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# VISCOUS FLUX MOTION IN ANISOTROPIC TYPE-II SUPERCONDUCTORS IN LOW FIELDS

Zhidong Hao and John R. Clem
Ames Laboratory-USDOE and Department of Physics, Iowa State University
Ames, Iowa 50011

## Abstract

The Bardeen-Stephen model of viscous flux motion in isotropic type-II superconductors is extended to the anisotropic case characterized by a phenomenological effective mass tensor $m_{ij}$. When the magnetic field is low and the vortex lines are aligned along one of the three principal axes, simple expressions for the viscosity tensor $\eta_{ij}$ of the viscous flux motion are obtained as functions of $m_{ij}$ and the normal state conductivity tensor $\sigma_{ij}$ for temperature $T$ close to the critical temperature $T_c$. For the high-temperature oxide superconductors the theory predicts that $\eta_b^{(a)} : \eta_b^{(c)} : \eta_c^{(a)} \approx 1 : 4\gamma : 3\gamma^2$, where $\eta_i^{(j)}$ is the viscosity for the motion along the $i$-axis of a vortex parallel to the $j$-axis and $\gamma = \sqrt{m_c/m_a}$ is the anisotropy parameter ($m_i$, $i = a, b, c$, are the principal values of the mass tensor satisfying $m_a \approx m_b \ll m_c$).

## Introduction

The new high-temperature oxide superconductors show many strongly anisotropic properties. We consider in this paper how the anisotropy affects the viscous motion of a flux-line lattice. For the isotropic case the physically most transparent model for viscous flux motion was developed first by Bardeen and Stephen (BS)[1]. The BS model makes the approximation that the superconductor is local and assumes that there is a normal core of radius $\xi$, the Ginzburg-Landau (GL) coherence length, and that the dissipation occurs inside and in the immediate vicinity of the vortex core. Although it is clearly only a simplifying approximation, the BS model yields expressions for the viscosity and the flux-flow resistivity which are nearly identical with the empirical results[2], and gives a simple and clear picture of the mechanism of viscous flux motion. Our purpose is to generalize the BS model to include the effect of anisotropy.

The anisotropy of superconductivity can be most easily described in the framework of the GL theory by introducing a phenomenological effective mass tensor $M_{ij}$[3,4], which has the principal values of $M_i$ ($i = 1, 2, 3$). It is convenient to define a normalized mass tensor $m_{ij} = M_{ij}/\bar{M}$ with principal values $m_i = M_i/\bar{M}$, where the mean mass $\bar{M} = (M_1 M_2 M_3)^{1/3}$, such that $m_1 m_2 m_3 = 1$. The penetration depths $\lambda_i = \lambda\sqrt{m_i}$ describe the decay of components of *supercurrent* along the principal directions, and the corresponding coherence lengths $\xi_i = \xi/\sqrt{m_i}$ characterize the spatial variation of the order parameter along these directions[5]. The scalars $\lambda = (\lambda_1\lambda_2\lambda_3)^{1/3}$ and $\xi = (\xi_1\xi_2\xi_3)^{1/3}$ can be considered as the mean penetration depth and the mean coherence length, and are given by the same expressions as for the isotropic case[6], except that the isotropic mass $M$ is replaced by $\bar{M}$. The GL parameter is $\kappa = \lambda/\xi$.

We confine ourselves to the case that the magnetic field $H$ is small compared with the upper critical field $H_{c2}$ ($H \ll H_{c2}$), so that a vortex in mixed state can be considered as isolated. We assume that the temperature $T$ is close to the critical temperature $T_c$ ($T_c - T \ll T_c$), so that the BS model can be further simplified as that in which there is a sharp discontinuity at the core boundary between a fully normal core and fully superconducting material[7]. Accordingly the London equations[8] are applied outside the core, Ohm's law is applied inside the core, and the electrical conductivity of the normal electrons in the superconducting state is approximated by the normal state conductivity.

## Viscous Flux Motion

The magnetic flux density $\mathbf{b}$ of an isolated vortex in an anisotropic type-II superconductor is given in Ref. 5 in the GL regime for the case when the vortex axis is parallel to one of the principal axes. In the London limit (the reduced order parameter is 1 outside the vortex core and 0 inside the core, and $\kappa \gg 1$) $\mathbf{b} = b_3\hat{\mathbf{x}}_3$ is given by

$$b_3 = \frac{\phi_0}{2\pi\lambda_1\lambda_2} K_0\left(\sqrt{\frac{x_1^2}{\lambda_2^2} + \frac{x_2^2}{\lambda_1^2}}\right) \tag{1}$$

for a vortex centered on the $\hat{\mathbf{x}}_3$-axis, where $K_n$ is a Bessel function of $n$-th order. We choose axes coinciding with the principal axes with unit vectors $\hat{\mathbf{x}}_i$ and position coordinates $x_i$ ($i = 1, 2, 3$). Outside the core, the superfluid velocity $\mathbf{v}_s$ obeys the Maxwell's equation

$$\mathbf{v}_s = -\frac{c}{4\pi e n_s}\nabla \times \mathbf{b} \tag{2}$$

$$\simeq \frac{\hbar\sqrt{m_3}}{2\bar{M}} \frac{\hat{\mathbf{x}}_1 m_2 x_2 - \hat{\mathbf{x}}_2 m_1 x_1}{m_1 x_1^2 + m_2 x_2^2} \tag{3}$$

for $|x_1| < \lambda_2$, $|x_2| < \lambda_1$, where $n_s$ is the superfluid density, and we have used $K_1(x) \simeq 1/x$ for small $x$ and $\lambda^2 = \bar{M}c^2/4\pi e^2 n_s$. We assume for the vortex core boundary the ellipse

$$\frac{x_1^2}{\xi_1^2} + \frac{x_2^2}{\xi_2^2} = 1. \tag{4}$$

Equations (3) and (4) and the adiabatic approximation

$$\mathbf{v}_s(\mathbf{x}, t) = \mathbf{v}_s(\mathbf{x} - \mathbf{V}t), \tag{5}$$

where $\mathbf{V}$ is the velocity of the flux motion, comprise the starting point for our calculation. As will be shown below, the dissipation that contributes to the viscous drag concentrates in the vicinity of the core, and we therefore

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use Eq. (3) for the whole space outside the core.

When the vortex moves with velocity $\mathbf{V}$, the electric field $\mathbf{e}_s$ induced outside the core can be obtained from the second London equation[8]. For the anisotropic case this gives

$$\mathbf{e}_s = -\frac{\bar{M}}{e}\frac{\partial}{\partial t}\left(\sum_i \hat{\mathbf{x}}_i m_i v_{si}\right) = \frac{\bar{M}}{e}\mathbf{V}\cdot\nabla\left(\sum_i \hat{\mathbf{x}}_i m_i v_{si}\right), \tag{6}$$

where Eq. (5) has been used. A straightforward calculation gives

$$\mathbf{e}_s = \frac{\hbar\sqrt{m_1 m_2}}{2e(m_1 x_1^2 + m_2 x_2^2)} \times \left\{\hat{\mathbf{x}}_1\left[V_2 - \frac{2x_2(m_1 x_1 V_1 + m_2 x_2 V_2)}{m_1 x_1^2 + m_2 x_2^2}\right] + \hat{\mathbf{x}}_2\left[-V_1 + \frac{2x_1(m_1 x_1 V_1 + m_2 x_2 V_2)}{m_1 x_1^2 + m_2 x_2^2}\right]\right\}, \tag{7}$$

where $V_i$ $(i = 1, 2)$ are the components of $\mathbf{V}$. The electric field $\mathbf{e}_c$ induced inside the core is

$$\mathbf{e}_c = e_c \hat{\mathbf{x}}_3 \times \hat{V}, \tag{8}$$

where the magnitude $e_c$ is obtained by the continuity of the tangential component of the electric field at the core boundary. Note that the tangential vector of the core boundary is parallel to $\mathbf{v}_s$. Therefore $\mathbf{e}_c \cdot \mathbf{v}_s = \mathbf{e}_s \cdot \mathbf{v}_s$, and we find

$$\mathbf{e}_c = \frac{\hbar}{2e\xi_1\xi_2}(-\hat{\mathbf{x}}_1 V_2 + \hat{\mathbf{x}}_2 V_1). \tag{9}$$

Figures (1*a*), (1*b*) and (1*c*) show the stream lines of the electric field in the core vicinity for vortex moving in different directions. The stream lines inside the core are straight and uniform; outside the core they are given by the solutions of the first order differential equation $dx_2/dx_1 = e_{s2}/e_{s1}$.

The rate of dissipation per unit length induced outside the core by the motion of the vortex is

$$D_s = \int_{\frac{x_1^2}{\xi_1^2}+\frac{x_2^2}{\xi_2^2}\geq 1} d^2x\,\sigma_{ij} e_{si} e_{sj} = \frac{\phi_0 H_{c2\|3}}{4c^2}\left[\sigma_1\left(V_1^2\frac{m_1}{m_2} + V_2^2\right) + \sigma_2\left(V_1^2 + V_2^2\frac{m_2}{m_1}\right)\right], \tag{10}$$

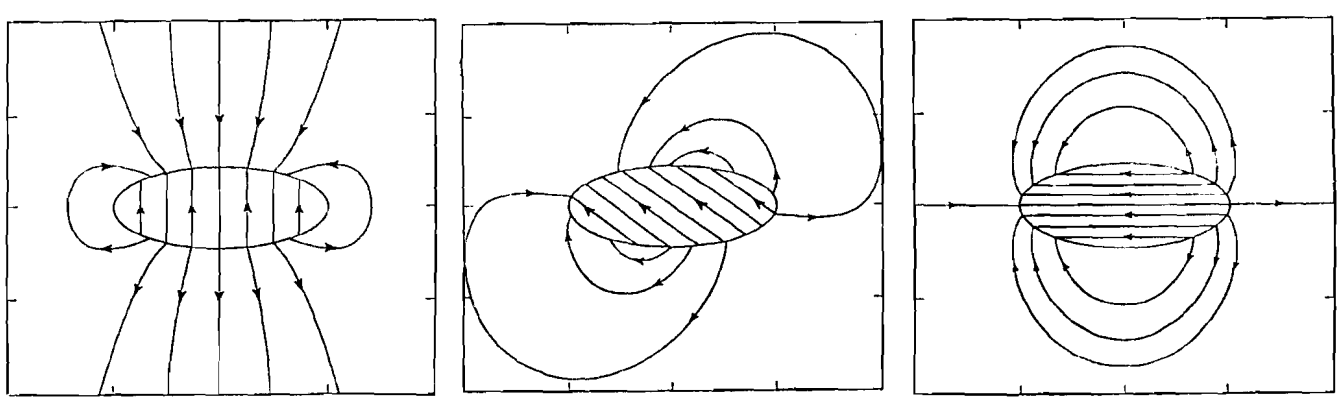
where $\sigma_{ij}$ is the normal state conductivity tensor with the principal values $\sigma_i$ $(i = 1, 2, 3)$, and

$$H_{c2\|3} = \frac{\phi_0}{2\pi\xi_1\xi_2} \tag{11}$$

is the upper critical field along the $x_3$-axis[3]. Here we have made the assumption that $\sigma_{ij}$ has the same principal axes as those of $m_{ij}$, so that $\sigma_{ij} = \sigma_i\delta_{ij}$, and we have approximated the conductivity of the normal electrons in the superconducting state by $\sigma_{ij}$, which is valid for $T$ close to $T_c$. The usual summation convention over the indices repeated twice is adopted hereafter. Note that the integrand in Eq. (10) is significant only near the edge of the core, because $e_s$ decays rapidly as the distance from the edge of the core increases. The rate of dissipation per unit length induced inside the core by the motion of the vortex is

$$D_c = \pi\xi_1\xi_2\sigma_{ij}e_{ci}e_{cj} = \frac{\phi_0 H_{c2\|3}}{2c^2}(\sigma_1 V_2^2 + \sigma_2 V_1^2). \tag{12}$$

The total rate of dissipation is



(a) (b) (c)

Figure 1. Stream lines of the electric field induced by a vortex moving along the direction forming an angle $\theta$ with respect to the horizontal, $x_1$-axis: (*a*) $\theta = 0$, (*b*) $\theta = \pi/4$ and (*c*) $\theta = \pi/2$. The vertical axis corresponds to $x_2$-axis, and the $x_3$-axis points out of the paper.

$$D = D_c + D_s. \tag{13}$$

The viscosity tensor $\eta_{ij}$ of the motion of the vortex is defined by

$$D = \eta_{ij} V_i V_j. \tag{14}$$

From Eqs.(10)–(14) we find

$$\eta_{ij} = \eta_i \delta_{ij}, \tag{15}$$

$$\eta_1 = \frac{\phi_0 H_{c2\|3}}{4c^2}\left(\frac{m_1}{m_2}\sigma_1 + 3\sigma_2\right), \tag{16}$$

$$\eta_2 = \frac{\phi_0 H_{c2\|3}}{4c^2}\left(3\sigma_1 + \frac{m_2}{m_1}\sigma_2\right). \tag{17}$$

Note that $\eta_i$ depends on both $\sigma_1$ and $\sigma_2$, because that the dissipative current has both components along the $x_1$- and the $x_2$-axes.

In the isotropic superconductors the flux-flow resistivity $\rho_f$ is related to the viscosity $\eta$ by (see Ref. 9)

$$\rho_f = \frac{\phi_0 B}{c^2 \eta}, \tag{18}$$

where $B$ is the averaged magnetic flux density. For the anisotropic case Eq. (18) is generalized as

$$\rho_{f1} = \frac{\phi_0 B}{c^2 \eta_2}, \tag{19}$$

$$\rho_{f2} = \frac{\phi_0 B}{c^2 \eta_1}, \tag{20}$$

where $\rho_{fi}$ $(i = 1, 2)$ are the principal values of the flux-flow resistivity tensor $\rho_{fij}$ (we have assumed that both $\rho_{fij}$ and $\eta_{ij}$ have the same principal axes as those of $m_{ij}$, so that $\rho_{fij} = \rho_{fi}\delta_{ij}$). Note that $\rho_{f1}$ $(\rho_{f2})$ is related to $\eta_2$ $(\eta_1)$ because that the $x_1$- $(x_2$-$)$ component of the macroscopic electric field is induced by the motion of the vortex in the $x_2$- $(x_1$-$)$ direction.

If the field is applied along the $x_1$- or the $x_2$-axes, the corresponding results can be obtained by cyclic permutation $(1 \to 2 \to 3 \to 1)$.

For the high-temperature oxide superconductors, we may neglect the anisotropy in the $ab$ plane and consider these materials as uniaxial, with much larger effective mass along the $c$-axis, *i.e.*, $m_a \approx m_b \ll m_c$ and $\sigma_a \approx \sigma_b \gg \sigma_c$, and it is generally found that the anisotropy in the electrical conductivity is much larger than that in the effective mass, *i.e.*, $\sigma_a/\sigma_c \approx \sigma_b/\sigma_c \gg m_c/m_a \approx m_c/m_b$. This is related to the fact that the high-temperature superconductors are in the clean limit as far as $ab$-plane transport properties are concerned. Using these facts we obtain the following. For the case that the vortices are parallel to the $c$-axis, flux motion is nearly isotropic, $\eta_a^{(c)}/\eta_b^{(c)} \approx 1$, where $\eta_i^{(j)}$ is the viscosity for the motion along the $i$-axis of a vortex parallel to the $j$-axis. For the case that the vortices are parallel to the $ab$-plane, however, flux motion is highly anisotropic,

$$\frac{\eta_b^{(a)}}{\eta_c^{(a)}} \approx \frac{1}{3\gamma^2}, \tag{21}$$

where

$$\gamma = \sqrt{\frac{m_c}{m_a}} \tag{22}$$

is an anisotropy parameter. We also find

$$\frac{\eta_b^{(c)}}{\eta_b^{(a)}} \approx 4\gamma, \tag{23}$$

and

$$\frac{\eta_b^{(c)}}{\eta_c^{(a)}} \approx \frac{4}{3\gamma}. \tag{24}$$

Equations (21), (23) and (24) can be combined into

$$\eta_b^{(a)} : \eta_b^{(c)} : \eta_c^{(a)} \approx 1 : 4\gamma : 3\gamma^2, \tag{25}$$

which is independent of $\sigma_{ij}$, implying that the anisotropy in the viscous flux motion is determined by the anisotropy only in the effective mass tensor.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