IP/BBSR/94-26
May '94

An Evaluation of the Binding Energy of a System of Non-Relativistic Self-Gravitating Particles

FERMILAB
AUG 1994
LIBRARY
Fermilab Library
0 1160 0037903 6

S. Mishra and D.N. Tripathy

Institute of Physics,Sachivalaya Marg, Bhubaneswar-751005

INDIA

Abstract

The ground state energy of a system of N self-gravitating particles is calculated taking product type of trial wavefunctions involving hydrogenic basis, and adopting variational procedure, for N = 3 and N = 4. Using these values, a generalized expression for the ground state energy, $E_0(N)$, of the system, valid for any N, is obtained by comparing this with the ground state energy of a free electron gas at T = 0K. The resulting expression for $E_0(N)$ is found to go as N^2 for large N, a behaviour which is considered to be most appropriate and qualitatively different from those of earlier theories. Using our data, we could also reproduce the dependence $E_0 \sim N^{7/3}$ for large N, which is the same as obtained by Levy-Leblond before for a system of self gravitating fermions, following a non-relativistic quantum mechanical calculation. As an application of the present formula, a brief discussion concerning the formation of the universe has been made.

IP-BBSR-94-26

I. Introduction

It was first pointed out by Chandrasekher and Landau [1] that a many-body system in its ground state composed of a number of self-gravitating particles is likely to undergo a gravitational collapse. Because of this, the importance of deriving the ground state energy of such a many-body system in the limit $N \to \infty$, has been recongnised for a long time. In this context, there was some interesting work done by Fisher and Ruelle [2] who had established a rigorous mathematical basis for the application of statistical mechanics to an infinite system of interacting particles. What they showed in this work was that it is impossible to define the usual thermodynamic variables of any infinite system unless the relevant forces are of saturating character. Prior to this, the proof concerning the stability of matter governed by screened Coulomb forces was given by Dyson and Lenaed [3] however, for the gravitational forces, it was knot an easy task. Around this time, there was an interesting work by Levy-Leblond [4] on a self-gravity system of particles where the forces are known to be of nonsaturating in character. In this work, for the first time both an upper and a lower bound to the ground state energy of the system were established. Leblond's result disproves the numerical result obtained by Ruffin and Bonnazzola [5] earlier, based on a non-relativistic Newtonian approximation and without going into the eqn. of state approach. In a much more recent work, Basdevant etal [6] proposed a new method to obtain a lower bound to the ground state energy of a system of self-gravitating non-relativisticv particles, which is found to be quite different from those known before. From the work by these authors, it is seen that for large N, their result for the upper bound of the ground state energy differs by within less than 14% from that obtained for the lower bound of the ground state energy, whereas for small N, the discrepancy becomes much larger.

In the present work, we have developed a new method for calculating the ground

state energy of the infinite system of self-gravitating particles. We have succeeded in calculating the ground state energies of system composed of particles upto N = 4, variationaly using hydrogenic basis. The resulting expression for energy is then generalized for any N, and the values of relevant parameters one comes across are determined after comparing it with the expression for the ground state energy of a nonrelativistic free electron gas at T = 0K. This is discussed in Sec.2 of the paper. In the last section, we have made a brief discussion of our result.

II. Calculation of the Binding Energy

Consider a system of N identical particles of mass m interacting through gravitational interactions. The Hamiltonian of the system (in the unit of $\hbar = 1$) is written as

$$H = -\sum_{i=1}^{N} \frac{\nabla_i^2}{2m} + \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} v(|\vec{X}_i - \vec{X}_j|), \tag{1}$$

where $v(|\vec{X}_i - \vec{X}_j|) = \frac{-g^2}{|\vec{X}_i - \vec{X}_j|}$, with $g^2 = Gm^2$, G being the wellknown gravitational constant. Confining to the case of N = 2, the ground state energy of the system is given to be

$$E_0(N=2) = -(\frac{1}{4})G^2m^5 \tag{2}$$

This is the energy corresponding to a positronium atom in the ground state with g^2 replaced by e^2.

Now considering the case of a 3-body system (N = 3) the Hamiltonian (1) has the form:

$$H = -\frac{\nabla_1^2}{2m} - \frac{\nabla_2^2}{2m} - \frac{\nabla_3^2}{2m} - \frac{g^2}{|\vec{X}_1 - \vec{X}_2|} - \frac{g^2}{|\vec{X}_1 - \vec{X}_3|} - \frac{g^2}{|\vec{X}_2 - \vec{X}_3|} \tag{3}$$

The above Hamiltonian, when transformed to the centre of mass and relative coordinates, assumes the form, in the centre mass frame of the three-particle system, as

$$H = \frac{\nabla_r^2}{2\mu} - \frac{\nabla_{r_0}^2}{2\mu'} - \frac{g^2}{r} - \frac{g^2}{|\vec{r}_0 + \vec{r}/2|} - \frac{g^2}{|\vec{r}_0 - \vec{r}/2|} \tag{4}$$

where $\mu = \frac{m}{2}$, $\mu' = \frac{2m}{3}$ are the reduced masses one comes across. Using (4), we evaluate the ground state energy of the system following standard variational procedure by choosing a trial wavefunction of the kind

$$\Psi(\vec{r}, \vec{r}_0) = e^{-(\lambda_1 r + \lambda_2 r_0)} \tag{5}$$

As one can notice from above, the trial wavefunction (5) is of a product type involving hydrogenic basis. It is to be mentioned here that nowhere spin is explicitely taken into consideration in our calculation. Both λ_1 and λ_2 denote the variational parameters whose values are determined by minimizing the average value of H over the state Ψ. Thus, we obtain the value of the ground state energy of the system as

$$E_0(N=3) = (-0..96708)G^2m^5, \tag{6}$$

corresponding to $\lambda_1 \simeq 0.78639$ and $\lambda_2 \simeq 0.91147$. Before we go to calculate the binding energy for the ground state of the system of particle having N = 4, which is obviously going to be a very cumbersome task, we try to re-do our calculation for the binding energy of the system for N = 3 from a different angle. For that, we recast the Hamiltonian (4) as

$$H = H_0 + H_1, \tag{7}$$

when

$$H_0 = \frac{\nabla_r^2}{2\mu} - \frac{\nabla_{r_0}^2}{2\mu'} - \frac{g^2}{r} - \frac{g^2}{r_0}, \tag{8}$$

we now approximate H_1 by writing

$$H_1 = \beta g^2(\frac{1}{r} + \frac{1}{r_0}) \tag{9}$$

The value of β is determined by taking the average of H_1 and the expression on the right side of (9) over the unperturbed eigenfunction Ψ_0 for ground state of the system,

which is written as

$$\Psi_0(\vec{r},\vec{r}_0) = Ae^{-[\frac{1}{2}r+\frac{2}{3}r_0]}. \tag{10}$$

where A is the proper normalization constant. This kind of technique has been already tested by us for the case of helium atom in the ground state [7], in which case, the interparticle potential is approximated to have the form as shown in (9). In the case of helium atom, the value of β happens to be 5/16. With this β, one reproduces the value for the effective nuclear charge $Z_{eff} = (2-\beta) = \frac{27}{16}$, which is exactly the same as obtained variationally. In the present case, it is found that β assumes a value $\beta = -0.27886$. With this β, the ground state energy of the gravitating system, consisting of 3 particles is obtained as

$$\tilde{E}_0(N=3) = (-0.95403)G^2m^5 \tag{11}$$

Thus, we find that the two result are in very close agreement with each other. This justifies the reliability of the new procedure to be followed by us hereafter for the calculation of the binding energy of a system.

We now proceed to calculate the ground state energy of the system consisting of N = 4 particles using the new method as cited above. For this case, the total Hamiltonian of the system in the centre of mass frame of the four particles, is written as

$$H = \frac{\nabla_r^2}{2\mu} - \frac{\nabla_{r_0}^2}{2\mu} - \frac{\nabla_s^2}{2\mu''} - \frac{g^2}{r} - \frac{g^2}{\vec{r}_0} - \frac{g^2}{r_s}$$
$$-\frac{g^2}{|\vec{s}+\frac{\vec{r}}{2}+\frac{\vec{r}_0}{2}|} - \frac{g^2}{|\vec{s}-\frac{\vec{r}}{2}+\frac{\vec{r}_0}{2}|} - \frac{g^2}{|\vec{s}+\frac{\vec{r}}{2}-\frac{\vec{r}_0}{2}|} - \frac{g^2}{|\vec{s}-\frac{\vec{r}}{2}-\frac{\vec{r}_0}{2}|} \tag{12}$$

where $\mu = \frac{m}{2}$, $\mu'' = m$

As before, we break up the above Hamiltonian into the unperturbed part plus the interaction part which in the present case is given as

$$H_1' = \frac{g^2}{s} - \frac{g^2}{|\vec{s}+\frac{\vec{r}}{2}-\frac{\vec{r}_0}{2}|} - \frac{g^2}{|\vec{s}-\frac{\vec{r}}{2}+\frac{\vec{r}_0}{2}|} - \frac{g^2}{|\vec{s}-\frac{\vec{r}}{2}-\frac{\vec{r}_0}{2}|} - \frac{g^2}{|\vec{s}+\frac{\vec{r}}{2}+\frac{\vec{r}_0}{2}|} \tag{13}$$

Now as before, we approximate H_1' by writing

$$H_1' = \beta g^2(\frac{1}{s} + \frac{1}{r} + \frac{1}{r_0}) \tag{14}$$

Taking the average value of the operator on both sides of (14) with respect to the unperturbed eigenfunction corresponding to the ground state of the four particle system, we find $\beta' = -0.48400$. With this β', we obtain the ground state energy of the system as

$$\tilde{E}_0'(N=4) \simeq -(2.20226)G^2m^5 \tag{15}$$

A calculation of the binding energy of this system has also been made by us using the variational procedure, which though very complicated, is trackable. This gives rise to a binding energy.

$$E_0'(N=4) = (-2.82177)G^2m^5 \tag{16}$$

It has also been found by us that an improvement of the technique used by us in the present calculation through the introduction of a parameter β can produce a better result for $\tilde{E}_0$ which is found to be closer to the variational result. Anyway, it is to be noted here that the variational result gives a value which is always lower than the value obtained by the method approximating the interaction part H_1. An evaluation of the binding energy of the system of particles with N = 5 using variational technique is almost impossible. However, one can do the calculation using the new technique proposed by us. This gives rise to a $\beta'' = -0.71286$ and the binding energy corresponding to this is found to be

$$\tilde{E}_0''(N=5) = (-4.10743)G^2m^5 \tag{17}$$

Following earlier discussions, the variational result is expected to be lower than this. It is to be further noticed that, our calculated result should constitute good upper bounds for the binding energy for the ground state energies for all N up to N = 5. This is mainly

due to our choice of trial wavefunctions using hydrogenic basis. For $N > 5$, we have not done any calculation of the binding energy. The upper bounds for the binding energies given by Basdevant et al [6] do not yield satisfactory result for small N, although for large N, they may be accepted to be reliable.

In order to obtain a compact expression for the ground state energy of the system for any N, we take into account the fact that for a system of N particles, the number of possible pairs that be formed out of N is given as N(N-1)/2. Let us now assume that the ground state energy spectrum of a system of N self gravitating particles is similar to that of a system of degenerate free electron gas. This amounts to considering the gravitating particles as fermions without explicitly taking the spin into consideration. The characteristic difference between a system of N gravitating particles and a N-electron system is that for the gravitating system of particles, the interparticle potential is always attractive, where as it is repulsive for the electron gas. Because of this attractiveness of the gravitational potential, energy spectrum of a self-gravitating system becomes bound. One may therefore, write the ground state energy of a system of N-gravitating fermions as

$$E_0 \leq -\frac{N(N-1)}{2}\left[\frac{2.21}{r_s^2} + \frac{0.9163}{r_s}\right] G^2 m^5 \tag{18}$$

where an overall negative sign refers to the fact that the total energy of the system is negative. The sign of the second term within the bracket in the above equation is supposed to be of opposite to that one encounters for a free electron gas. This is because, here, we have the interparticle potential which is attractive and the very terms account for the first order corrections. In the above equation the quantity $(G^2 m^5)$ represents the value of energy in an unit equivalent to the atomic unit. From analogy with a system of free electron gas, the expression in the right hand side (rhs) of (18) can be considered to represent the value of its ground state energy evaluated within the Hartree-Fock

approximation. Now in the rhs of (18) let us introduce a parameter ν by writing

$$\left(\frac{2.21}{r_s^2}\right) = \nu, \tag{19}$$

where the value of ν is determined by comparing the first term with the binding energy of a pair of gravitating particles each of mass m. This is equivalent to that of a polsitronium atom if g^2 is replaced by e^2. For N = 2, we find that $\nu = (\frac{1}{4})$. Thus, we write

$$E_0 \leq -\frac{N(N-1)}{8}[1 + 1.23275\alpha_N]G^2 m^5, \tag{20}$$

where an extra factor α_N has been multiplical with second term which has been done so to account for the variation with N. The value of α_N for various N are determined by comparing (20) with our calculated binding energies for $N \leq 4$. Using all these values, of α_N upto $N \leq 4$, we arrive at a generalized expression for α_N which is to be valid for any N. It is to be noted here that for N = 2 we set $\alpha_2 = 0$ and for $N \geq 3$, α_N have finite values.

For N = 3, we have found that $E_0(N=3) = (-0.96708)G^2 m^5$, the value variationally obtained by us. Comparing this with the expression on the rhs of (20), we arrive at $\alpha_3 = 0.23471$. Similarly, taking $E_0(N=4) = (-2.82177)g^2 m^5$, the variational result for N = 4, we obtain $\alpha_4 = 0.71493$. Using these two values of α we now try to express α_N by writing it as

$$\alpha_N = 3\alpha_3[1 + (\frac{N-1}{N})^\delta] \; for \; N \geq 4 \tag{21}$$

when $\alpha_3 = 0.23471$. Out of very many choices, the above form of α_N seems to be the best one. The exponent δ is determined by comparing the value from the rhs of (21) for N = 4 with that of α_4 quoted earlier. It is found that δ assumes a value of $\delta = 14.5$. In order to check how good the formula (21) for α_N, we consider the case of N = 5. This gives rise to $\alpha_5 = 0.73183$. Using this value, from (20) we obtain the ground state energy

of the system as $E_0(N=5) = (-4.7554)G^2m^5$. This compares reasonably well with the variational result for N = 5 which is supposed to be lower than $E_0 < (-4.10743)G^2m^5$. Furthermore, one can see that the very result obtained through the use of the parameter β is found to be close to the value $E_0 = (-4.3360)G^2m^5$ that follows from the generalized expression for the upper bound of E_0 given by Basdevant et al. Applying the formulae (21) and (20) to N = 6, we find $\alpha_6 = 0.75419$ and the binding energy of the system corresponding to this becomes $E_0(N=6) = (-7.23650)G^2m^5$. The corresponding value of E_0 for N = 6 obtained by Basdevant et al becomes $E_0(N=6) = (-8.130)G^2m^5$ which is lower than our present result. It is therefore apparent that as N increases beyond $N \geq 6$, the discripancies between our result and that of Basdevant et al will be increasing gradually. Anyway from all the discussions made above, it looks that our calculated values for the binding energy are quite resonable and hence this justifies the correctness of our choices of the formulae (21) and (20) for α_N and $E_0(N)$ respectively. For $N \leq 4$, we claim that our result should be considered to be more reliable than those of Basdevant et al [6] since we have used hydrogenic basis in our calculations.

Using our calculated values for the binding energy of the system consisting of particles $N \leq 4$, we have also tried to fit these numbers with a formula of the kind.

$$E_0(N) \leq A\ N(N-1)^{4/3}G^2m^5 \tag{22}$$

The value of the constant A in the above equation is fixed by comparing the ground state energy with that of a positronium atom having e^2 replaced by $g^2 = Gm^2$. which gives $A = (\frac{1}{8})$. Using this, the final expression for $E_0(N)$ becomes

$$E_0(N) \leq [\frac{N(N-1)^{4/3}}{8}]G^2m^5 \tag{23}$$

with the help of equation (23), we have calculated E_0s for $N \leq 5$, which is given in table I. From the table one finds that the values of E_0 obtained from (23) are consistently

higher then our previous set of numbers upto a certain N, beyond which they lie below those given by (20). Besides we also notice that these new results not very far off our earlier results. This shows that the formula for $E_0(N)$ given in (23) is a reasonably good one. From this, it follows that for $N \to \infty$,

$$E_0(N) \leq (\frac{1}{8}N^{7/3})G^2m^5 \tag{24}$$

This $N^{7/3}$ dependence of $E_0(N)$ for large N is also being exhibited from the work given by Levy-Leblend based on a derivation of the ground state energy of a non-relativistic quantum mechanical system of N gravitating particles treated as fermions Levy-Leblonds expression for large N differs from that shown in (24) with respect to the coefficient factor only.

While calculating $E_0(N)$ for $N \leq 4$, using variational procedure we have chosen product type of trial wave functions for averaging. Since this does not satisfy antisymmetric property, one may not correspond to a system of fermions. Since with our choice of a product type of trial wave functions, our calculated values for $E_0(N)$ fit reasonably well with a formula having a $N^{7/3}$ dependence which is the same as that follows from the calculation given by Levy-Leblend using an antisymmetric state type of wavefunction, one may therefore say that a $N^{7/3}$-dependence in perhaps realizable irrespective of whether the system of particles are bosons or fermions. Leaving aside the fact that the expression for $E_0(N)$ given by Levy-Leblond as $N^{1/3}(N-1)^2$ for finite N, since the coefficient associated with his $E_0(N)$ does not reproduce the correct binding energy of the system for N = 2 (which corresponds to two fermions of equal mass interacting through an attractive gravitational potential), the formula (23) may be considered to be a better choice for $E_0(N)$ than the one obtained by Levy-Leblond. This can be further seen from the calculated values of $E_0(N)$ for $N \leq 4$. Comparing the present formula (23) with the one given by us in (20), we feel that (20) is no doubt better because this is established

by an exact fit with the results obtained variationally.

III. Discussion of Results

The expression for the ground state energy of the system of N self-gravitating particles as given by us in (20) denotes the sum of energies of all possible pairs that can be chosen out of N and those arising out the interactions between the pairs. Therefore, even if one starts with a system of fermions, the composite objects that one has to ultimately deal with constitute an assembly of bosons. Using our result, a crude estimate of the critical mass of the boson star can be made by equating the expression for $E_0(N)$ with $-Nmc^2$, where m denotes the mass of the elementary fermion. Thus, in the limit $N \to \infty$ (20) becomes

$$E_0(N) = -[(0.342)N^2]G^2m^5, \tag{25}$$

following which we obtain

$$N \simeq (1.9)G^{-2}m^{-4} \tag{26a}$$

in the unit $\hbar = c = 1$. This is further written as

$$N \simeq 2.9(\frac{M_{plank}}{m})^4, \tag{26b}$$

where M_{plank} denotes the well-known planck mass [8] whose value is $\sim 2.2 \times 10^5 g$. Using (26b), we obtain the critical mass of the system as

$$M_c = Nm \simeq 2.9(\frac{M_{plank}}{m})^4 m \tag{26c}$$

A refinement of the above result is made by using the semirelativistic Hamiltonian of the N-particle system and the procedure followed by A. Martin [9]. With this, the mass of the ground state of the N-particle system (because c = 1) is given by

$$M_N < N(\frac{m^2}{2\mu} + \frac{1}{2}\mu) - \frac{N(N-1)}{8}[1 + 0.86802\{1 + (\frac{N-1}{N})^{\delta}\}]G^2m^4\mu \tag{27}$$

where μ is an arbitrary positive parameter minizing the above expression with respect to μ, we obtain for $N \to \infty$, $M_N = -\infty$ if $N >\sim 1.5G^{-2}m^{-4}$; that is the collapse of the system occurs if

$$N >\sim 1.5(\frac{M_{plank}}{m})^4 \tag{28}$$

Using m to be equal to 10 GeV, which corresponds to the mass of the photino (a fermion having spin 1/2), we have $(\frac{M_{plank}}{m}) \simeq 10^{18}$. This gives rise to the limiting value for $N > (\sim 1.5 \times 10^{72})$, for which $M_N = -\infty$. We now consider the situation when $N < (\sim 1.5(\frac{M_{plank}}{m})^4)$. In this case, we have for $N \to \infty$,

$$M_N\{(-0.684N^2)G^2m^4\}^{1/2} \tag{29}$$

Minizing this expression, with respect to N , we obtain

$$M_{max} < [\sim (0.6G^{-2}m^{-4})m] = 0.6(\frac{M_{plank}}{m})^4 m \tag{30}$$

This is the semi-relativistic result for the maximum value of the boson star, which can be made stable. Looking at our expression for M_{max}, we find that it is proportional to $(\frac{M_{plank}}{m})^4 m$, a behaviour which is qualitatively different from that of the result obtained by Ruffin and Bonnazzola (5) based on a relativistic calculation. For a m = 10 GeV, M_{max} assumes a value of $M_{max} = 1.2 \times 10^{49} g$. This is the maximum mass of the bosonic system (star) that can be formed out of a large number of photinos ($N_r \sim 1.5 \times 10^{72}$), beyond which the system is unstable. The photino which is considered to be the supersymmetric partner of a photon has been speculated to be a possible candidate for the presence of dark matter in the universe. An object or a star having $M_{max} \sim 1.2 \times 10^{49} g$, could most probably refer to the mass of a super cluster of galaxies or a massive black hole etc. One of the popular views on the constituents of todays universe is the existence of super clusters, cluster of galexies, galaxies and the other light emitting system, which are

directly related to the presence of dark matter in the universe. Substituting the value of $(\frac{M_{plank}}{m}) \sim 10^{18}$, Ruffin and Bonnazzola's calculation gives rise the maximum value of the mass of the boson star as $M_{max} \simeq 1.3 \times 10^{13} g$ which is much smaller than the solar mass $M_{\odot}(\sim 2 \times 10^{13})$. It is well known that if there was a boson star or a blackhole of mass $M \leq 10^{15} g$ present in the early universe, this would have by now radiated away all its mass by the Hawking process. Therefore primodial black holes with $M \geq 10^{15}$g are most likely to exist in today's universe. The value of M_{max} to be an order of $10^{49} g$, which we have reported here, is certainly a very high number since it is almost 10^{16} times the solar mass $M_{\odot}$. Such a heavy mass could be mostly associated with the formation of a supercluster of galaxies in the universe Abell and Vaucoulers [10] have even predicted superclusters of masses $10^{15} \sim 10^{17}$ $M_{\odot}$ and from their studies it has been estimated that 90% of the galaxies belong to clusters and supercluster. Assuming some of the galaxies to be spiral in structure, there have been calculations of the masses of such galaxies using virial theorem and the measured data on their rotational velocities. From those calculations, masses of the order of $10^{9} \sim 10^{12}$ $M_{\odot}$ have been reported. The famous Andromida galaxy, known as M31, has been found to have a mass of $\sim 10^{7} M_{\odot}$. A small quadrupole anisotropy that has been recently observed in the cosmic microwave back ground radiation [11] is expected to be due to the existence of cluster of galaxies in the universe.

Using our calculated values for the mass of the supercluster which gives $M \sim 0.6 \times 10^{16} M_{\odot}$ and assuming that it is globular in structure, we have estimated the density ρ_0 of the cluster by choosing the radius R of the cluster to be roughly equal to 20 mpc (1 mpc $= 3.086 \times 10^{24} cm$). This gives $\rho_0 \sim 1.015 \times 10^{-29} g/cm^3$. This may be considered to be the mean density of the universe, if one assumes that the cosmic matter is mainly constituted of the so the called globular superclusters which refer to the dark matter. An estimation

of the critical density [12] ρ_c has been made using the formula $\rho_c = H_0^2[(\frac{8\pi}{3})G]$, choosing the best value of the Hubble constant known at present $H_0 = 150/(Sec \times 10^7 light\ year)$, one finds that $\rho_c \sim 5 \times 10^{-30} g/cm^3$. Since from the present calculation we find $\rho_c > \rho_0$, from this one may infer that the universe is finite, closed and eventually collapsing. However, this seems to be contrading the mostly common belief that the universe is infinite, open and ever expanding.

Finally, we want to see whether there is any possibility of formation of a massive black hole by the collapse of an object having $M_m \sim 1.2 \times 10^{49}$g. From General Theory of relativity, it follows that if a massive star has to form a black hole its Schwarzchild radius Rs [11] has to be $Rs \simeq 1.8 \times 10^{21} cm$. The question now arises, can one think of such a radius ? In order to answer this, let us consider the Bohr radius a_0 of a pair of gravitating particles of mass m, which is given as $a_0 = \frac{\hbar^2}{\mu g^2} = 8.4 \times 10^{20} cm$. Assuming that one has a system consisting of a large number of particles ($N = 1.5 \times 10^{72}$) at T = OK, one can write, in analogy with a system of electron gas, the interparticle separation r_0 between a pair of particles as $r_0 = a_0 r_s$, where r_s is a dimensionless parameter which can be thought of to be propertional to [number density]$^{-1/3}$ of the particles in the system. The Schwarzchild radius can be calculated using the relation

$$\frac{4\pi Rs^3}{3} = \frac{4\pi}{3} r_0^3 (\frac{N}{2}) \tag{31a}$$

where r_0 is the average interparticle separation within the medium and the factor of 2 comes because of pairing.

From this we obtain

$$Rs = r_0 (\frac{N}{2})^{1/3} \tag{31b}$$

Taking $N \sim 1.5 \times 10^{72}$, one finds

$$R_s \sim 1.8 \times 10^{21}$$

for a $r_0 \simeq 2 \times 10^{-3}$

Corresponding to this r_s, the value of r_s becomes $r_s \sim 0.24 \times 10^{-23}$, which is obviously much less than unity. Therefore, in analogy with a system of free electron gas this could refer to a very high dense system of self gravitating fermions. All these discussions made in this section are seen to strictly follow from our expression for the binding energy of a system of N gravitating fermions given in (20) which for $N \to \infty$, behaves as N^2.

References

[1] S. Chandrasekhar, Monthly Notices Roy. Astron Sco. 91 (1931) 456; L. Landau, Phys. Z. USSR 1 (1932) 285.

[2] M.E. Fischer and D. Rulle, J. Math. Phys. 7 (1966) 260.

[3] F.J. Dyson and A. Lenord, J. Math. Phys. 8 (1967) 423.

[4] J.M. Levy-Leblond, J. Math. Phys. 10 (1969) 806.

[5] R. Ruffirn and S. Bonnazzola, Phys. Rev. 187 (1969) 1767.

[6] J. L. Basdevant, A. Martin and J.M. Richard, Nucl. Phys. B343 (1990) 60.

[7] D.N. Tripathy in Abstracts of contributed papers, Vol-II 18th International Conference on the physics of electronic and atomic collisions, AARHUS University. Denmark (1993), ed. T. Anderson et al.

[8] Stuart L. Shapiro and Saul A. Teukosky, Black Holes, Write Dwarfs and Neutron stars (John Wiley and Sons, Inc. New York, 1983) p.366. 5.

[9] A. Martin, Phys. Lett. B 214 (1988) 561.

[10] G.O. Abell, Ap. J. Suppl 3 (1958) 211.
G. de Vaucouleurs, Science, 167 (1970) 1203.

[11] G. Contopoulos and D. Kotsakis, Cosmology (Springer - Verlag, Berlin Heidelberg, 1987) P.71, 93.

[12] Fang Li Zhi and Lishuxian, Creation of the Universe (World Scientific, Singapore, 1989) p.61.

Table Caption

Table I Binding energy of the system for different values of N measured in the unit of G^2m^5.

NAME	I.D.No.	M.S.

RETURN TO FERMILAB LIBRARY

TABLE 1

N	$E_0[from(23)]^*$	$E_0[from(20)]^*$
2	-0.25	-0.25
3	-0.94494	-0.96708
4	-2.16337	-2.82177
5	-3.96850	-4.10743
10	-23.40094	-11.25001