} + A\tilde{\theta} + 2B \tag{18}$$

$$\mathbf{a}^2 = (a\dot{a}B^2 + b\dot{b}C^2 + 2d\dot{d}BC - \tilde{\theta}A^2 - 2AB)^2(A^2-1) \times \left(\frac{\lambda-1}{A^2 + (\lambda-1)(1-A^2)}\right)^2 \tag{19}$$

$$\omega^2 = \frac{1}{2gA^2}(b^2(A^2-1) - gB^2) \tag{20}$$

$$\sigma^2 = A(A^2-1)(\dot{B}C - B\dot{C})(b^2C + d^2B) + \frac{1}{2}g(\dot{B}C - B\dot{C})^2 + A^4\omega^2 + (a^2B\dot{B} + b^2C\dot{C} + d^2(\dot{B}C + B\dot{C}))A^3B +$$

$$
\begin{aligned}
&2A^2(a\dot{a}B\dot{B} + b\dot{b}C\dot{C} + d\dot{d}(\dot{B}C + B\dot{C})) + (\tilde{\theta}A + B)^2 + \\
&2A(a\dot{a}B^2 + b\dot{b}C^2 + 2d\dot{d}BC)(B - \dot{A} + \frac{1}{2}\frac{\dot{g}}{g}A) + A^2 + B^2 - \\
&A^3(A^2 - 1)\dot{B} - \frac{\dot{g}}{g}\frac{\dot{c}}{c}A^2 - \frac{\theta^2}{3} - \\
&2\frac{A^3}{g}((ab\dot{a}\dot{b} - d^2\dot{d}^2)A + (a\dot{a}b^2 - d^3\dot{d})B + (b^2d\dot{d} - b\dot{b}d^2)C) \quad (21)
\end{aligned}
$$

If in equation (21) we put $A = 1$ and $B = C = 0$ we get the shear scalar in the frame of the normals ($\tilde{\sigma}$).

3 Asymptotic solutions

Before introducing the numerical simulations, we shall derive some asymptotic solutions which approximately describe the model near the initial singularity and for large values of t and which are illustrative of the way rotation may influence the expansion of the universe. In this section the scalar field will not be taken into account; the fluid will be given by the energy-momentum tensor (1) with the equation of state (2)

As in previous instances, we look for solutions of the power-law form

$$a = a_1 t^{\alpha}\ , \quad b = b_1 t^{\beta}\ , \quad c = c_1 t^{\gamma}\ , \quad d = d_1 t^{\delta}\ . \tag{22}$$

We begin analysing the behaviour of the model near the initial singularity, $t \to 0$. Substituting (22) into the Einstein equations (7) to (10) we find the following generalization of the well-known Kasner relations

$$\alpha^2 + \beta^2 + \gamma^2 = \alpha + \beta + \gamma = 1 \tag{23}$$

and

$$\delta(1 + \alpha + \beta - \gamma - 2\delta) - 2\alpha\beta = 0 \tag{24}$$

while equations (16) and (17) demand that $\alpha = 1/3$ and $\delta = \beta$; thus, as long as the r.h.s. of the Einstein equations can be consistently neglected, we find the asymptotic power-law solution to be valid, with

$$\alpha = \frac{1}{3}\ , \quad \beta = \delta = \frac{1}{3}(1 + \sqrt{3})\ , \quad \gamma = \frac{1}{3}(1 - \sqrt{3}) \tag{25}$$

where $2\delta > (\alpha + \beta)$ as necessary for the consistency of the approximations we made. This solution also applies when we have an equation of state $p = -\rho$. We were not able to find a power-law solution for the matter terms ρ, B and C consistent with (25); thus we assume that such terms become negligible, compared with the l.h.s. of the equations, following some complicated non-power-law type of behaviour. However, a power-law solution is also possible for the matter terms, but then solution (25) has to be slightly modified. We assume that $A \to 1$ faster then $1 + t^{\epsilon}$ ($\epsilon > 0$). For equations for $\dot{B}$ and $\dot{C}$ to be consistent we must take $B = -C$; looking for power-law solutions of the form

$$B = B_1 t^{\xi} , \quad C = C_1 t^{\chi} , \quad \rho = \rho_1 t^{r} , \tag{26}$$

this means that

$$B_1 = -C_1 , \quad r = -\lambda , \quad \xi = \chi = \lambda - 1 - 2\alpha , \quad \text{for } 1 \leq \lambda \leq 2 . \tag{27}$$

Then, going back to the equations for R_{01}, R_{02} and the remaining Einstein equations, we find that although the r.h.s. of equations (7) to (10) can be consistently neglected, this is not so with the r.h.s of equation (16). After defining $x \equiv a_1^2 B_1 \rho_1 \lambda$, we get

$$\alpha = \frac{1}{3}(1 - x) \tag{28}$$

$$\beta = \delta = \left(\frac{1}{3} + \frac{x}{6}\right) \pm \frac{\sqrt{3}}{3}\left(1 - 3s\rho_1 - \frac{x^2}{4}\right)^{1/2} \tag{29}$$

$$\gamma = \left(\frac{1}{3} + \frac{x}{6}\right) \mp \frac{\sqrt{3}}{3}\left(1 - 3s\rho_1 - \frac{x^2}{4}\right)^{1/2} \tag{30}$$

where s is 1 for stiff matter ($\lambda = 2$) and 0 in all the other cases. These solutions are subject to the constraints

$$1 \leq x \leq 2 , \quad \lambda > 1 + \alpha , \quad \text{when } s = 0$$

and

$$\left.\begin{array}{ll} 0 \leq \rho_1 < \frac{1}{3} , & 2\alpha - \beta < 1 \\ -\sqrt{1 - 3\rho_1} < x \leq 2\sqrt{1 - 3\rho_1} & \text{for the } + \text{ solution} \\ \sqrt{1 - 3\rho_1} < x \leq 2\sqrt{1 - 3\rho_1} & \text{for the } - \text{ solution} \end{array}\right\} \text{when } s = 1 .$$

The interesting consequence of this solution is that $\omega^2 \to 0$ on the approach to the origin, which means that, at least for some initial conditions, the rotation increases when we come away from the origin and the universe expands; this suggests that in such cases it might be possible for rotation to take sufficient energy away from the inflating universe, leading to a premature stop of the inflation. This is similar to what was found in a previous study of a closed Bianchi-*IX* model for $\lambda > 3/2$ but, now as then, such fears were not confirmed by numerical simulations. Such behaviour of the rotation has been noticed before by Barrow, based on dimensional considerations [22].

We turn now to the large t behaviour, in order to derive how rotation and shear vary in this limit. We have a variety of possible approximate solutions, obtained under different simplifying assumptions, all of them numerically checked.

Neglecting the r.h.s of equations (7) to (10) and taking into account the curvature term, we find a power-law solution $a_i = a_{1i} t^{\alpha_i}$ ($i = 1$ to 4) with

$$\alpha_i \equiv \alpha = 1 \ , \quad a_{11} = 1 \text{ and the others } a_{1i} \text{ arbitrary} \ . \tag{31}$$

Numerically we checked that this solution acts as an atractor; the fact that the shear in the frame of the normals ($\bar{\sigma}$) goes to zero with t^{-1} does not necessarily mean that an observer comoving with the fluid sees an isotropic universe because the shear in the frame of the velocities (σ) may increase with the expansion of the universe for those solutions in which B and C grow with time. In fact, such an observer may not even see an homogemeous universe, because, the hipersurfaces normal to his four velocity are diferent from the hipersurfaces of homogeneity and intersect an infinite number of these [13,23].

This solution applies in those cases where the r.h.s. decreases faster than t^{-2}. We found several consistent power-law solutions for the fluid variables ρ, B and C of which we give now a representative example. For solutions of the form (26) and (31) we find from the equations for $\dot{B}$, $\dot{C}$ and $\dot{\rho}$:

$$r = -3\lambda \ , \quad \xi = \chi = 3\lambda - 5 \ , \quad A \to 1 \ , \quad \text{for } 1 \leq \lambda < \frac{4}{3} \tag{32}$$

and a similar solution, albeit more complicated, which starts from a value of λ slightly below 4/3, with $\xi = \chi > -1$ and becoming increasingly positive

as r becomes more negative and smaller than -6. There is a jump between the two solutions, with r going from -3λ to approximately -6 within a small interval of λ. At $\lambda = 4/3$, $\xi = \chi = 0$ and B and C quickly approach a constant. This has important consequences as constant or increasing ($\lambda > 4/3$) velocity components and shear will conflict with observations (existence of large-scale condensations), as noticed by Collins and Hawking [19], a problem whose solution may require an inflationary period.

In all these cases, the rotation $\omega \to 0$ (see (20)) as the universe expands.

If the universe is matter dominated, $\lambda = 1$ and $\xi = \chi = -2$, it is not difficult to see that $\omega \sim t^{-2} \sim R^{-2}$ ($R = (abc)^{1/3}$). For $\lambda = 4/3$ — radiation domination — and $\xi = \chi = 0$, $\omega \sim t^{-1} \sim R^{-1}$. If we have a more rigid fluid ($\lambda > \frac{4}{3}$), as it may happen in the hadronic or earlier eras, again $\omega \sim t^{-1} \sim R^{-1}$. In the first two cases the behaviour of ω with R is the same as in an FLRW background.

Assuming $R \sim T^{-1}$ at $T = T_{Pl}$ we have $\omega_i = \omega_1 T_{Pl}/T_i \approx 10^{32}\omega_1$ (ω_1 is the present value of rotation). Because when $T_i \approx T_{Pl}$ and $\rho_i \approx \rho_{Pl}$, R_i is many orders of magnitude larger than l_{Pl}, the typical natural value of ω with which to compare ω_i is not ω_{Pl} (usually defined from $\omega_{Pl} l_{Pl} \equiv c$), but rather something like $\overline{\omega} R_i = c$, giving $\overline{\omega} \approx 10^{-30}\omega_{Pl}$; taking a present upper limit of $\omega_1 \approx 10^{-14}$ arcsec/century we find that $\omega_i \approx 10^{-12}\overline{\omega}$ which, although not as severe as usually quoted, is still a value unnaturally low (remember that near the origin ω decreases as we go backwards in time, supporting our order of magnitude calculation). This in turn means that taking an ω_i nearer to the typical value $\overline{\omega}$, we should now have an ω many orders of magnitude above the observed value.

Inflation is also needed to solve this problem, although its solution is not independent of the solution of the isotropy problem.

A word of caution is necessary. Our results concernig ω only apply to near-isotropic situations, but in its favour we can mention the fact that the solution (32) seems to be an attractor with the system falling rather quickly into it. To investigate situations with a high degree of initial anisotropy and including inflation, we have to resort to numerical computations.

4 Numerical simulations

The numerical simulations were carried out using a fourth-order Runge-Kutta method with step control. The accuracy of the numerical solution was checked with the help of the constraint equations (15) to (17).

We made several simulations without the scalar field in order to confirm the existence of the asymptotic solutions (32). Figure 1 shows the asymptotic variation of ξ, χ and r with λ. The exact value of λ for which the predicted transition between the two kinds of solution takes place depends on the initial values B_0 and C_0. For some values of B_0 and C_0 this rapid transition does not happen and we have another type of solution with ξ and χ approaching a constant for rigid fluids and r increasing slightly with λ. All these solutions behave like attractors.

We now add the scalar field, with the energy-momentum tensor given by equation (5) and a potential of the form (6).

There is an upper bound to the mass of the inflaton, $m_\phi < 10^{-6}$, following from the requirement that the energy density fluctuations $\delta\rho/\rho$ must be of the order of $10^{-4} - 10^{-5}$ at the horizon crossing time [2]. We found that initial values of ϕ of the order of 3 Planck masses were enough to obtain sufficient inflation ($R \sim 10^{30}$). It was verified that during inflation, the value of the rotation was lowered by about 35 orders of magnitude (figure 2). This number is fairly independent of the initial conditions, even for very large initial values of the rotation.

The influence of rotation as well as that of shear on the total rate of expansion of the universe was seen to be always smaller than about 4% of the total e-folding for a range in the initial values of rotation and shear of about 10^7.

For very rigid fluids, however, we found some curious and unexpected results concerning the velocity components and the shear. For $\lambda = 1.5$, the components of the velocity not only survive inflation as they may, during or after inflation, either approach a constant or, for still larger values of λ, start to grow with a finite rate that increases with the rigidity of the fluid (fig. 3). This result seems to be independent of the initial conditions — even for $B_0 \approx C_0 \approx 10^{-7}$ we obtain such a behaviour after inflation.

In conclusion, we studied a Bianchi-V model with shear and rotation. A number of asymptotic power-law solutions were found near the singular-

ity and for large t. For a large range of initial conditions we also found that inflation is not affected by the presence of shear and rotation in the primordial fluid.

The velocity components and the shear can survive inflation, if the equation of state is sufficiently rigid. We might think this would be an argument against such a rigid equation of state; in fact it is not, as ρ refers to a pre-inflationary fluid which will be completely diluted at the end of the inflationary period, while most of the present day ordinary matter is thought to have been created throught the couplings of the scalar field to the ordinary matter during the oscillations of this field around the minimum of $V(\phi)$.

Related to this, it is important to notice that the only vector we can form from a scalar field is a gradient whose curl is zero. Any matter arising from this field should not, then, have any vorticity. The observation of a global, large-scale, rotation in the universe today would, we believe, be a serious problem to at least the usual versions of the inflationary model.

Acknowledgement

The authors would like to thank P. M. Sá for useful conversations.

References

[1] A. H. Guth, *Phys. Rev.* D23 (1981), 347.

[2] E. W. Kolb and M. S. Turner, "The Early universe", Lecture Note Series - Frontiers in Physics - Vol. 69, (Addison-Wesley, Reading, MA, 1990);

A. Linde, "Particle Physics and Inflationary Cosmology", Contemporary Concepts in Physics, Vol. V, (Harwood Academic Publishers, Chur, 1990).

[3] R. M. Wald, *Phys. Rev.* D28 (1983), 2118.

[4] A. Zardecki, *Phys. Rev.* D31 (1985), 718.

[5] Ø. Grøn, *Phys. Rev.* D32 (1985), 2522.

[6] E. Martinez-Gonzalez and B. J. T. Jones, *Phys. Lett.* B167 (1986), 37.

[7] J. D. Barrow, *Nucl. Phys.* B296 (1988), 697.

[8] A. B. Burd and J. D. Barrow, *Nucl. Phys.* B308 (1988), 929.

[9] L. E. Mendes and A. B. Henriques, *Phys. Lett.* B254 (1991), 44.

[10] A. B. Henriques, J. M. Mourão and P. M. Sá, *Phys. Lett.* B256 (1991), 359.

[11] R. A. Matzner, *Ann. Phys.* 65 (1971), 438.

[12] M. P. Ryan Jr., *Ann. Phys.* 65 (1971), 506.

[13] A. R. King and G. F. R. Ellis, *Commun. Math. Phys.* 31 (1973), 209.

[14] T. Rothman and M. S. Madsen, *Phys. Lett.* B159 (1985), 256.

[15] T. Rothman and G. F. R. Ellis, *Phys. Lett.* B180 (1986), 19.

[16] R. Matzner, T. Rothman and G. F. R. Ellis, *Phys. Rev.* D34 (1986), 2926.

[17] A. K. Raychaudhuri and B. Modak, *Class. Quantum Grav.* 5 (1988), 225.

[18] P. M. Sá and A. B. Henriques, *Phys. Lett.* B287 (1992), 61.

[19] S. W. Hawking, *Mon. Not. R. Astr. Soc.* 142 (1969), 129;
C. B. Collins and S. W. Hawking, *Mon. Not. R. Astr. Soc.* 162 (1973), 307.

[20] J. D. Barrow, R. Juszkiewicz and D. H. Sonoda, *Mon. Not. R. Astr. Soc.* 213 (1985), 917.

[21] J. Ellis and K. A. Olive, *Nature* 303 (1983), 679.

[22] J. D. Barrow, *Mon. Not. R. Astr. Soc.* 179 (1977), 47P.

[23] M. Demiański and L. P. Grishchuk, *Commun. Math. Phys.* 25 (1972), 233.

[24] M. S. Turner, *Phys. Rev.* D28 (1983), 1243.

Figure Captions

Fig. 1 – Asymptotic variation of the exponents ξ, χ and r ($\xi = \chi$), defined in equation (26), with λ for different initial values of B_0 and C_0, obtained by numerical simulation. The initial conditions for the simulations were: $a_0 = 2.0$, $b_0 = \dot{b}_0 = c_0 = d_0 = 1.0$ and $\rho_0 = 0.4$.

Fig. 2 – Variation of the rotation scalar with time, with and without inflation. The initial conditions were $a_0 = 1.1$, $b_0 = \dot{b}_0 = c_0 = d_0 = 1.0$, $\rho_0 = 0.4$, $\lambda = 4/3$, $B_0 = 0.0$.

Fig. 3 – Variation of the velocity component B with time (the qualitative behaviour of C is the same), for $\lambda = 1.44$, 1.5 and 1.58 obtained with $a_0 = 2.0$, $b_0 = \dot{b}_0 = c_0 = d_0 = 1.0$, $\rho_0 = 0.4$, $B_0 = 0.5$ and $C_0 = -0.5$.

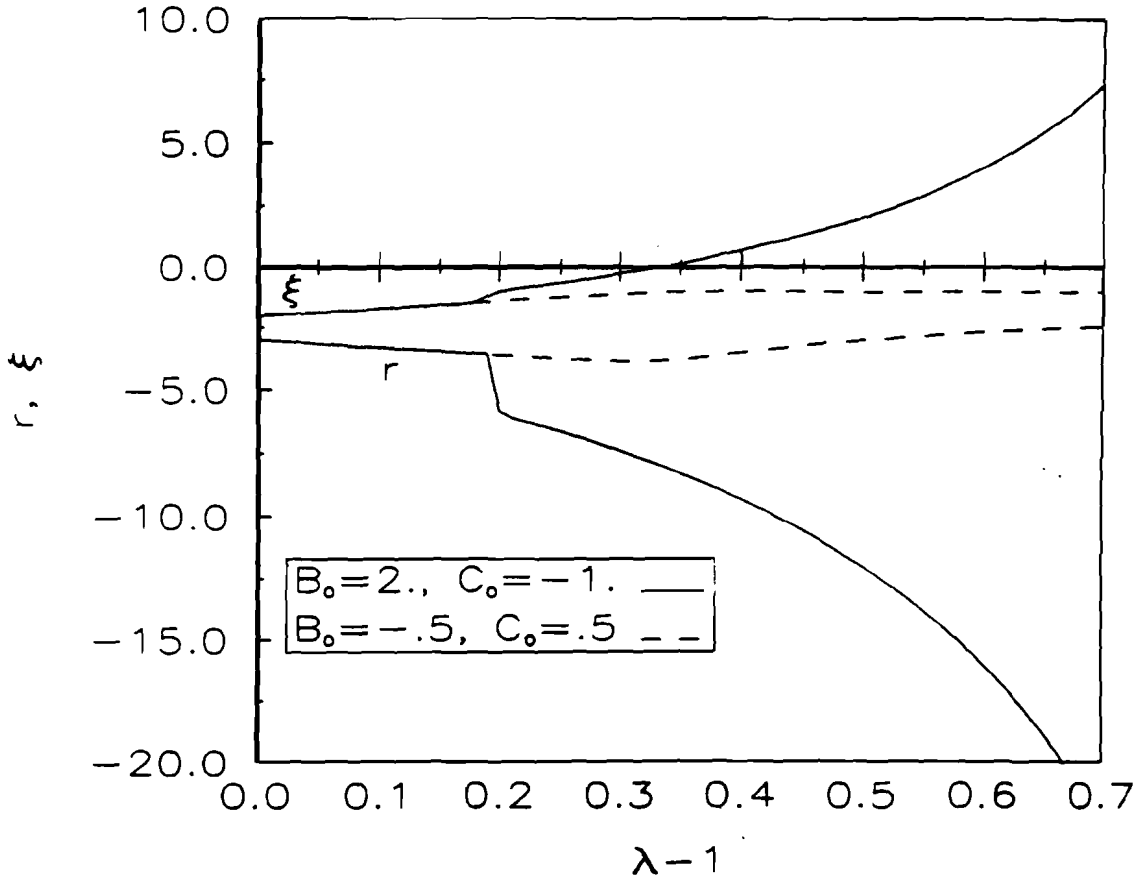

Figure 1

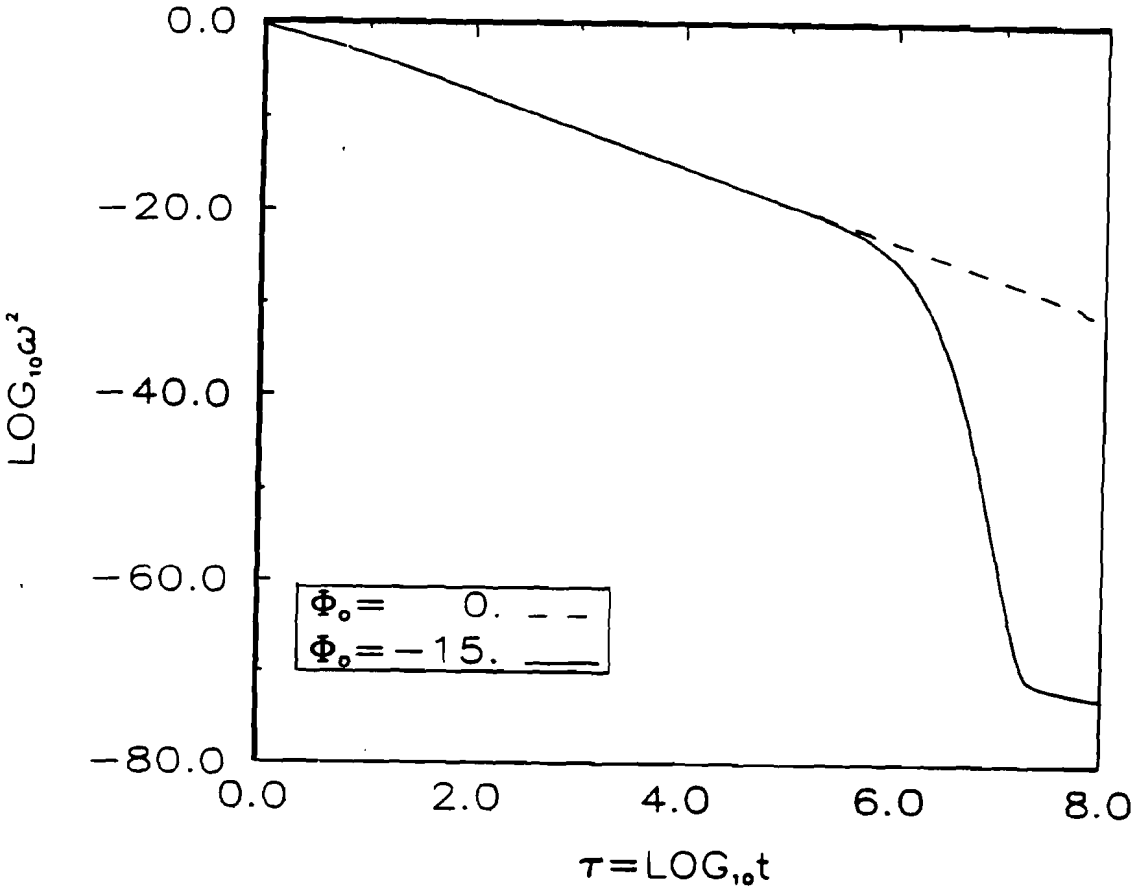

Figure 2

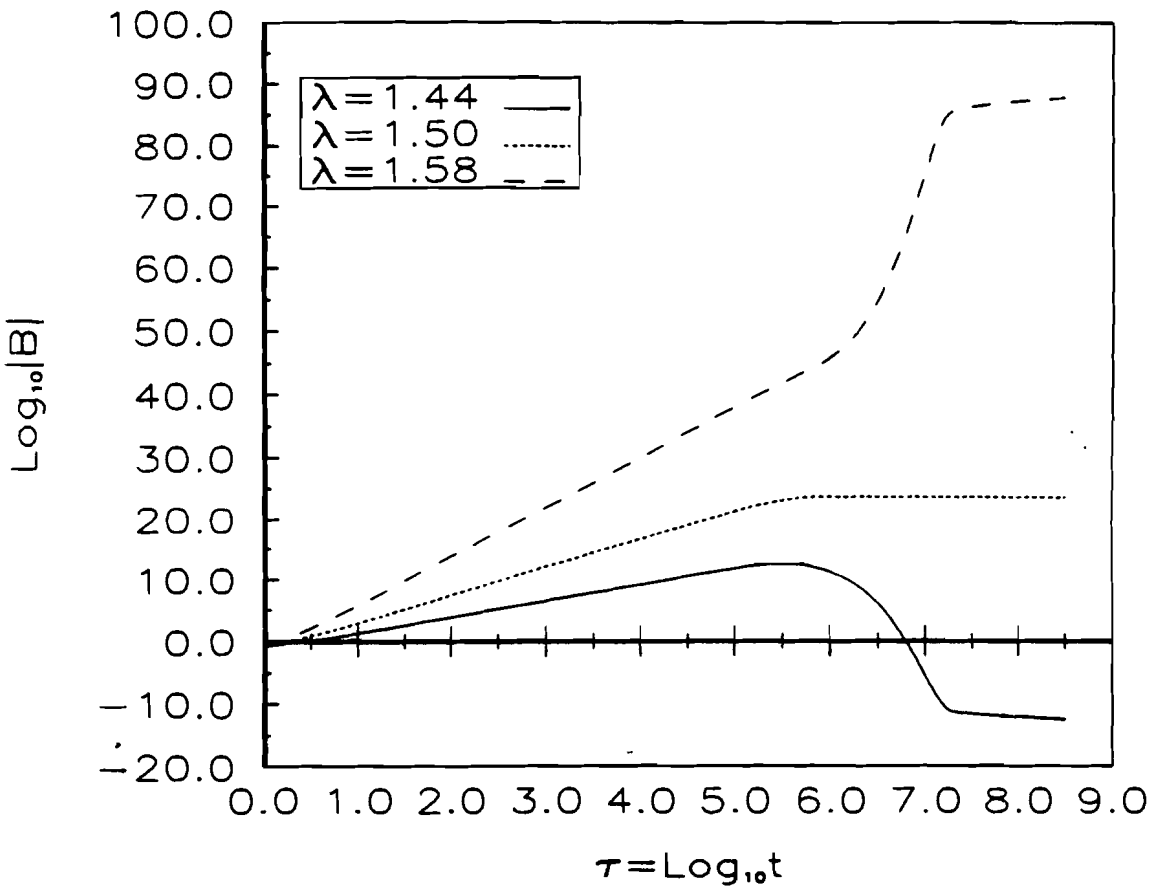
λ=1.44
λ=1.50
λ=1.58
100.0
90.0
80.0
70.0
60.0
50.0
40.0
30.0
20.0
10.0
0.0
−10.0
−20.0
Log₁₀|B|
0.0 1.0 2.0 3.0 4.0 5.0 6.0 7.0 8.0 9.0
τ=Log₁₀t

Figure 3