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Pairing Hamiltonian by a Path Integral Monte Carlo Procedure

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Abstract

A Monte Carlo approach is presented for the treatment of the pairing force in nuclear systems. Our method is computationally efficient and is very simple to implement. Numerical results are given and compared to an exact calculation and to the predictions of Bardeen-Cooper-Schrieffer (BCS) theory.

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1. Introduction

Monte Carlo methods have been used to study a variety of problems in condensed matter, nuclear, and high-energy physics. They have proven to be powerful tools for investigating the properties of quantum systems with many degrees of freedom including liquid helium [1-3], molecules [4], the electron gas [5], nuclear systems [6-10], and lattice gauge theories [11-15].

In this Letter, we propose a Monte Carlo technique which is very efficient for treating the pairing-force Hamiltonian in nuclei. Pairing forces are traditionally used to describe various properties such as the gap in the excitation spectrum of even-even nuclei, moments of inertia, even-odd mass differences, etc... In practice, the corresponding Hamiltonian is treated using the BCS method or improvements thereof, such as projection methods, the Lipkin-Nogami method, etc... [16]. It is well known that in some cases, these methods lead to significant errors. On the other hand, our Monte Carlo method makes possible an exact calculation, at least in principle. In what follows, we first introduce the model Hamiltonian, then discuss our Monte Carlo method. We close by comparing our results with exact values obtained for certain soluble systems. In practice, accuracies better than one percent are accessible within reasonable computation times. More realistic cases are presented in a second paper [17].

We consider here the case of a many-body system described by the Hamiltonian

$$H = \sum_{k>0}^{\Omega} \epsilon_k (a_k^\dagger a_k + a_{\bar{k}}^\dagger a_{\bar{k}}) - \sum_{k,k'>0}^{\Omega} G_{k,k'}\ a_{k'}^\dagger a_{\bar{k}'}^\dagger a_{\bar{k}} a_k \ ,$$

where k and $\bar{k}$ are time-reversed conjugate states with energy ϵ_k. The indices k, k' run from 1 to Ω, where Ω is the total number of conjugate orbit pairs. Our purpose is to calculate the exact N-particle ground state energy of H, without resorting to various assumptions regarding the functional form of the wave function (such as the BCS ansatz, etc...). For simplicity, this Letter treats the case of a pure pairing force of constant strength (i.e., $G_{k,k'} = G$, with $G > 0$). We have chosen this model in order to compare with exact results [18,19], and thus to check the accuracy of our Monte Carlo method. However, our approach is not restricted to G constant (see [17]).

Without loss of generality, one can assume that all particles are paired. We then work in a Fock space where the pair configurations denoted by $|C>$ constitute a basis. We define

$$|C> = |n_1, n_2, ...n_\Omega> \quad \text{with} \quad \sum_{k=1}^{\Omega} n_k = n \ , \quad n_k = 0 \ \text{ or } \ 1 \ ,$$

where n_k are the pair occupation numbers and $n = \frac{N}{2}$ is the total number of pairs.

Consider a direct calculation of the ground state energy or other observable. The ground state $|\Psi_0>$ of H can be expressed in the basis $\{|C>\}$ as

$$|\Psi_0> = \sum_C \alpha_C |C> \ ,$$

where the coefficients α_C are to be calculated to obtain the ground state energy E_0. Since the dimension of our Fock space is $\binom{\Omega}{n}$, even writing this sum is impracticable as it involves a huge number of terms for usual model spaces. It is thus appropriate to consider a Monte Carlo approach, since a sampling procedure may avoid a complete enumeration of the whole set of configurations $\{|C>\}$. Note that as the configurations $|C>$ contain the correct number of pairs n, the calculated ground state will also have exactly n pairs. This is a great advantage compared to the BCS formalism (without projection) where the number of pairs is fixed only on average which is a well known source of errors [16].

Let us apply the Hamiltonian to a given configuration $|C>$,

$$H|C>= E_C^{1-b}|C> -G \sum_{C'} |C'> ,$$

where $E_C^{1-b} = \sum_{k=1}^{\Omega} 2\epsilon_k \; n_k(C)$ denotes the one-body energy of the configuration, and the summation, corresponding to the two-body part of H, runs over all the configurations $|C'>$ obtained from $|C>$ by moving at most one pair. There are $n(\Omega - n + 1)$ such terms corresponding to all the possible pair diffusions, of which $n(\Omega - n)$ describe real diffusions (with $k \neq k'$) whereas the n remaining terms count the virtual diffusions associated to the diagonal terms ($k = k'$) of the double summation in H. Thus, separating the one-body and two-body parts, the matrix elements of H are clearly given by

$$< C'|H^{1-b}|C> = \begin{cases} E_C^{1-b} & \text{if } C = C' , \\ 0 & \text{otherwise,} \end{cases}$$
$$< C'|H^{2-b}|C> = \begin{cases} -G & \text{if } C \text{ and } C' \text{ are connected,} \\ 0 & \text{otherwise,} \end{cases}$$

where we consider C and C' as connected configurations if they differ at most by the position of one pair. It is easy to show that the components α_C of $|\Psi_0>$ are positive:

$$\begin{aligned} E_0 \cdot < \Psi_0|\Psi_0 > &=< \Psi_0|H|\Psi_0 > \\ &= \sum_C \sum_{C'} \alpha^*_{C'} \alpha_C \; < C'|H|C > \\ &= \sum_C |\alpha_C|^2 \; E_C^{1-b} \; - G \sum_{\text{connected } C,C'} \alpha^*_{C'} \alpha_C \; . \end{aligned}$$

We may impose the α_C to be real; then changing α_C to $|\alpha_C|$ can only decrease the energy E_0, since $G > 0$. Thus the ground state $|\Psi_0>$ must have its coefficients $\alpha_C \geq 0$, and a Monte Carlo sampling procedure may be applied wherein each α_C is associated to the occurrence probability of the configuration $|C>$. According to this probability interpretation, we impose here the normalization condition $\sum_C \alpha_C = 1$, slightly different from the standard $\sum_C |\alpha_C|^2 = 1$. So, the probabilities appearing in the Monte Carlo process are not the physical probabilities which are the $|\alpha_C|^2$.

2. The Monte Carlo Procedure

In order to calculate the ground state properties of the Hamiltonian of interest, we use a random walk algorithm based on the imaginary-time method (see [15] for a review). One applies the imaginary-time evolution operator $e^{-t(H-E)}$ to filter the ground state $|\Psi_0>$ from an arbitrary initial state $|\Phi>$. Then, when t is large,

$$e^{-t(H-E)}|\Phi> \sim e^{-t(E_0-E)} \ <\Psi_0|\Phi> \ |\Psi_0> ,$$

where $\hbar = 1$ and E is a normalization energy which is adjusted to keep the norm approximately constant during the time evolution of $|\Psi(t)>$. (Thus, once equilibrium is reached, $E \sim E_0$.) By reexpressing $e^{-t(H-E)}$ in terms of a product of infinitesimal evolution operators and by inserting complete sets of configurations $|C>$, one obtains

$$e^{-t(H-E)}|\Phi> = \prod_{s=1}^{N} e^{-\Delta t(H-E)}|\Phi> = \sum_{C_N} \cdots \sum_{C_0} |C_N><C_N|e^{-\Delta t(H-E)}|C_{N-1}>$$
$$\cdots <C_1|e^{-\Delta t(H-E)}|C_0><C_0|\Phi> ,$$

yielding a multiple summation representation for $|\Psi_0>$. This allows the evaluation of observables via multiple integrals (in fact summations in our discrete configuration space) which are calculated stochastically using a random walk algorithm. This procedure is called the Path Integral Monte Carlo method [15].

One essential problem is the difficulty of handling the infinitesimal evolution operator $e^{-\Delta t(H-E)}$ in our configuration space. Indeed, applying $e^{-\Delta t\ H}$ to a given configuration $|C>$ obviously gives

$$e^{-\Delta t\, H}|C> = |C> - \Delta t\ H|C> + \frac{\Delta t^2}{2!} H^2|C> + \cdots ,$$

where the second term in the r.h.s. contains configurations connected by 1 step (1 pair is moved), the third term contains configurations connected by 2 steps, etc... Thus it finally yields all the $\binom{\Omega}{n}$ other configurations. Of course one may truncate the expansion to the first-order term in Δt, just keeping the one-step diffusions, but this is a good approximation only if $\Delta t \cdot E_{max} \ll 1$ where E_{max} is the largest energy of the system. This constraint on Δt depends on the most excited states, even if one is only concerned with the ground state, so that in general the method is impracticable.

We solve this problem by doing an appropriate breakup of H in analogy with the work of Hirsch et al. [20]. We are then able to have an accurate approximation to the time evolution operator, and furthermore the method provides a simple stochastic generation of the configurations with the correct distribution. We write the Hamiltonian in the form $H = H^{1-b} + H^{2-b}$ and use the expression

$$e^{-\Delta t\ H} = e^{-\frac{\Delta t}{2}\ H^{1-b}} \cdot e^{-\Delta t\ H^{2-b}} \cdot e^{-\frac{\Delta t}{2}\ H^{1-b}} + \mathcal{O}(\Delta t^3) .$$

This is a symmetrized form of the more common breakup [15]

$$e^{-\Delta t\, H} = e^{-\Delta t\, H^{1-b}} \cdot e^{-\Delta t\, H^{2-b}} + \mathcal{O}(\Delta t^2)\ .$$

(Note that H^{1-b} and H^{2-b} do not commute.) The advantages of our breakup are: (i) the matrix elements of $e^{-\frac{\Delta t}{2}\, H^{1-b}}$ are trivial, (ii) the operator $e^{-\Delta t\, H^{2-b}}$ can be treated *exactly* (though stochastically), and (iii) the increased accuracy (one order in Δt) is obtained at essentially no cpu cost. For the one-body part, we have obviously

$$e^{-\frac{\Delta t}{2}\, H^{1-b}}|C>= e^{-\frac{\Delta t}{2}\, E_C^{1-b}}|C>\ .$$

For the two-body part, we have

$$\begin{aligned} e^{-\Delta t\, H^{2-b}}|C>= |C> &+G\Delta t \sum_{k,k'>0} a^\dagger_{k'} a^\dagger_{\bar{k}'} a_{\bar{k}} a_k |C> \\ &+\frac{(G\Delta t)^2}{2!} \sum_{k,k',k'',k'''>0} a^\dagger_{k'''} a^\dagger_{\bar{k}'''} a_{\bar{k}''} a_{k''} a^\dagger_{k'} a^\dagger_{\bar{k}'} a_{\bar{k}} a_k |C> + \cdots, \end{aligned}$$

where the successive terms in the r.h.s. correspond to the contributions of 0-, 1-, 2-steps diffusions, etc... In our method the matrix element $< C'|e^{-\Delta t\, H^{2-b}}|C >$ is not obtained directly, rather $|C'>$ is generated stochastically with the correct distribution. To do so, write

$$< C'|e^{-\Delta t\, H^{2-b}}|C >= \mathcal{N}(C) P(C', C)$$

where P is a probability distribution in $|C'>$. Remembering that there are $n(\Omega - n + 1)$ possible destinations for a one-step diffusion (including diffusions where the pair comes back to its initial orbit), one gets

$$\begin{aligned} \mathcal{N}(C) &= \sum_{C'} < C'|e^{-\Delta t\, H^{2-b}}|C > \\ &= 1 + n(\Omega - n + 1) G\Delta t + n^2(\Omega - n + 1)^2 \frac{(G\Delta t)^2}{2!} + \cdots \\ &= \exp \nu\ , \end{aligned}$$

where we have defined $\nu = n(\Omega - n + 1) G\Delta t$. The fact that $\mathcal{N}(C)$ is independent of $|C>$ is convenient though not essential for our Monte Carlo method [17]. Thus, defining the probability of going from $|C>$ to $|C'>$ as $P(C', C) =< C'| \cdot |C> / \mathcal{N}$, one finds out that the probability distribution of the number of steps L is a Poisson distribution of parameter ν:

$$\mathcal{P}(L) = \frac{\nu^L \exp(-\nu)}{L!}\ .$$

The variable ν thus represents the average number of steps per time interval Δt. The stochastic process giving rise to the transition probability $P(C', C)$ is the following : firstly, the number of steps L is chosen according to this Poisson law; secondly, for each (real or

virtual) step, one hop is chosen randomly among the $n(\Omega - n + 1)$ possible pair diffusions. This random process exactly simulates the 2-body part of the evolution operator, i.e. pair diffusion, up to a multiplicative factor.

Now, the matrix elements of the approximate time evolution operator can be written as

$$
\begin{aligned}
< C'|e^{-\frac{\Delta t}{2}(H^{1-b}-E)}e^{-\Delta t H^{2-b}}e^{-\frac{\Delta t}{2}(H^{1-b}-E)}|C > \\
&= e^{-\frac{\Delta t}{2}(E_{C'}^{1-b}-E)} \cdot e^{\nu} \cdot P(C',C)e^{-\frac{\Delta t}{2}(E_{C}^{1-b}-E)} \\
&= w(C') \cdot P(C',C) \cdot w(C) \ ,
\end{aligned}
$$

where we have defined $w(C) = e^{-\frac{\Delta t}{2}(E_C^{1-b}-E_{sen}-E)}$, and $E_{sen} = n(\Omega - n + 1)G$. This is the expression for the energy in the seniority scheme [16], which is the pairing energy for a single-shell system (when all the states are degenerate). Thus applying (our approximation of) $e^{-\Delta t(H-E)}$ to a given $|C>$ is equivalent to a random diffusion according to $P(C',C)$, followed by a multiplication by the weight $w(C')w(C)$ depending on the one-body energy of the two configuration $|C'>$ and $|C>$. This is analogous to the well-known Feynman procedure [21] related to the separation of H into a kinetic and a potential part. Here, H^{2-b} plays the role of a kinetic energy by spreading out $|\Psi(t)>$ in the configuration space, whereas H^{1-b} plays the role of a potential energy, concentrating $|\Psi(t)>$ into components with low one-body energies E^{1-b}.

Consider now the stochastic evaluation of the multiple integral appearing when calculating an overlap with a given state $|\zeta>$ (see [15]). We have

$$
\begin{aligned}
< \zeta|e^{-t(H-E)}|\Phi > \simeq \sum_{C_N} \cdots \sum_{C_0} < \zeta|C_N > \ w(C_N)\ P(C_N,C_{N-1})\ w^2(C_{N-1}) \\
\cdots w^2(C_2)\ P(C_2,C_1)\ w^2(C_1)\ P(C_1,C_0)\ w(C_0)\ < C_0|\Phi > \ .
\end{aligned}
$$

Let us take an ensemble of N_{ens} initial configurations $\{C_0^i\}$ selected according to $\Phi(C_0) = < C_0|\Phi >$ taken here as a probability distribution. The algorithm consists in evolving the configurations C_s^i according to the matrix elements $w(C_{s+1}^i)\ P(C_{s+1}^i, C_s^i)\ w(C_s^i)$ so that a new ensemble $\{C_{s+1}^i\}$ is generated. Each C_{s+1}^i is chosen in accordance with the distribution $P(C_{s+1}^i, C_s^i)$, and replicated stochastically by a factor $\tilde{w} = w(C_{s+1}^i)w(C_s^i)$. In practice, one selects a random number ξ uniformly distributed in the interval [0,1], and then C_{s+1}^i is replicated $[\tilde{w} + \xi]$ times where $[x]$ stands for the greatest integer less or equal to x (see [4]). In the limit of a large ensemble, the average number of replicates will be $\tilde{w}$, so that the evolution of the ensemble $\{C_s^i\}$ is correctly reproduced stochastically. After this random walk process, the final ensemble $\{C_N^i\}$ will be distributed according to $\prod_{s=1}^{N} w(C_s^i)\ P(C_s^i, C_{s-1}^i)\ w(C_{s-1}^i)\Phi(C_0)$, and we will have (if E is set so that the ensemble size is stable),

$$
< \zeta|\Psi_0 > \simeq \frac{1}{N_{ens}} \sum_{i=1}^{N_{ens}} < \zeta|C_N^i > \ .
$$

The ensemble $\{C_N^i\}$ represents a sample of $|\Psi_0>$ and thus each $|C>$ appears with probability α_C. The above formula enables one to estimate the α's by setting $|\zeta>=|C>$:

$$\alpha_C =< C|\Psi_0 > \simeq \frac{1}{N_{ens}} \sum_{i=1}^{N_{ens}} \delta_{C,C_N^i} ,$$

which is indeed the frequency with which the configuration $|C>$ appears in the Monte Carlo simulation.

The ground state energy can be obtained from $|\Psi_0>$ via

$$\begin{aligned} E_0 &= \sum_{C'} < C'|H|\Psi_0 > \\ &= \sum_{C,C'} \alpha_C \ < C'|H|C > \\ &= \sum_{C} \alpha_C \ [E_C^{1-b} - E_{sen}] , \end{aligned}$$

where we have used the normalization condition $\sum_C \alpha_C = 1$. Thus the energy E_0 is simply given by the average of the $[E_C^{1-b} - E_{sen}]$ weighted by the Monte Carlo sampling probabilities α_C. This formula is implemented by calculating the estimator

$$E_0 \simeq \frac{1}{N_{ens}} \sum_{i=1}^{N_{ens}} E_{C_N^i}^{1-b} - E_{sen} .$$

The value of the energy can also be determined from the normalization energy which appears in the time evolution operator. Indeed, if E_N is defined as the value of E which keeps the size of the ensemble of configurations stable, it is clearly another estimation of E_0. The agreement between these two methods constitutes a nontrivial consistency test of the Monte Carlo algorithm (see [15]). It's interesting to note that, in the case of a single-shell system, all the configurations $|C>$ are degenerate ($E_C^{1-b} = 0$) and play the same role, so that both methods clearly yield $E_0 = E_N = -E_{sen}$ as expected.

This Monte Carlo algorithm also makes possible the calculation of the exact pair occupation probabilities. Indeed, the expectation value of the pair number operator n_k is given by

$$< n_k >= \frac{< \Psi_0|a_k^\dagger a_{\bar{k}}^\dagger a_{\bar{k}} a_k|\Psi_0 >}{< \Psi_0|\Psi_0 >} = \frac{< \zeta|e^{-t(H-E)}|a_k^\dagger a_{\bar{k}}^\dagger a_{\bar{k}} a_k|\Psi_0 >}{< \zeta|e^{-t(H-E)}|\Psi_0 >} ,$$

where $|\zeta>$ is an arbitrary state, and t should be taken large enough to eliminate contributions from excited states. Using the fact that the n_k operator is local in our configuration space and taking $< C|\zeta >= 1$ for all $|C>$, we have

$$< n_k >= \frac{\sum_{C,C'} < C'|e^{-t(H-E)}|C > \ n_k(C) \ \alpha_C}{\sum_{C,C'} < C'|e^{-t(H-E)}|C > \alpha_C} .$$

This relation can be used as follows. Starting at time t_I with an ensemble of configurations distributed according to $|\Psi_0>$, apply the local operator n_k to the ensemble, i.e. weight each $|C>$ by its occupation number $n_k(C)$. Then evolve the ensemble to the time t_F. We take as estimator:

$$< n_k > \simeq \frac{1}{N_{ens}} \sum_{j=1}^{N_{ens}} n_k(C_I^{i(j)}) \ ,$$

where $n_k(C_I^{i(j)})$ is the ($\mathrm{k^{th}}$) occupation number of the configuration C_I^i, progenitor of C_F^j. Thus, it is necessary to keep track of each ensemble member's history in order to calculate the occupation probabilities.

3. Results

As an illustration, we apply our method to the case of equispaced doubly-degenerate single-particle levels. This problem has been solved exactly [18,19] for some model spaces with different values of the interaction strength reproducing typical values of the nuclear pairing energy.

In order to get good statistics for the ground state energy and pair occupation probabilities, we average our estimates of E_0 and n_k over time, after removing the initial transients so that the excited states have decayed away from the wave function. It is convenient to define a reduced time $\tau = n(\Omega - n + 1)Gt$ which has the advantage of corresponding to the average expected number of steps (pair exchanges) along the random walk of length t. Thus we average our estimators from τ_i to $\tau_i + T$ using steps of $\nu = n(\Omega - n + 1)G\ \Delta t$.

We have investigated the convergence of our estimate for the ground state energy E_0 as a function of Δt (or ν). Figure 1 shows the dependence of the measured E_0 on ν for a test case of 8 particles in 8 levels (i.e. n=4, $\Omega = 8$) with $G = 1.0$. We express energies and the pairing strength G in units of the single particle level spacing. The runs were such that statistical errors are of the same size as the symbols. We show: (a) the case for the symmetrized operator (i.e., Δt^3 breakup); (b) the case of the non-symmetrized operator (i.e., Δt^2 breakup). When going from the infinitesimal evolution operator to the finite time evolution operator, on the order of $1/\Delta t$ steps are required, so that the accuracy is reduced by one order in Δt. Thus, as confirmed by the numerics, the systematic error (due to $\Delta t \neq 0$) in the estimator of E_0 is proportional to Δt^2 in the case of the symmetrized operator while it is proportional to Δt for the non-symmetrized case. This clearly exhibits the accuracy gained when using the Δt^3 breakup, while the extra computation time is negligeable.

We show in table 1 our computed ground state energies along with the exact values [18,19] in the case of 24 particles placed in 24 equispaced single-particle levels ($n = 12$, $\Omega = 24$). Also shown are the corresponding BCS (without projection) energies. It is interesting to note that the number of configurations in this model space is on the order of 3 10^6, so that a Monte Carlo sampling is clearly appropriate. We took the following values for the parameters: $\nu = 0.3$, $N_{ens} = 10000$, $\tau_i = 250$, and $T = 3000$. Our computed energy E_0 exhibits a very good agreement with the exact calculation in the whole range of G values. (In fact in all of our runs, the results were consistent with the exact answers within

statistical errors.) The two different Monte Carlo estimations of the energy also agree very well. (We have found that they are very correlated and their difference is smaller than the statistical errors of either.) Thus we only display E_0 which is considered the better estimate of the ground state energy (see [9]). We have also indicated for reference the predictions of the BCS equations in order to demonstrate the precision gained with our approach. For both Monte Carlo and BCS calculations of E_0, the last columns show the quantities

$$Q = \frac{E - E_{MF}}{E_{exact} - E_{MF}} ,$$

where $E = E_{MC}$ or E_{BCS}, and $E_{MF} = n \cdot G$ stands for the mean-field energy (that part of the pairing energy which can be incorporated into the one-body energy). It is well known that BCS accounts only for part of the two-body pairing energy, and thus $Q_{BCS} < 1$, especially for low G. On the contrary, the Monte Carlo method gives $Q_{MC} \simeq 1$ within statistical errors.

Finally, we show in table 2 the occupation probabilities for three levels above and below the Fermi level in the case $n = 8$, $\Omega = 16$, and $G = 0.48$. For comparison, we indicate the result of our Monte Carlo computation, the exact values [18,19], and the predictions of BCS equations (i.e., v_k^2). We increased the integration time T compared to table 1 in order to reduce the statistical noise. Note that in order to implement our time averaging for the n_k, we have to keep in memory an associated n_k for each configuration of the ensemble during a given time (long enough to get relaxation). We calculate a running average of the n_k from the ensemble, using a time difference $t_F - t_I$ of 100 steps. This choice results from a compromise. For small values of $t_F - t_I$, the contributions from excited states have not sufficiently decayed, whereas for large values the statistical errors in the estimator for $< n_k >$ become important. This behavior was worsened when $n = 12$, $\Omega = 24$, so that no good compromise was found in that case.

4. Conclusions

Our Monte Carlo algorithm is very appealing for its conceptual simplicity. Pair diffusion is simulated stochastically, and the corresponding sequence of random walk processes followed by replications corresponds very well to one's intuitive view of the processes involved. In particular, it is interesting to note that the pair occupation probabilities which are simply the $v_k^2 = < BCS|a_k^\dagger a_{\bar{k}}^\dagger a_{\bar{k}} a_k|BCS >$ in the BCS formalism appear in our method as actual probabilities in the Monte Carlo process. The method is also very efficient from the computational point of view. A comparison with other methods (such as BCS with refinements, Lipkin-Nogami method, ...) and the extension to a general interaction matrix G_{ij} will be presented in a subsequent article [17].

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Table 1

G	E_{exact}	E_{MC}	E_{BCS}	Q_{MC}	Q_{BCS}
0.38	7.487	7.471 ± 0.050	5.532	0.9945	0.3321
0.42	9.083	9.010 ± 0.050	6.819	0.9819	0.4400
0.46	10.950	10.879 ± 0.050	8.390	0.9869	0.5285
0.50	13.096	13.123 ± 0.050	10.251	1.0038	0.5991
0.54	15.518	15.492 ± 0.050	12.398	0.9971	0.6548

Table 2

k	$< n_k >_{exact}$	$< n_k >_{MC}$	v_k^2
+3	0.0929	0.0926 ± 0.0010	0.0899
+2	0.1570	0.1598 ± 0.0010	0.1693
+1	0.3056	0.3050 ± 0.0010	0.3551
−1	0.6944	0.6952 ± 0.0010	0.6449
−2	0.8430	0.8430 ± 0.0010	0.8307
−3	0.9071	0.9068 ± 0.0010	0.9101

Table captions

Table 1:

Pairing energy for the case ($n = 12$, $\Omega = 24$) as a function of G. The quantity Q defined in section 3 is also presented. The energies and G are expressed in units of the single particle level spacing.

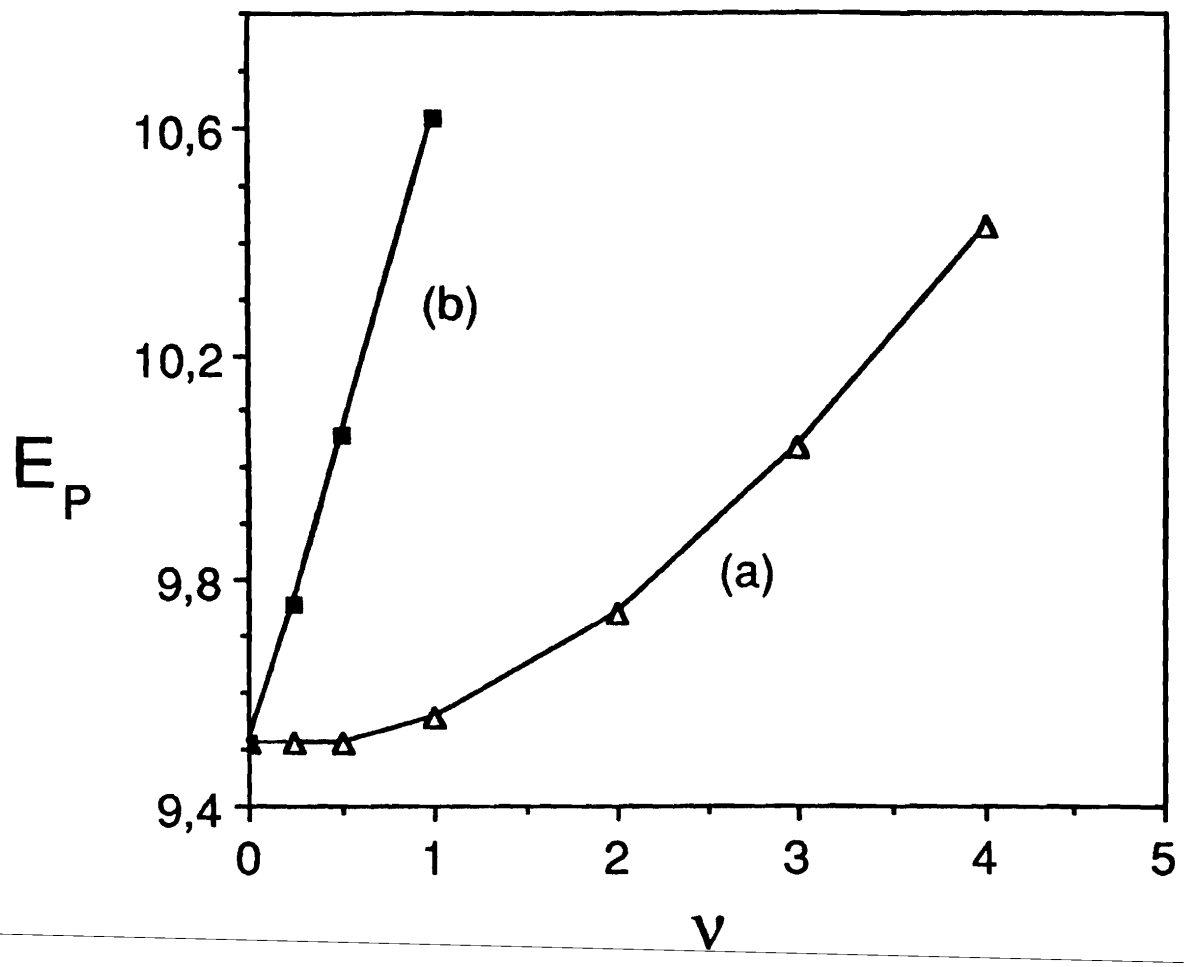
Table 2:

Pair occupation probabilities of the 3 levels above and below the Fermi level for the case ($n = 8$, $\Omega = 16$, $G = 0.48$).

Figure captions

Figure 1:

Systematic error in the pairing energy estimate E_P due to a finite Δt, as a function of ν for the case ($n = 4$, $\Omega = 8$, $G = 1.0$). (The exact value is 9.513)


10,6
10,2
9,8
9,4
E_P
(b)
(a)
0
1
2
3
4
5
ν