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Effects of interfragment nuclear interaction and statistical approach to multifragmentation phenomena on

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Abstract: Nuclear and Coulomb mean fields seen by a fragment, in the assembly of interacting nuclear species, are calculated following a statistical method, and used in a grand canonical model to describe the multifragmentation process. Nuclear mean field enhances the yield of neutron and other light mass fragments dramatically, without significantly affecting the yields of relatively heavier fragments. It enhances the multiplicities of these particles in a big way, and hence provides an additional mechanism, other than dynamical and deformation effects usually assumed to explain the data. Nuclear interaction is found to lower the temperature to a physically accepted value. The role of Coulomb interaction in producing a minimum in the mass yield distribution, which has been questioned recently, is reaffirmed by extensive calculations.

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After the advent of heavy-ion reactions, it has been possible to produce hot nuclei in the laboratory and study their properties. With this, a new frontier in nuclear physics has opened up. Several questions like, what is the maximum temperature upto which a nucleus can be heated, how does a very hot nucleus decay, and what are the properties of hot nuclear matter etc.. remain to be answered satisfactorily. This has become an area of intense investigation in recent years. A hot nucleus can be produced, when a high energy proton strikes a heavy-ion. At low temperature (< 1 or 2 MeV), a heavy hot nucleus undergoes binary fission, which has been well studied over the years. At higher temperature of 4 or 5 MeV, the nucleus decays into many fragments. It is not yet clearly established[1], whether the large number of fragments one observes in this process is the end product of a series of binary decays or it is a one step process in which, the nucleus breaks up into many fragments simultaneously. The latter mechanism is usually termed as multifragmentation. During the last decade, in the high energy proton induced heavy-ion reactions, the decay of the hot heavy nuclei has been assumed to be of this nature and accordingly many models have been proposed. Due to the complexity of the problem arising out of the interplay of the large number of degrees of freedom, descriptions based on statistical approach[2-8] have been usually attempted. The general picture of this process is that a high energy proton strikes a target nucleus and passes through it imparting some energy. The targt gets excited and also some what expanded with imparted energy, and decays statistically into various fragments, solely governed by available phase space. The assembly of fragments acquired statistical equilibrium with temperature $k_\beta T$ and volume V, corresponding to a subnuclear density. If the interfragment interactions are ignored, then the complexity is greatly reduced and one obtains a simple tractable one-body picture in the grand canonical model. Gross and collaborators[2] were the first to recognize the importance of the interfragment Coulomb interaction in the dynamics of multifragmentation phenomena

and incorporated it, into the frame work of grand canonical model, here after referred to as I and showed through the practical application, that the a minimum in the mass yields, the isotopic yields etc.. observed in the experiments can be described. Since interfragment nuclear interaction is short range and more complex, it was generally ignored, with the hope that it would not bu important. However, lately, attempts have been made to take it into account in several model studies using canonical and microcanonical ensembles. Koonin and Randrup[9], in their microcanonical model, have attempted to take into account the nuclear interfragment interaction by idealizing the fragments as hard spheres. They arrived at an ordered crystaline state, which was regarded as unphysical. Randrup et al.[10] have examined various approximations with a view to incorporate the inter fragment interaction effectively. Satpathy et al.[11] have also attempted to incorporate the interfragment nuclear interaction in a phenomenological manner with the meagre qualitative success. Recently we have developed a statistical approach to calculate the mean field originating from both the Coulomb and the nuclear interfragement interactions and used it in the framework of grand canonical model. A preliminary report showing the practical application of this approach to experimental mass yield distribution in some reactions has appeared elsewhere[12]. A significant result obtained there by the introduction of the nuclear interaction in the model, which is successful in describing Purdue-Fermilab data[13] on $p + Kr$ and $p + Xe$ reactions with a lower temperature of about 6 MeV.

An important way to study the properties of hot nuclei is to detect the evaporated neutrons and other light nuclei. Recently, these measurements[14-16] are being carried out in various laboratories. This provides a good prospect for, not only, unraveling the mechanism of the process, but also, for testing the various presently available models. Analysis of such data carried out in the frame work of the usual statistical models show that these models are inadequate to describe them. It has been generally felt[14] that the dynamical effects, and deformation degrees of freedom are likely to play important role in

the emission of these particles. It is also believed by many workers [17,18] in this fi that the light particles are produced in a separate mechanism called cleavage or shatteri where a part of the target nucleus somehow shot away, which decays into the observed li fragments. In view of this, in the present paper, we would like to investigate in det the role of nuclear interaction in the production of the light particles. In our prelimin study[12], we have already observed significant enhancement of the neutron cross-sect because of the introduction of the nuclear interaction. In the present study we have furtl improved the meanfield by specifically taking the neutron, proton, deuteron, triton a helion optical potentials as their intefragment nuclear interaction rather than the proxim potential[19] taken earlier.

Recently, Deangelis et al.[20] have critically examined the role of Coulomb interacti in producing a minimum in the mass yield distribution, in a model based on the principle minimal information similar to that of Gross et al.[2]. As stated earlier, Gross and colla orators have demonstrated the essentiality of the Coulomb interaction in the production 'U'-shaped mass yield which has been widely accepted. Deangelis et al.[20] have tested th assertion and have concluded that it is not necessary to include the Coulomb energy in ord to produce a minimum in the mass yields. We reexamine this issue and find that in the analysis they have not taken the self Coulomb energy into account, which is an essenti component in the binding energy of a fragment. When one switches off the interfragme Coulomb interaction and retains the self Coulomb energy of the fragment, the minimu in the mass yields disappears. A further examination of this issue is carried on with o improved statistical model including the nuclear interaction.

In section.2, we have presented the outline of the multifragmentation model with a account of the statistical method for the evaluation of the Coulomb and the nuclear mea fields. Section.3 contains application of our improved model to various experimental dat In section.4, we have investigated the role of the Coulomb and the nuclear interactio

in producing a minimum in the mass yield distribution. Summary and conclusions are presented in section.5.

2. Multifragmentation model and the mean field

For completeness and clarity of this paper, we recount the essential equations of this model here. We follow the grand canonical model proposed earlier in I, in collaboration with Gross et al.. Our notation is similar to that of references[2,21]. We consider an assembly of fragments at statistical equlibrium with temperature $k_\beta T$, interacting through both the nuclear and Coulomb interaction. In this model, the logarithm of the grand partition function ξ, in terms of the proton and neutron chemical potentials μ_p, μ_n and the inverse temperature $\beta = 1/k_\beta T$ is obtained as

$$ln\xi(\mu_n, \mu_p, \beta) = \sum_i W_i(\mu_n, \mu_p, \beta) \tag{1}$$

where

$$W_i(\mu_n, \mu_p, \beta) = \left(\frac{MA_i}{2\pi\hbar^2\beta}\right)^{3/2} \left(\frac{4\pi r_0^3 A}{3}\right) \phi_i(\beta) e^{-\beta\{fA - B_i + \overline{C}_i + \overline{V}_i - \mu_n(A_i - Z_i) - \mu_p Z_i\}} \tag{2}$$

Here $\frac{4\pi r_0^3 A}{3}$ is the volume and f is the binding energy per nucleon of the target. B_i is the binding energy of the fragment nucleus i. $\overline{C}_i$ and $\overline{V}_i$ are respectively the mean Coulomb and nuclear interaction seen by the fragment i in the assembly of other fragments. $\phi_i(\beta)$ is the internal partition function of the excited but particle stable states of fragment i.

The mean potential $\overline{C}_i$ and $\overline{V}_i$ are the most difficult quantities to be computed. In reference[2], only $\overline{C}_i$ was calculated in a self consistent manner in a model in which, the interacting pair was assumed to touch one another. In that calculation, it was neccessary to have the Coulomb radius constant r_{oc} as a free parameter, whose value was taken to be 2.36 fm. In the present study as described in the following, we have developed a statistical method to calculate both $\overline{C}_i$ and $\overline{V}_i$, in which the necessity of using the parameter r_{oc} does

not arise. For the fragment i, the Coulomb and nuclear potentials $C_i(r)$ and $V_i(r)$, in terms of fragment-fragment interaction can be written as

$$C_i(r_i) = \sum_j \frac{e^2 Z_i Z_j}{|\mathbf{r}_i - \mathbf{r}_j|} \tag{3a}$$

$$V_i(r_i) = \sum_j V(|\mathbf{r}_i - \mathbf{r}_j|) \tag{3b}$$

where $V(|\mathbf{r}_i - \mathbf{r}_j|)$ is the heavy-ion potential between the fragment i and j with charge Z_i and Z_j, located at coordinates $\mathbf{r}_i$ and $\mathbf{r}_j$ respectively. The evaluation of the above two quantities is quite complex, because these depend upon the locations of the fragments j, which continuously go on changing. Further, $C_i(r_i)$ and $V_i(r_i)$ depend upon the location of the fragment i inside the assembly and as such can not be used in the expression for W_i. One has to compute the appropriate mean potentials $\overline{C}_i$ and $\overline{V}_i$, which a fragment sees and it should be independent of the position of the fragment inside the assembly. We have used a well defined statistical approach to evaluate $\overline{C}_i$ and $\overline{V}_i$, given below.

We view the interaction of the fragment i, with all other fragments to be equivalent to its interaction with all the rest put together as a single fragment called the complementary fragment C. Thus if the fragment i has mass number A_i and the charge number Z_i, then the complementary fragment will have the mass number $A - A_i$ and charge number $Z - Z_i$. The system is considered to be in a heat bath at temperature $1/\beta$. The probability $P(R)$ of the location of the fragment i relative to the complementary fragment C at a distance R will be given by

$$P(R) = \frac{e^{-\beta\{\frac{p_i^2}{2m_i} + \frac{p_c^2}{2m_c} + U(R)\}}}{\int\int\int\int d^3r_i d^3r_c d^3p_i d^3p_c e^{-\beta\{\frac{p_i^2}{2m_i} + \frac{p_c^2}{2m_c} + U(R)\}}}$$

where $\mathbf{p}_i$, $\mathbf{p}_c$ are the momenta and $\mathbf{r}_i$, $\mathbf{r}_c$ are the coordinates of the fragment i and complementary fragment C respectively and $R = |\mathbf{r}_i - \mathbf{r}_c|$. $U(R)$ is the total fragment

interaction comprising of Coulomb and nuclear parts. Hence the mean potential energy $\overline{U}$ for the fragment i is given by

$$\begin{aligned}\overline{U} &= \int\int\int\int U(R)P(R)d^3r_i d^3r_c d^3p_i d^3p_c \\ &= \frac{\int e^{-\beta U(R)}U(R)d^3R}{\int e^{-\beta U(R)}d^3R}\end{aligned} \tag{4}$$

The integration in above equation is over all possible values of R starting from the point corresponding to the touching configuration of two fragments and covering the entire volume of assembly.

3. Application to various reactions

We have applied the above model to study the various proton induced heavy-ion reactions. In this model, there is only one parameter, namely, the temperature of the decaying target. In order to describe the mass yield distribution for widely varying target masses, ranging from Cu to Au, it was necessary to use different break-up radius constant r_o. The values of r_o quoted in our previous study[12] was somewhat inaccurate, which we have now corrected. In the multifragmentation process, the mass yield comprises all types of fragments ranging from pure nucleons to heavy fragments, with mass number close to that of the target. It is desirable to have a potential, which should be realistic enough to describe the interactions between all possible combination of fragments in the assembly. The Coulomb interaction between any two fragments is well defined and is given by the equation (3a), which has been used in our calculation. As per the nuclear part of the interfragment interaction, one has to make a choice. The proximity potential[19] is a widely used heavy-ion potential which we have choosen to use in our previous[12] and also present studies. This potential is generally quite good for heavy fragments. It is unlikely to be accurate for very light fragments. In our treatment, lighter a fragment is, the heaviour becomes the corresponding complimentary fragment. In reference[12], we have taken proximity potential for all combinations of fragments occuring in our model. We have demonstrated its goodness in

the case of a very small fragment like deuteron by comparing it with the actual optical p tential. To examine this point again, we have plotted in figure 1 the actual optical potent and the proximity potential between deuteron and ^{108}Ag as solid and dashed lines respe tively. The parameters used for the optical potential are the set A in the table 3 of Per and Perry[22]. The arrow indicates the point corresponding to the touching configurati and the region from this point onwards, is the relavent region. In our previous study[1 we had used the proximity potential for all fragments, including the neutrons and the pr tons. Undoubtedly, this has given rise to a lot of simplifications in our calculations. No realizing the fact that the mass yields of light fragments especially of neutrons is of cruci importance, both from the theoretical and experimental points of view, we would like to s if the proximity potential is reasonable to represent the interactions in case of the neutro and the protons. In figure 2(a) and 2(b), the exact optical potential (solid lines) and t proximity potential (dashed lines) for the neutron and the proton with ^{108}Ag respective are shown. The parameters of the optical potentials are taken from the reference[23]. comparision between the figure 1 and figure 2 shows that the descripancy between the op cal potential and the corresponding proximity potential has been substancially reduced f deuteron compared to that of neutron or proton. Still it is about 30% at the touching poi in the case of deuteron. Therefore, to improve the accuracy in the present study, we ha taken the actual optical potential for neutron, proton, deuteron, triton and helion, and t proximity potential for all other fragments. The parameters of the optical potentials f triton and helion are taken from the reference[24].

3.1 Study of the mean field

Taking the above interfragment interactions, we calculate the nuclear and the Coulon meanfield $\overline{V}_i$ and $\overline{C}_i$ for the fragment i using the expressions given in equation (4). As typical example, we have presented in figure 3, $\overline{V}_i$, $\overline{C}_i$ and $\overline{V}_i + \overline{C}_i$ as dotted, dashed a solid curves respectively for the β-stable fragments in the $p + Ag$ reaction for $k_B T$=3 Me

For the fragment mass number $A = 1$, we have plotted the potentials for neutron. Since neutron does not have any Coulomb interaction, it sees only the nuclear attraction, which is about -5.8 MeV in the present case. Because of this, its cross section will be dramatically enhanced by a factor $e^{-\beta \overline{V}_i} = 6.9$ in the present calculation compared to those in which the nuclear interaction is not taken into account. This is quite evident from the exponential part of the expression for W_i in equation (2). To show this effect in the cases of other light particles, we have plotted the variation of $\overline{V}_i$, $\overline{C}_i$ and $\overline{V}_i + \overline{C}_i$ with mass number A, for the systems with $N = 1$, $N = 2$, $Z = 1$ and $Z = 2$ in figures 4(a), 4(b), 5(a) and 5(b) respectively. It can be seen that for $Z = 1$, the total interaction changes sign from repulsive to attractive as A increases from 1 to 3, rendering the enhancement of cross section for heavior isotopes. Same trend is seen for the system $Z = 2$. For the cases with $N = 1$ and $N = 2$ systems, the trend of the total mean field is just opposite to that of the $Z = 1$ and $Z = 2$ system. While in the former, the meanfield increases with the rise of mass number A, in the later it decreases. This is due to the progressive enhancement of the Coulomb interaction, which dominates the total interaction. In the former cases of $Z = 1$ and $Z = 2$, the Coulomb interaction remains nearly constant even though the mass number increases. So the nuclear meanfield governs the variation of the total mean field with the increase of the mass number and influences the relative yields of the various isotopes significantly. In fact, for $Z = 2$ system, as one moves from ^4He to ^9He, the binding energy varies within 2 MeV, while the nuclear meanfield varies from -9.5 to -17.5 MeV. Thus, in this case, the yields of various isotopes are governed more decisively by the nuclear interaction. This calculation predicts that in the case of He, the yield will increase with the increase of the mass of the isotopes, which could be checked experimentally. This trend will progressively become weaker with increase of Z because, for the heavier fragments with larger Z values, the nuclear interaction is relatively much smaller, compared to binding energies and hence, it will not influence the yield that effectively, as in the case of lighter ones. In the following,

we have done a model calculation, to see the influence of the interaction on the mass yield.

3.2 Calculations of mass yields

Before we embark upon the practical application of this model to describe specific data, we would like to make some model study to see the effect of the nuclear interaction on the fragmentation dynamics. For this purpose we have chosen $p + Ag$ reaction. We have performed calculations of mass yield for $k_\beta T = 3$ MeV, first with Coulomb interaction only, and then, with Coulomb plus nuclear interaction. The corresponding results are presented in figure 6 by dashed and solid histograms. It may be seen that the cross sections for light mass fragments are dramatically enhanced by the factors ~ 7 and ~ 37 in the mass bean $1 - 4$ and $4 - 8$ respectively due to the nuclear interaction. Of special significance is the cross section of the neutron, which has been enhanced by a factor ~ 6. For heavy mass fragments, close to the target mass, the cross sections are somewhat decreased due to the nuclear interaction. Thus, the nuclear interaction has the effect of enhancing the light mass cross sections in a very impressive manner, with slight lowering for the cross section of heavy fragments. This is quite understandable as stated earlier, from the expression of W_i, where for smaller fragments, the interaction $\overline{V}_i$ is a dominating term in the exponential [equation (2)] compared to the binding energy, which is not the case for heavy fragments. Thus the nuclear interaction has the effect of enhancing the cross-section of lighter fragments without seriously affecting the cross-section of heavier ones.

We would like to make another model study, to see how does the mass yields varies with the change of the temperatures. In figure 7, we have shown the mass yields for $p + Ag$ reactions at $k_\beta T = 2$, 4.5 and 7 MeV. The solid, dashed and dotted histograms represent the corresponding mass yield distributions. We find a shoulder like distribution for the lower temperature $k_\beta T = 2$ MeV, the 'U'-shape for the medium temperature $k_\beta T = 4.5$ MeV and the exponential shape for the high temperature $k_\beta T = 7$ MeV which are similar to the trends seen by Gross et al. in their earlier work[2] with parameterised Coulomb

interaction. The only difference is the enhancement of the cross-section for light fragments, which is a very wellcome feature. Thus, it is reassuring that the introduction of a new element into the dynamics like nuclear force which is quite opposite in nature compared to the Coulomb interaction usually taken into account in the description, has the desirable physical behaviour.

We have now applied the present model to several proton induced reactions, ranging from the low mass ^{63}Cu to the high mass ^{197}Au. In figures 8, 9, 10 and 11, the solid histogram represents the mass yield distribution, obtained in our present model for $p+Cu$, $p+Ag$, $p+Ta$ and $p+Au$ reactions respectively. The solid dots are the experimental data taken from the respective references[25-28]. The break up radius constant r_o is taken as 1.85 fm for $p+Cu$ and $p+Ag$ reactions involving relatively light nuclei, and 3.35 fm for $p+Ta$ and $p+Au$ reactions, involving the heavy nuclei. The respective temperatures are 4.05, 5.11, 3.95 and 3.92 MeV. In all these four cases, our mass yield distributions compare well with the experimental data. These results can be compared with the Monte-Carlo calculations carried out in the frame work of microcanonical ensemble by Ban-Hao et al. [29]. They have taken the Coulomb interaction of the fragments only, and have treated the neutron channel separately. In general, we predict much higher cross-sections for the light fragments in all the four cases. In fact, in the mass bean 1-4, the yields obtained in our calculation are about 10 times higher than theirs for $p+Ag$, $p+Ta$ and $p+Au$ reactions. The data on the cross-sections for the light mass fragments are not yet available and the interest for such measurements has been growing recently.

In our preliminary report[12] we had applied the model to the description of the Purdue-Fermilab data[13] on $p+Kr$ and $p+Xe$ reactions and had got encouraging results. Since we have improved the treatment of nuclear interaction by specially incorporating the exact optical potential for the neutron and proton, we would like to recalculate the mass yield for these reactions. The break up radius constant r_o is taken as 1.85 fm and 2.75 fm for

$p+Kr$ and $p+Xe$ reactions respectively. We get a reasonably good fit with the dat which are presented in figures 12 and 13 for the $p+Kr$ and $p+Xe$ reactions respectivel The fits are obtained for temperatures $k_BT = 6.6$ and 6.225 MeV respectively. The dash curves connecting open circles represent the present calculation while the filled circles a the experimental data and the solid curves correspond to the description of the therm liquid drop model of Hirsch et al.[30]. It can be seen that the quality of the fit of our resul with experiment is quite reasonable. As expected, the fit has improved considerably f the light mass fragments. To see the effect of nuclear interaction, we calculate the ma yield for $p+Kr$ reaction, taking into account only the Coulomb part of interfragme interaction. We find that to get a somewhat reasonable fit, a much higher temperatu of $k_BT = 9$ MeV is necessary. This is shown in figure 14. We have reported in ou previous work [21] that, a still higher temperature of more than 10 MeV was necessar for a comparable fit with these data using earlier model of Gross et al. in I. Thus th inclusion of the nuclear interaction reduces the temperature from a highly unphysical valu of about 10 MeV to 6 MeV, which is in accord with the experiments[31]. The highes temperature of a thermally equilibrated compound nucleus has been found to be aroun 5-6 MeV. The Coulomb interaction being repulsive in nature, will make the fragments i the assembly, fly away from one another increasing their kinetic energies, and consequentl raising the temperature. The nuclear interaction being attractive in nature will act in th opposite direction and lower the temperature. This clearly brings out the role of nuclea interaction in the dynamics of the multifragmentation process. Ban-Hao and Gross hav attempted to fit the same data of $p+Kr$ and $p+Xe$ reactions with lower temperature o 4.65 and 4.53 MeV respectively in their Monte-Carlo calculations[29] in the frame work o microcanonical ensemble, without taking the nuclear interaction. But they got 'U'-shap mass yield distributions for both the cases.

The break up density enters explicitely in the calculation of multiplicities of the emitte

fragments. It gives spatial extension of the hot system before it undergoes multifragmentation. This quantity has an important bearing on the deexcitation mechanism. Since it is not known, it is usually treated as a parameter to be determined from the fit of the experimental mass yields. In our calculation we have taken break up radius constant r_o = 1.85 fm for $p+Cu$, $p+Kr$ and $p+Ag$ reactions, r_o = 2.75 fm for $p+Xe$ reaction and r_o = 3.35 fm for $p+Ta$ and $p+Au$ reactions, which correspond to the densities $\rho_o/4.5$, $\rho_o/15$ and $\rho_o/26$ respectively, where $\rho_o = 0.17$ fm^{-3}, is the normal nuclear matter density. Ban-Hao and Gross have used the break up density $\rho_o/6$ to $\rho_o/8$ in their calculations[29] for mass yield distributions in various proton induced reactions. Barz et al.[1] have analysed the two particle velocity correlation data of intermediate mass fragments for the $^{18}O+^{nat}Ag$ reaction at E/A = 84 MeV, to throw some light on the decay mechanism. They have concluded that for a highly excited nucleus to undergo instantaneous multifragmentation rather than sequential binary decay, a low break up density of about $\rho_o/10$ is necessary for this system. In a recent study of the mechanism of the multifragmentation process, Hagel et al. have analysed the decay mode of $^{40}Ca+^{40}Ca$ reaction at bombarding energy of 35 MeV/nucleon, using various models[32]. From a comparative analysis of the data, they have found that, a low density of $\rho_o/6$ is required in the model of Ban-Hao and Gross[29] to get the best fit with the experimental data. They have concluded that the data implies the deexcitation from a hot and very expanded system. It is indeed satisfying that our present study points clearly, the similar low break up density for multifragmentation process in the low and medium mass region. We feel, the break up density will decrease with the increase of mass number of the system.

It is particularly important to find the relative importance of nuclear interaction and Coulomb interaction in various systems, when the break up densities are low. The average nuclear meanfield and the average Coulomb meanfield seen by a fragment in the assembly

can be defined as

$$\overline{V}_{av} = \frac{\sum_i W_i \overline{V}_i}{\sum_i W_i}$$

and

$$\overline{C}_{av} = \frac{\sum_i W_i \overline{C}_i}{\sum_i W_i}$$

where $\overline{V}_i$ and $\overline{C}_i$ are the mean nuclear and Coulomb potentials defined above and the W_i is mean multiplicity given by equation 2. These quantities may provide a measure of their relative strengths. We have calculated these quantities for all the reactions, using the respective $\overline{V}_i$, $\overline{C}_i$ and W_i. As representative cases, we present here the results for $p+Ag$, $p+Xe$ and $p+Au$ for which the break up densities are $\rho_o/4.5$, $\rho_o/15$ and $\rho_o/26$, respectively. For $p+Ag$ reaction, $\overline{C}_{av}$ = 18.19 MeV and $\overline{V}_{av}$ = -10.69 MeV, for $p+Xe$ reaction, $\overline{C}_{av}$ = 10.31 MeV and $\overline{V}_{av}$ = -2.0 MeV and for $p+Au$ reaction, $\overline{C}_{av}$ = 10.49 MeV and $\overline{V}_{av}$ = -0.85 MeV. While for $p+Ag$ reaction, the magnitude of the $\overline{V}_{av}$ is 58% of the $\overline{C}_{av}$, it is only 19% and 8% for $p+Xe$ and $p+Au$ reactions, respectively. Thus with the decrease of the break up density, the relative importance of the nuclear interaction progressively weakened. However, even though for $p+Au$ reaction, it is only 8%, still it is not negligibly small. We would like to emphasize here that although on the average, the contribution from nuclear interaction is small, still it has significant effect on the relative yield for the various isobars, for a given mass number. For example, in the $p+Au$ reaction, for the mass number A = 158, the two isobars with Z = 70 and 74, the $\overline{C}_i$ and $\overline{V}_i$ are 50.05 MeV, -0.39 MeV and 30.03 MeV, -11.89 MeV, respectively. In the former case the higher Coulomb interaction reduces the probability for two fragments coming closer, making the nuclear interaction weaker. Obviously, this has very significant effect on the corresponding mass yields of the respective fragments.

3.3 Multiplicity of light mass fragments

During the last few years, the study of the properties of hot nuclei has been an area of intense investigation. In particular, we have not yet acquired adequate knowledge regarding the decay modes of a very hot nucleus. It has been felt that, a possible way of getting the information about the hot nuclei is by detecting the multiplicity and energy of the neutron, proton and other light fragments. In this context, many measurements have been done for heavy ion induced fission and fusion processes. However, in case of multifragmentation process, such measurements, at present are rather scanty. In general, in the heavy ion induced fission and fusion channels, the neutron multiplicities have been found to be rather very large, which can not be explained using a standard statistical model. Recently, Gonin et al.[14] have made such measurements in $^{60}Ni + ^{100}Mo$ reaction at 550 and 655 MeV bombarding energies. The compound nucleus has been formed at 251 and 293 MeV excitation energies. They have made comparison of the experimental multiplicities for neutron, proton and α-particle, with that of the statistical models like PACE2[33] and CASCADE[34]. They found that these models are not able to describe the overall trend of the experimental data. The statistical model calculations underestimate the multiplicities of those particles, in particular of neutron by about a factor four compared to the experiment.

This strikingly low neutron multiplicity of the statistical models has created a serious problem in this area of study. Gonin et al.[14] have concluded that the standard statistical model calculations can not explain the multiplicity and it is essential to perform dynamical calculations taking into account the deformation. The low multiplicities of the charge particles also point out towards a similar conclusions, as it has been found that the emission barrier has to be somewhat reduced to gain enhancement for matching the data. In this context, we feel the usual statistical models are inadequate to describe the process, because these do not include the nuclear interaction of the fragments. In the preceeding section we have shown that the nuclear interaction plays very important role in the mass yield

distribution of light particles. It has the desirable effect of enhancing their cross section. For $p + Ag$ reaction, we have calculated in our model the multiplicities for n, p, and α-particle as a function of temperature with nuclear plus Coulomb interaction and with only Coulomb interaction. The corresponding results are shown as solid and dashed curves for neutron in figure 15 and for proton and α particle in figure 16. It is indeed gratifying to find that, due to the inclusion of nuclear force, there is an enhancement of the neutron multiplicity by a factor about 10 at temperature 3 MeV. This enhancement factor gradually decreases to 2.5 when the temperature increases to 6 MeV. The fall in the enhancement is due to the relative increase of the kinetic energy with respect to the potential energy with the rise of temperature. In case of proton and α-particle, the multiplicities are also substantially enhanced. This enhancement can be interpreted to be due to the reduction of the emission barrier due to the nuclear interaction. Thus in the frame work of statistical model, the neutron and charged particles multiplicities are very substantially enhanced without resorting to other mechanism like dynamical and deformation effects etc..., but by taking into account nuclear inter fragment interaction, which is an integral part of nuclear dynamics, hitherto unaccounted.

4. Role of Coulomb interaction on the mass yield

The importance of Coulomb interaction in the multifragmentation process was recognised quite early by Gross et al.[2] through the studies carried out in their grand canonical model, in which the effect of Coulomb interaction was taken into account in a mean field approximation. It has also been emphasized on several occassions in the past, that the mutual Coulomb interaction among the fragments gives rise to the 'U'-shape form in the mass yield distribution. However, recently Deangelis et al.[20], have claimed on the basis of their calculations in a model, similar to a grand canonical one, that the Coulomb interaction is not essential to produce a minimum in the mass yield distribution curves. In their work, they have obtained a minimum in the mass yield curve even without taking the

Coulomb interaction. Therefore in this section, we have reexamined this issue by performing the calculation for both symmetric and asymmetric systems using the model in I and also our present model.

We find that Deangelis et al.[20], in their calculations, have not only ignored the mutual Coulomb interaction between the fragments but also, the self Coulomb energy of the fragments, which is an integral part of the binding energy. The self Coulomb energy is quite essential for a well defined physical nucleus which occurs as a fragment in the assembly. From our extensive calculations, we observe that the earlier conclusion of Gross et al.[2] remains quite valid and Deangelis et al.[20] arrive at a contradictory result because of ignoring the self Coulomb energy of the fragments. In the following, we present our calculations, which corroborates the contension of Gross et al.[2].

To compare the results with Deangelis et al.[20], we have chossen the same symmetric system of mass number $A = 100$ and have performed the following three sets of calculations for the mass yield distribution using the model of Gross et al.[2];

(a) both the self Coulomb energy and the mutual Coulomb interaction of the fragments are included,

(b) only the self Coulomb energy of the fragments is taken into account, and

(c) both the self Coulomb energy and the mutual Coulomb interaction of the fragments are ignored.

The results for (a), (b), and (c) are presented as solid, dotted and dashed curves respectively, for temperature $k_BT = 5$ and 10 MeV in figure 17. It can be seen that for $k_BT = 5$ MeV, the dashed curve has a minimum as the solid curve, which is exactly what, the calculations of Deangelis et al.[20] have shown in their work. The dotted curves obtained taking only the self Coulomb interaction does not show a minimum. For $k_BT = 10$ MeV, all the three sets of calculations show sharply falling mass yield distributions. Since one can not ignore the self Coulomb interaction for describing well defined nuclear fragments, the

present calculation clearly coroborates the importance of the mutual Coulomb interaction in producing a minimum in the mass yield distribution curve as has been claimed by Gross et al.[2] earlier. To further ascertain this fact, we have repeated the same set of calculations for a realistic heavy system like ^{108}Ag and have obtained similar results for temperature $k_BT = 4$ and 10 MeV, as shown in figure 18.

Using our present model, in which both the Coulomb and the nuclear interactions, are taken into account, we have shown in figure 7 that the 'U'-shape is produced for the medium temperature and a sharply falling one at high temperature for $p + Ag$ reaction. However, to show again the importance of Coulomb interaction in producing a minimum in the mass yields, we have performed the same sets of calculations (a),(b), and (c) as above using the present model, ignoring the nuclear interaction. The results obtained for temperature $k_BT = 5$ and 10 MeV, are shown in figure 19, which are quite similar to figures 17 and 18. Thus it is clearly established that the Coulomb interaction is essential in order to produce a minimum in the mass yield distribution.

5. Summary and conclusion

In summary, we have presented a new statistical method to calculate the mean field of an interacting assembly of nuclear fragments, and have used it in the frame work of grand canonical ensemble, to obtain the mass yield distributions. Both the Coulomb and the nuclear interfragment interactions have been taken into account and are treated in the same footing in calculating the respective mean fields without introducing any free parameter. The proximity potential[19] has been used as the nuclear interfragment interaction for all fragments except for neutron, proton, deuteron, triton and helion, for which the exact optical potentials have been taken, to represent their nuclear interaction with the complementary heavy mass fragments.

In I, it was necessary to use the large value of 2.36 fm for the Coulomb radius constant r_{oc} to fit the mass yield distribution. This large value had the effect of reducing the inter-

action. In the present model, we have no necessity of having this parameter. It appears that by the introduction of the nuclear interaction, which is opposite in nature to Coulomb, similar reduction to a large extent in the mean field has been achieved, which was done phenomenologically in references[2,21], by using large value of r_{oc}. This is undoubtedly a very satisfying feature of the present method for calculating the mean field. For light particles, especially for neutrons, the mean fields have been drastically modified by the introduction of nuclear interaction. This has enhanced their cross-section to a great extent without substantially affecting the yield of medium and heavy mass fragments. With the introduction of nuclear interaction, the necessity for treating the neutron channel separately, which has been customarily done in the improved microcanonical models[9,29], does not arise in the present model. Thus all fragments starting from nucleon to the heaviest one, are treated on equal footing. Therefore, the propositions of many authors[17,18] regarding a separate mechanism for production of neutron and other light particles by shattering and cleavage etc... may not be warranted.

We have shown that by the inclusion of the nuclear interaction, the unphysical temperature of about 10 MeV, which was required to describe the Purdue- Fermilab data[13] on $p+Kr$ and $p+Xe$ reactions, comes down to a much lower value of about 6 MeV, which is about the maximum temperature observed for a compound nucleus. This reduction of temperature is a very desirable physical effect. It is shown that the neutron multiplicity is enhanced several times and the multiplicities of light mass fragments are also substantially increased by the introduction of the nuclear interaction. Therefore, the dynamical effect and the large deformation which have been invoked to explain the high multiplicities observed in experiments are not the essential mechanisms. The nuclear interaction incorporated in the statistical model gives a large enhancement. Thus, the inadequacy of the usual statistical models is circumvented to a large extent by the introduction of the nuclear interaction.

It is satisfying to find that the present study favours low break up density in the range

of about $\rho_o/4.5$ to $\rho_o/15$u or the low and medium mass nuclei, which is in agreement wi the recent observations[1,29,32]. For heavy mass nuclei like Ta and Au. our calculati gives a lower density of about $\rho_o/26$, which needs experimental confirmation.

Recent controversy raised by some authors[20] claiming that Coulomb interaction not necessary to produce a minimum in the mass yield is critically examined in this pap It is shown that, because of ignoring the self Coulomb energies of the fragments, they cou obtain a minimum in the mass yield even though the mutual Coulomb interactions we not included. The self Coulomb energy forms an integral part of the binding energy of fragment nucleus. One can not neglect it in describing the physics of the real fragmen By a series of calculations carried out in the present model and also in the model in I, is clearly shown that once the self Coulomb energies of fragments are taken into accour which can not be ignored, then in that case, the mutual Coulomb interaction is necessary order to produce a minimum in the mass yields. This vindicates the earlier claim of Gro and his collaborators in I.

In view of the above, we would like to conclude that nuclear interaction plays a important role in the dynamics of the multifragmentation process. This study presents practical method to calculate the nuclear and Coulomb mean fields and use them in tl grand canonical model. We hope this provides an improved and useful description of tl multifragmentation phenomena.

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Figure Caption

Figure 1. Comparison of proximity potential and the optical potential between the deuteron and the complementary fragment ^{108}Ag are represented by dashed and solid curves, respectively. The parameters of the proximity and the optical potentials are from reference [19] and reference [22], respectively. The arrow indicates the distance at which the two nuclei touch each other.

Figure 2. (a) Same as in Figure 1, except for the neutron. (b) Same as in Figure 1, except for the proton. The parameters of the optical potentials for neutron and proton are taken from reference [23].

Figure 3. The mean potential energy of various β-stable fragments of the Ag nucleus in the $p + Ag$ collision calculated in the present model at $k_\beta T = 3$ MeV. Coulomb, nuclear and Coulomb + nuclear potentials are represented by dashed, dotted and solid curves, respectively. See the text for details.

Figure 4. The mean potential energy of fragments with the same N-value, in the $p + Ag$ collision calculated in the present model at $k_\beta T = 3$ MeV. Coulomb, nuclear and Coulomb + nuclear potentials are represented by dashed, dotted and solid curves, respectively. (a) For the fragments with $N = 1$ and (b) for the fragments with $N = 2$.

Figure 5. Same as in Figure 4, except for the fragments with the same Z-value. (a) For the fragments with $Z = 1$ and (b) for the fragments with $Z = 2$.

Figure 6. Calculated mass yield distributions for the $p + Ag$ reaction. The dashed and solid histograms represent the results obtained in the present model with only Coulomb interaction and with Coulomb + nuclear interation, respectively.

Figure 7. Calculated mass yield distributions in the present model for the $p + Ag$ reaction at three different temperatures. See the text for details.

Figure 8. Calculated mass yield distributions in the present model for the $p + Cu$ reaction. Data are taken from reference [25].

Figure 9. Calculated mass yield distributions in the present model for the $p + Ag$ reaction. Data are taken from reference [26].

Figure 10. Calculated mass yield distributions in the present model for the $p + Ta$ reaction. Data are taken from reference [27].

Figure 11. Calculated mass yield distributions in the present model for the $p + Au$ reaction. Data are taken from reference [28].

Figure 12. The fragment mass yield versus the fragment mass number A_f for the $p + Kr$ reaction is plotted. The filled circles are the experimental data taken from reference [13]., the solid curve is a power-law fit to the same data by Hirsch et al.[30]. The open circles represent the results obtained in the present calculation, which have been connected by dashed lines.

Figure 13. Same as in Figure 12, except for the $p + Xe$ reaction.

Figure 14. Same as in Figure 12 but included only the Coulomb interaction in the present calculation.

Figure 15. The neutron multiplicity versus temperature in $p + Ag$ reaction. The dashed and solid curves are the results obtained in the present calculations with only Coulomb interaction and with the Coulomb + nuclear interaction, respectively.

Figure 16. Same as in Figure 15, except for the multiplicities of proton (p) and alpha-particle (α).

Figure 17. The fragment mass yield versus the fragment mass number A_f at two different temperatures for a symmetric system with $A = 100$, obtained in the model of reference [2]. The dashed curves are the yields calculated, ignoring both the self Coulomb energy and the mutual Coulomb interaction, the solid curves represent the yields calculated, including both the self Coulomb energy and the mutual Coulomb interaction. The dotted curves are the yields calculated, including only the self Coulomb energy.

Figure 18. Same as in Figure 17, except for a realistic asymmetric nucleus ^{108}Ag in the $p + Ag$ reaction at two different temperatures. The results presented here are obtained in

model of reference [2].

Figure 19. Same as in Figure 18, but the results presented here are obtained in the present calculation at two different temperatures.

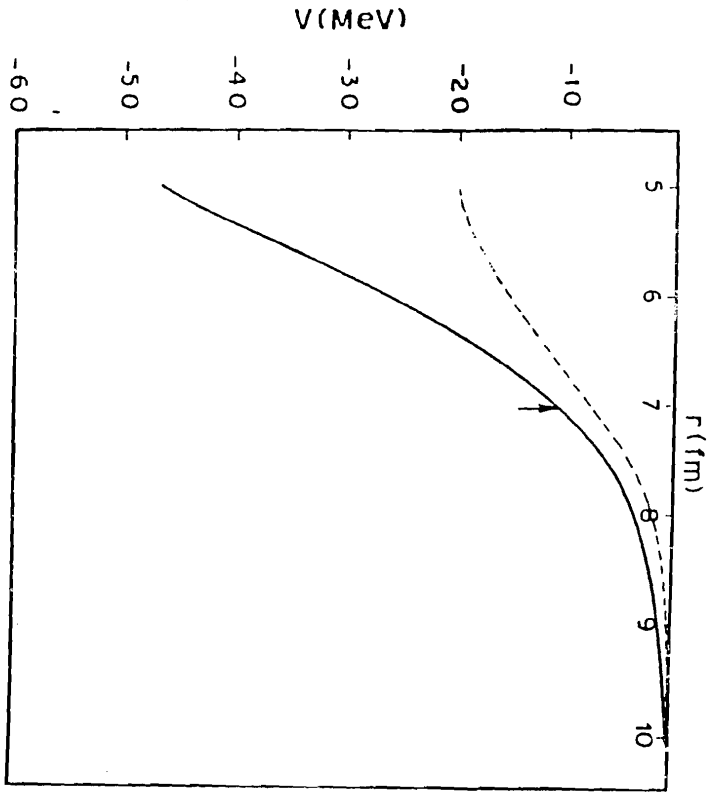
V(MeV)
-10
-20
-30
-40
-50
-60
5
6
7
8
9
10
r(fm)

Fig.1.

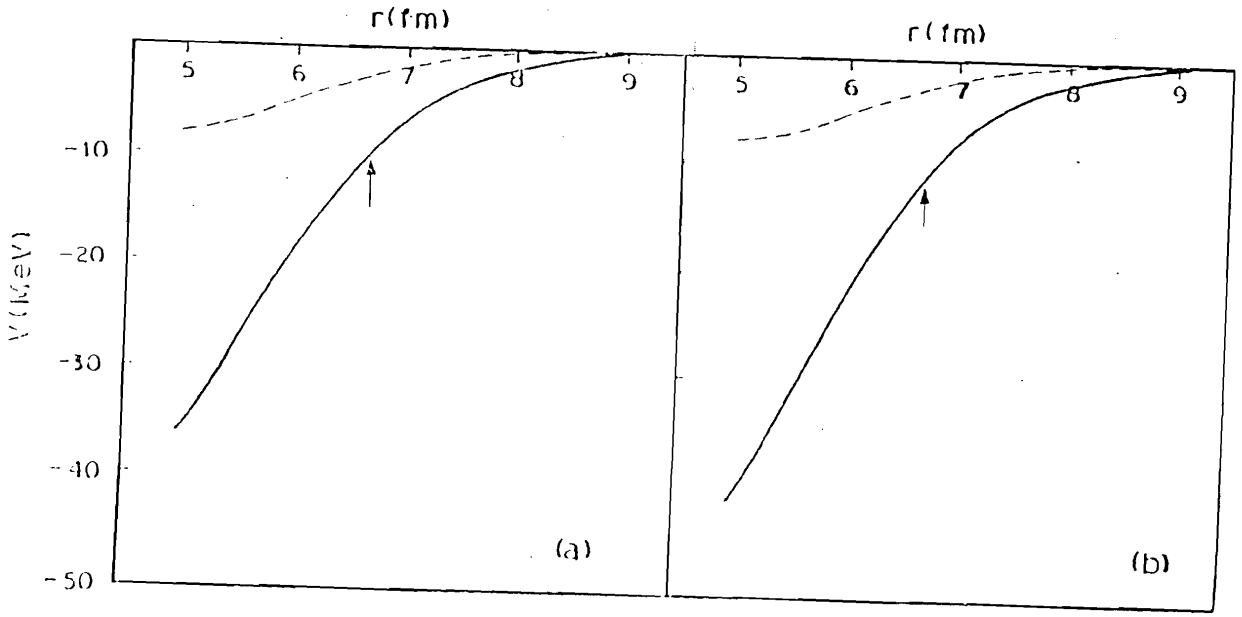
V(MeV)
-10
-20
-30
-40
-50
r(fm)
5
6
7
8
9
(a)
(b)

Fig. 2.

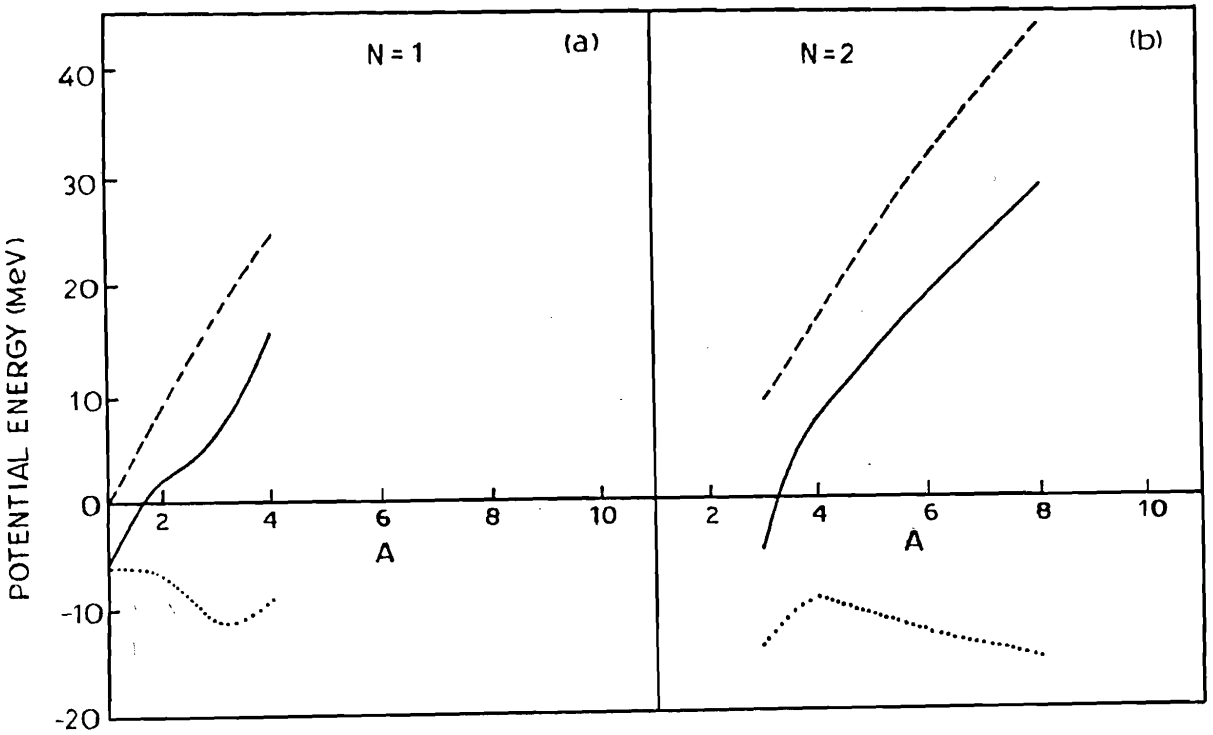
N = 1
(a)
N = 2
(b)
POTENTIAL ENERGY (MeV)
40
30
20
10
0
-10
-20
2
4
6
8
10
A

Fig. 4

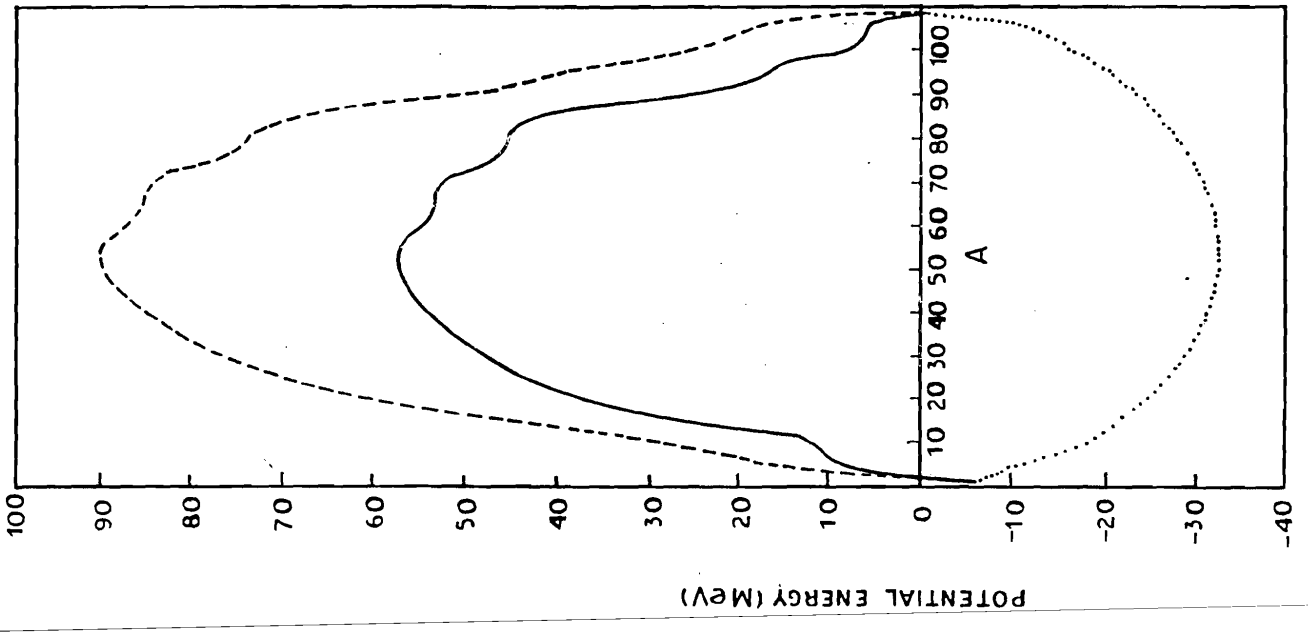
POTENTIAL ENERGY (MeV)
100
90
80
70
60
50
40
30
20
10
0
-10
-20
-30
-40
10 20 30 40 50 60 70 80 90 100
A

Fig.3

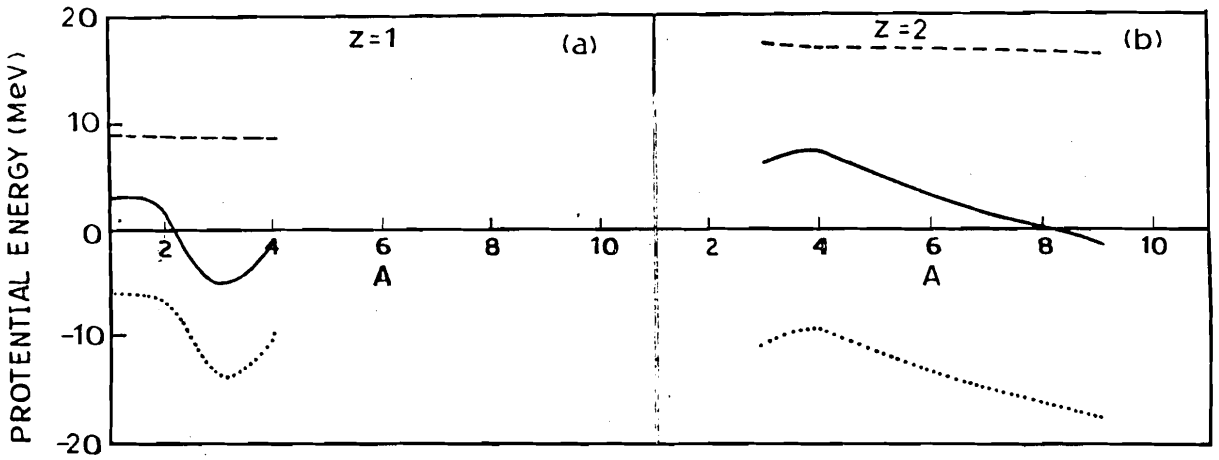
PROTENTIAL ENERGY (MeV)
20
10
0
-10
-20
Z=1
(a)
Z=2
(b)
2
4
6
8
10
A

Fig.5

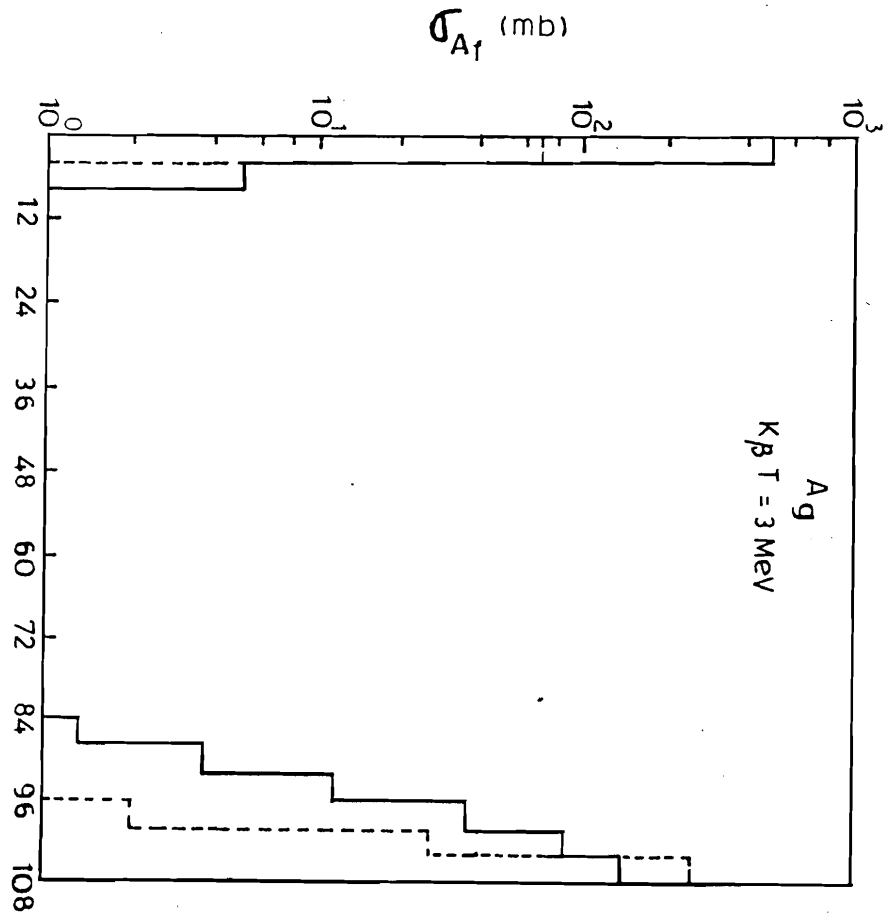
σ_{A_f} (mb)
10^0
10^1
10^2
10^3
12
24
36
48
60
72
84
96
108
A_f
A_g
K_β T = 3 MeV

Fig.6

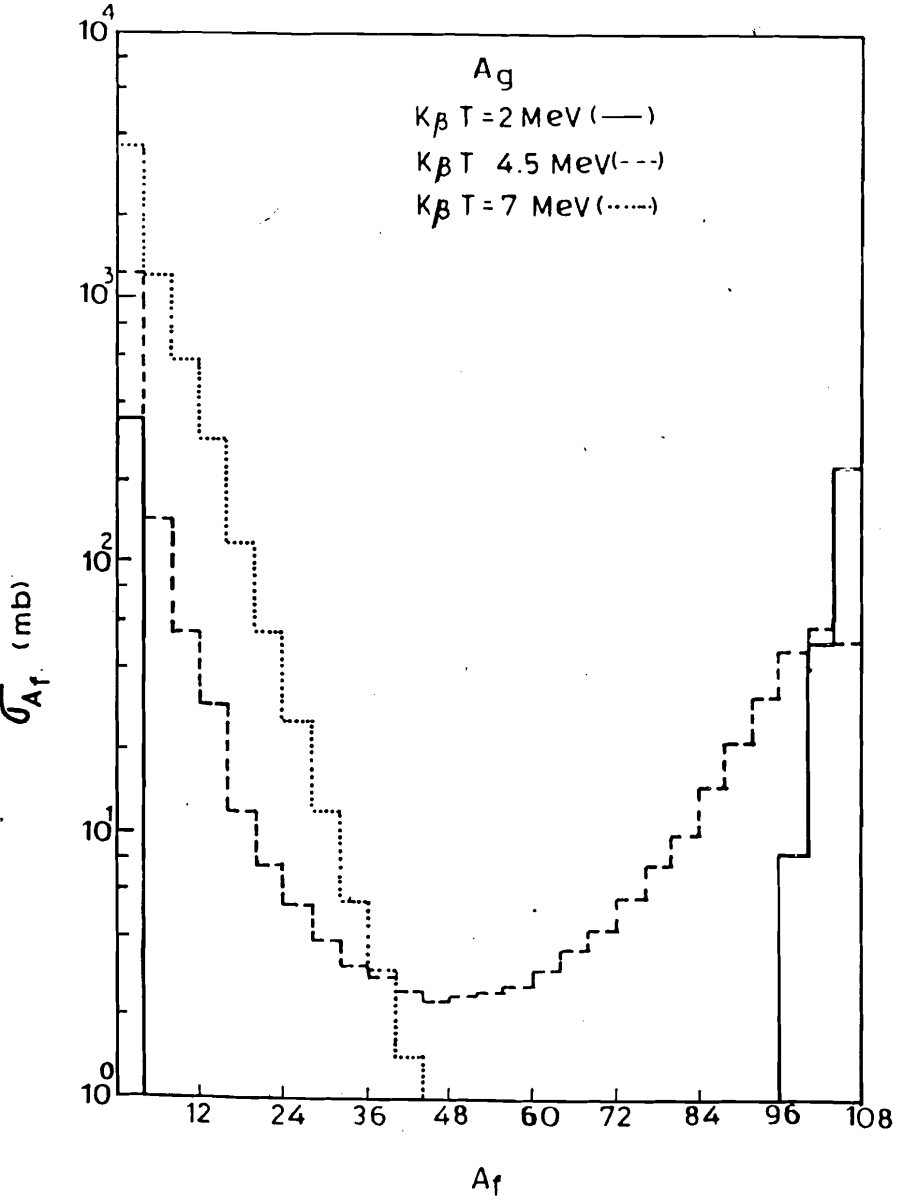
Ag
Kβ T = 2 MeV (——)
Kβ T 4.5 MeV(- - -)
Kβ T = 7 MeV (·····)
σAf (mb)
Af
10^4
10^3
10^2
10^1
10^0
12
24
36
48
60
72
84
96
108

Fig.7

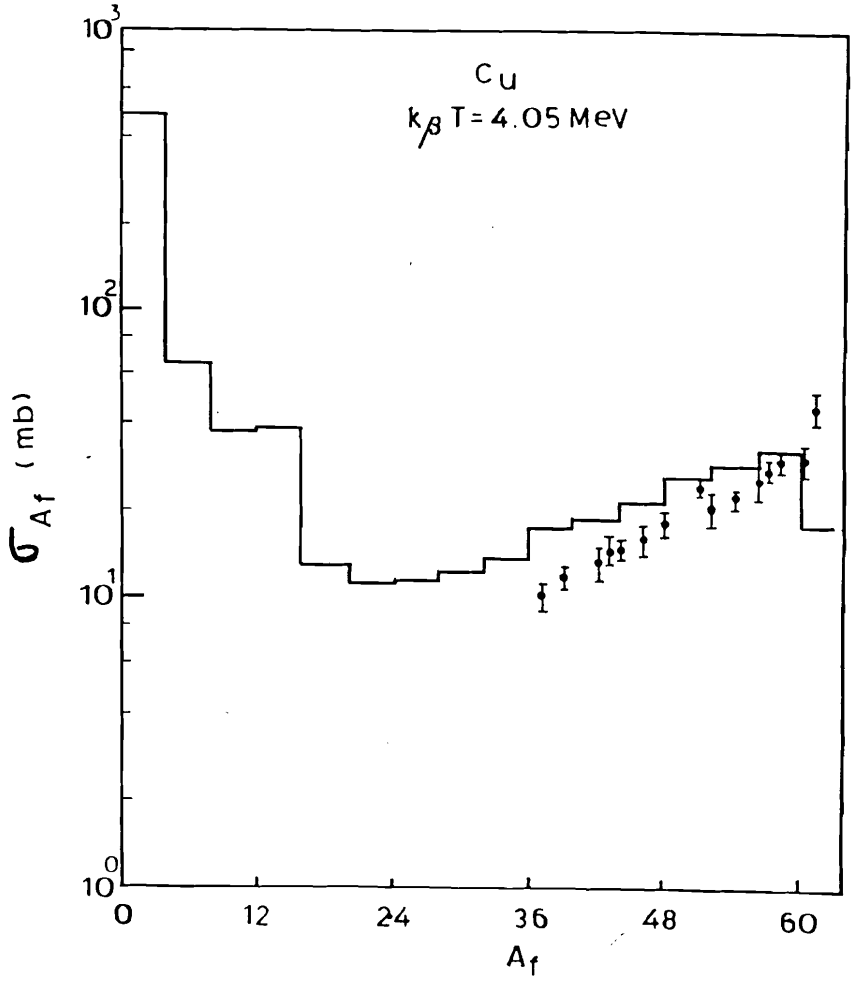
Cu
kβ T = 4.05 MeV
σAf (mb)
Af
10^3
10^2
10^1
10^0
0
12
24
36
48
60

Fig.8

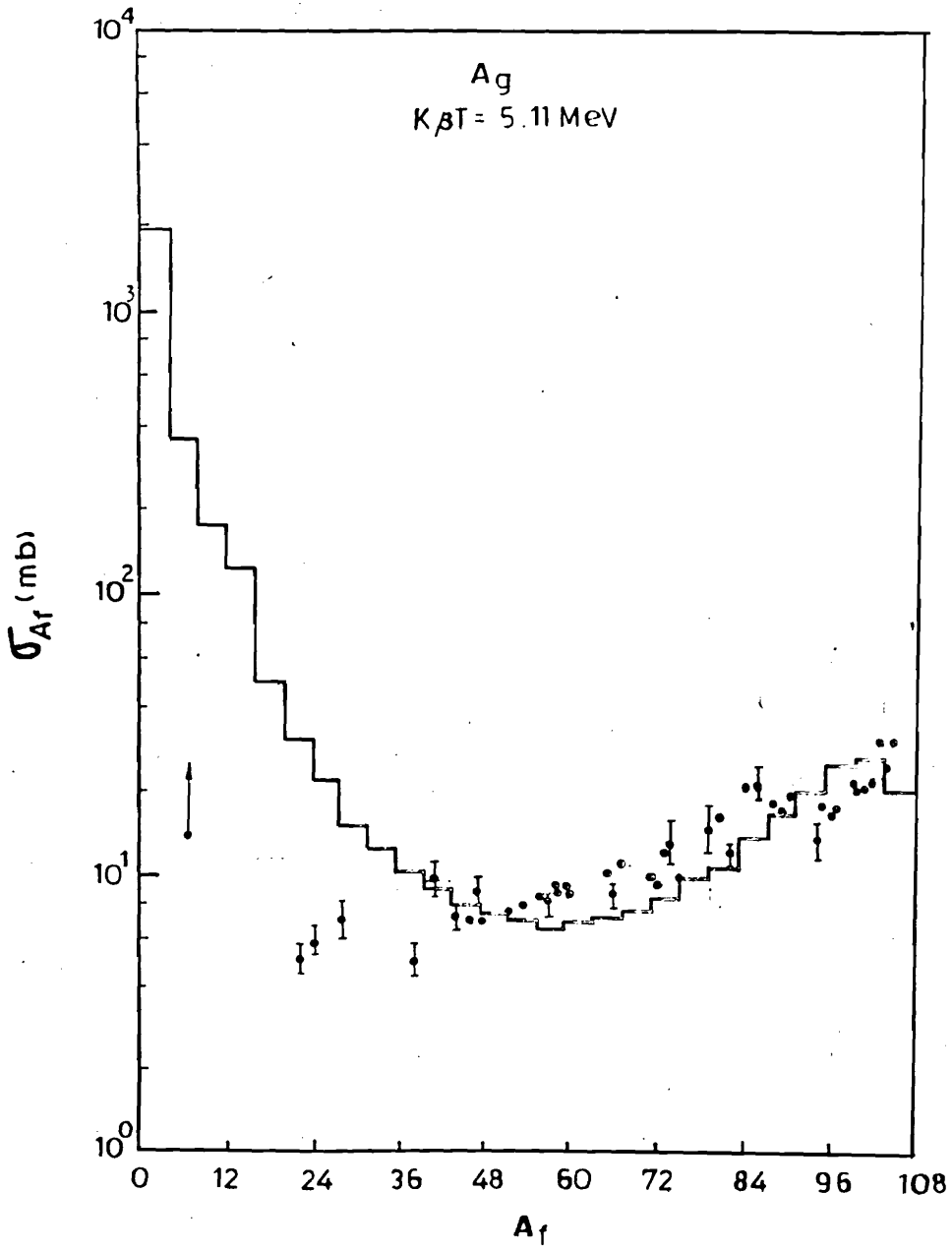

Fig.9

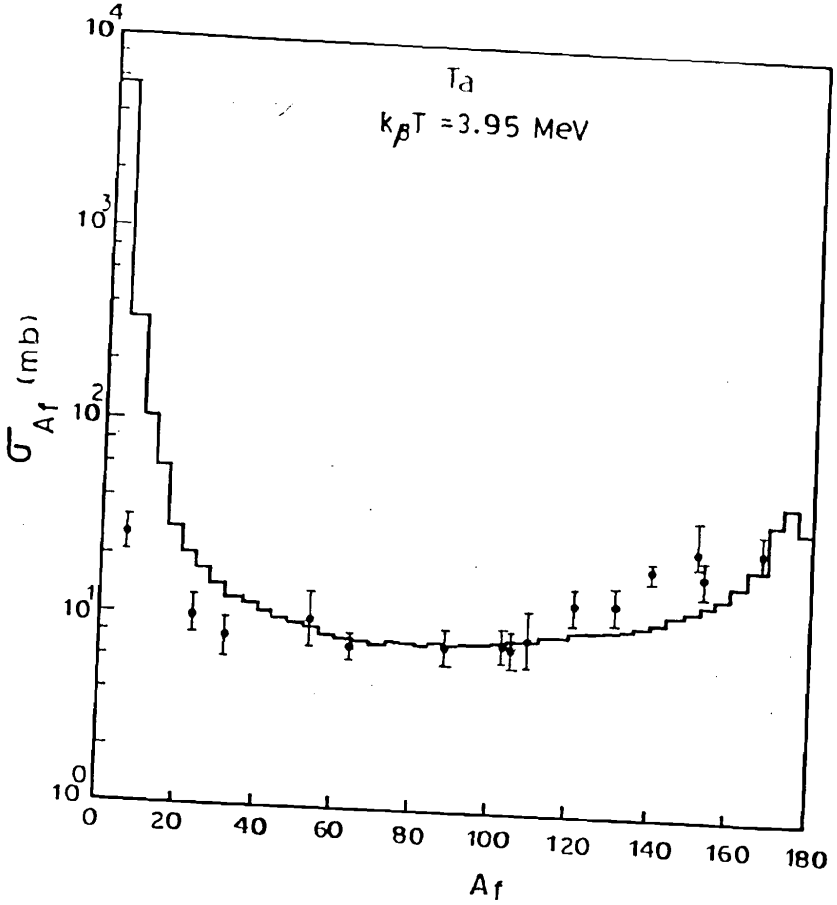

Fig.10

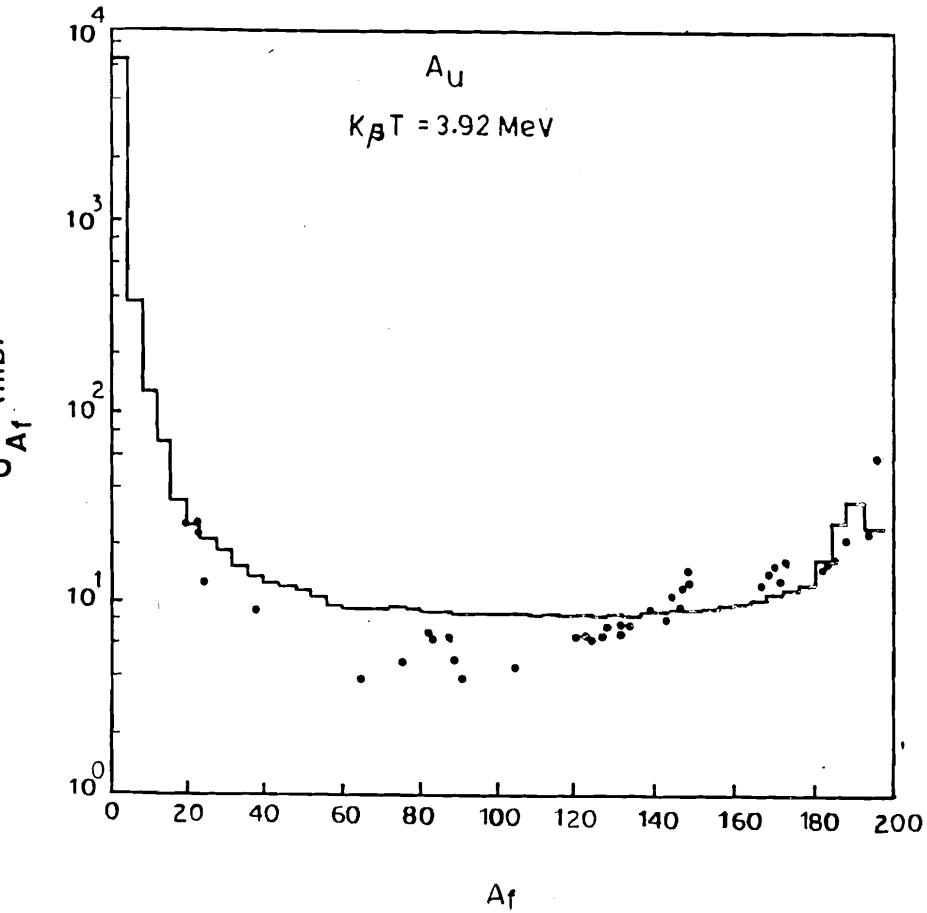
A_u
$K_\beta T$ = 3.92 MeV
10^4
10^3
10^2
10^1
10^0
0
20
40
60
80
100
120
140
160
180
200
A_f

Fig.11

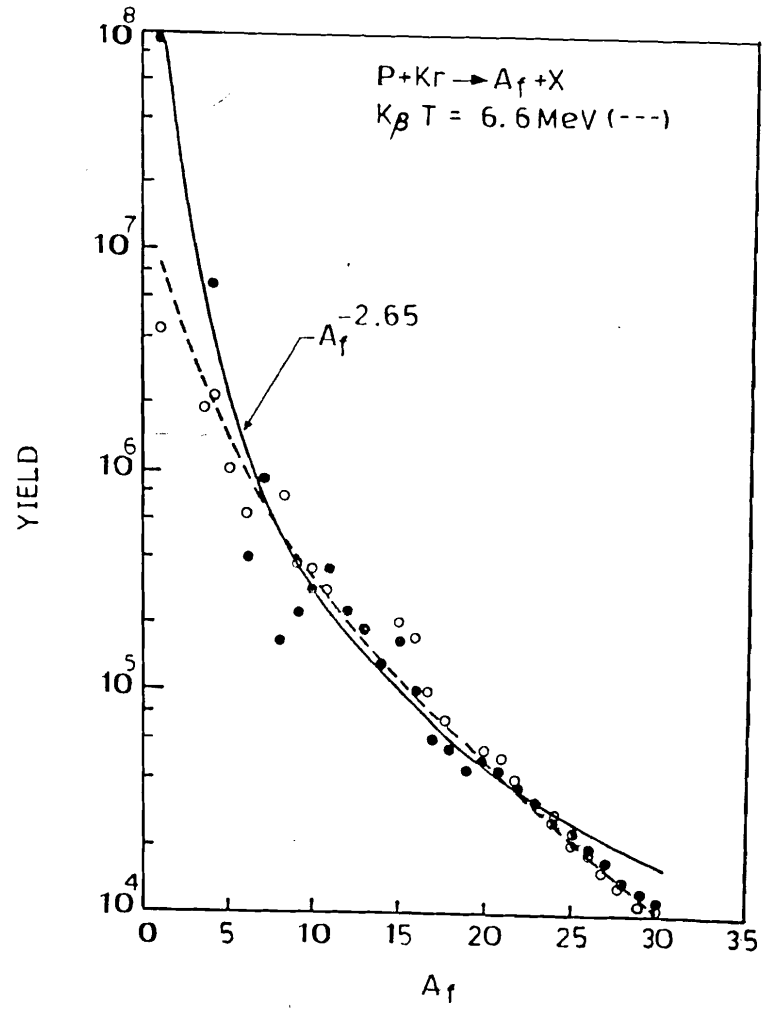
P+Kr → A_f +X
K_β T = 6.6 MeV (---)
$A_f^{-2.65}$
YIELD
10^8
10^7
10^6
10^5
10^4
0
5
10
15
20
25
30
35
A_f

Fig.12

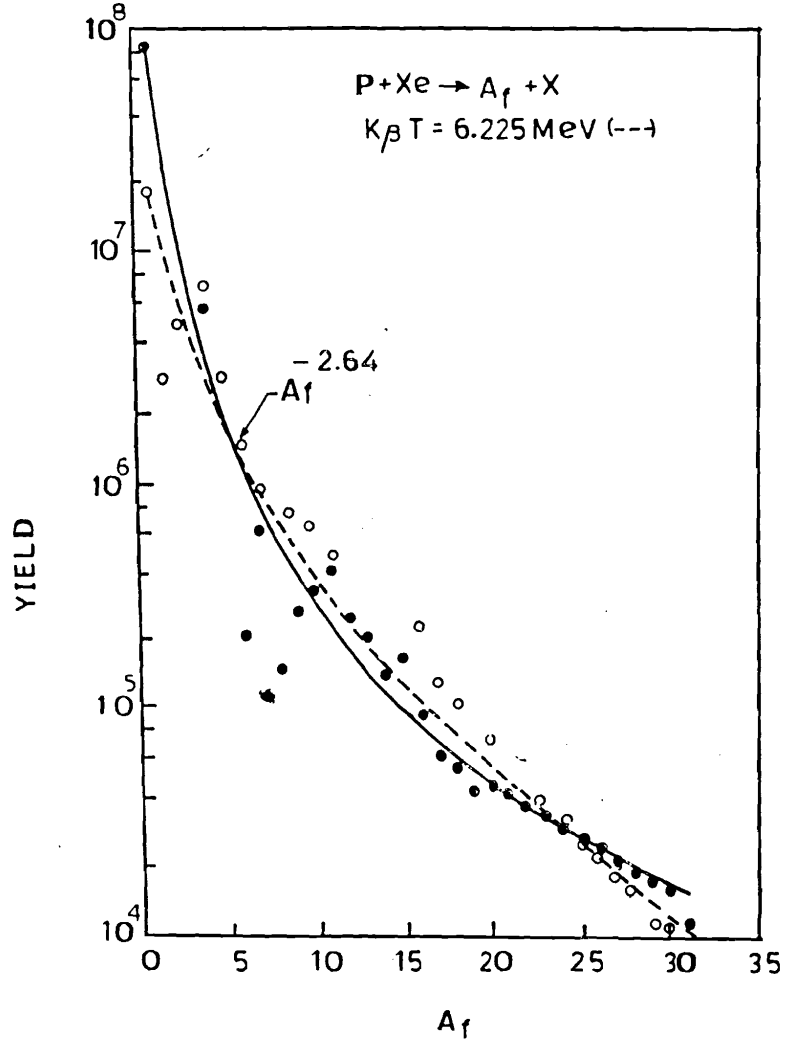
P+Xe → Af +X
Kβ T = 6.225 MeV (---)
Af^-2.64
YIELD
Af

Fig.13

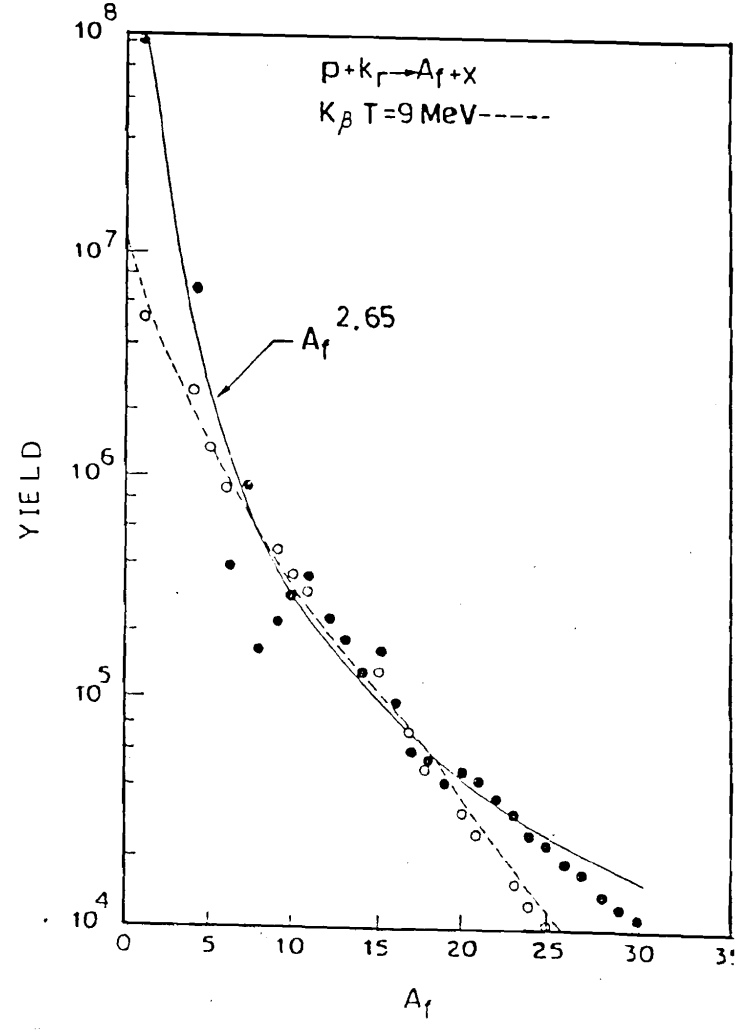
p+kr→Af+x
Kβ T=9 MeV-----
Af^2.65
YIELD
Af

Fig.14.

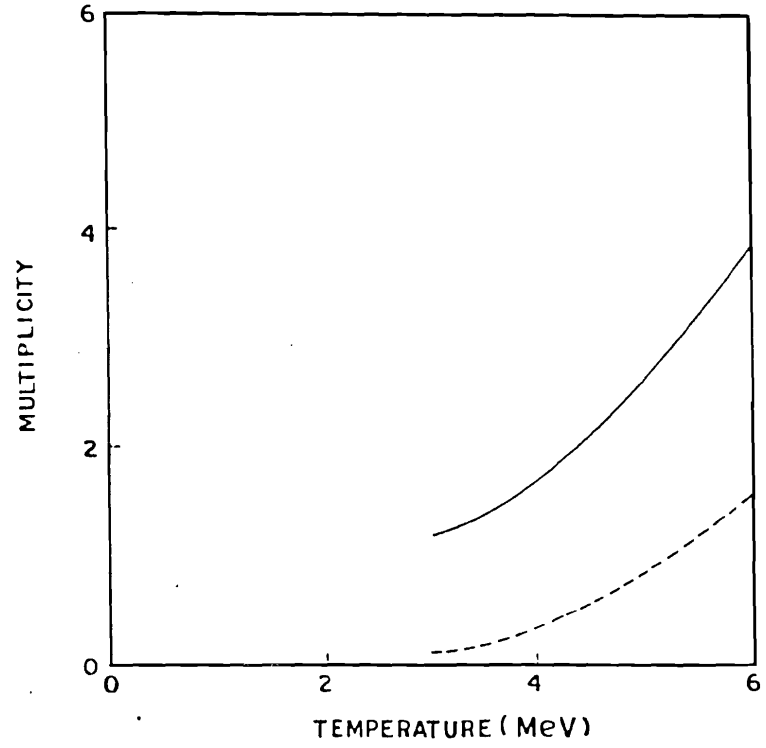
MULTIPLICITY
TEMPERATURE (MeV)
0
2
4
6

Fig 15.

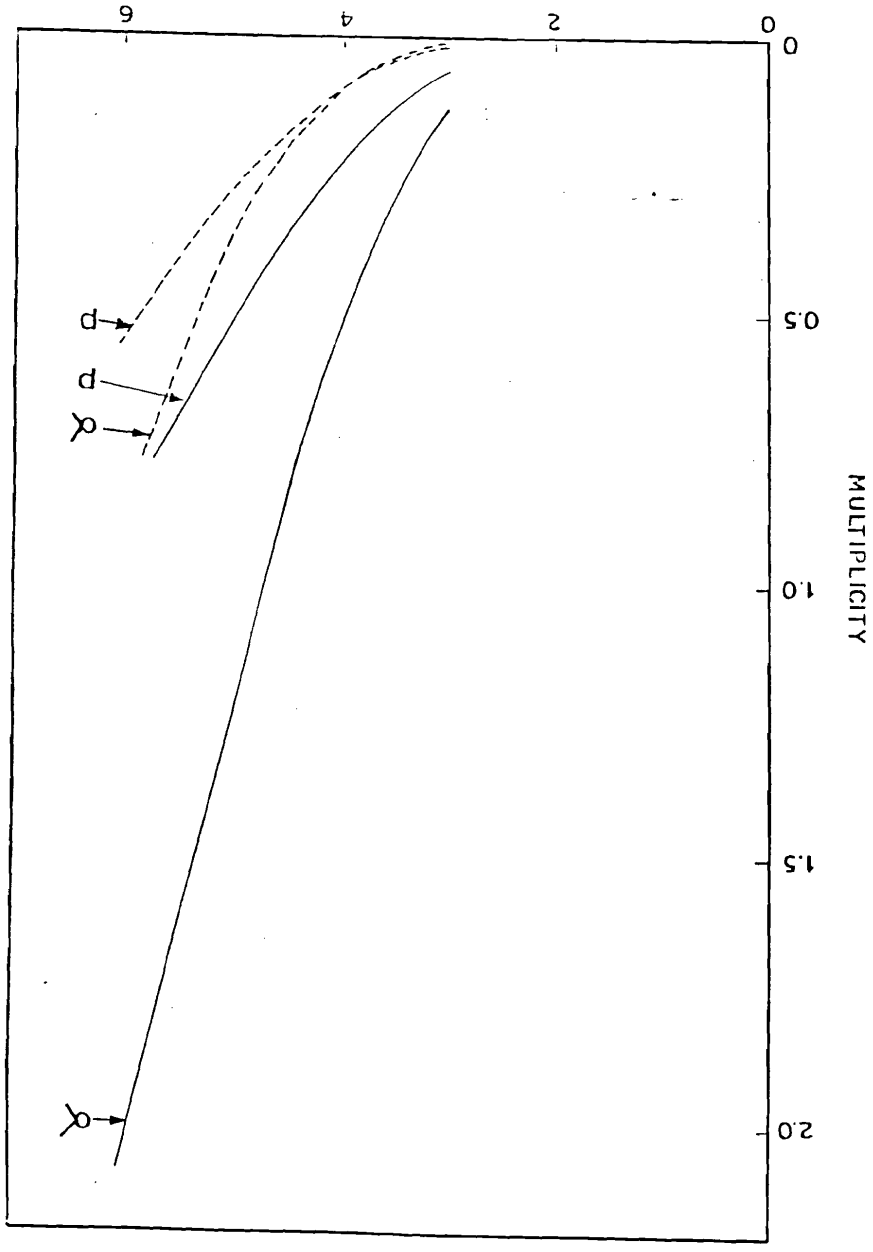
MULTIPLICITY
TEMPERATURE (MeV)
0
0.5
1.0
1.5
2.0
2
4
6
α
p
d
p

Fig.16

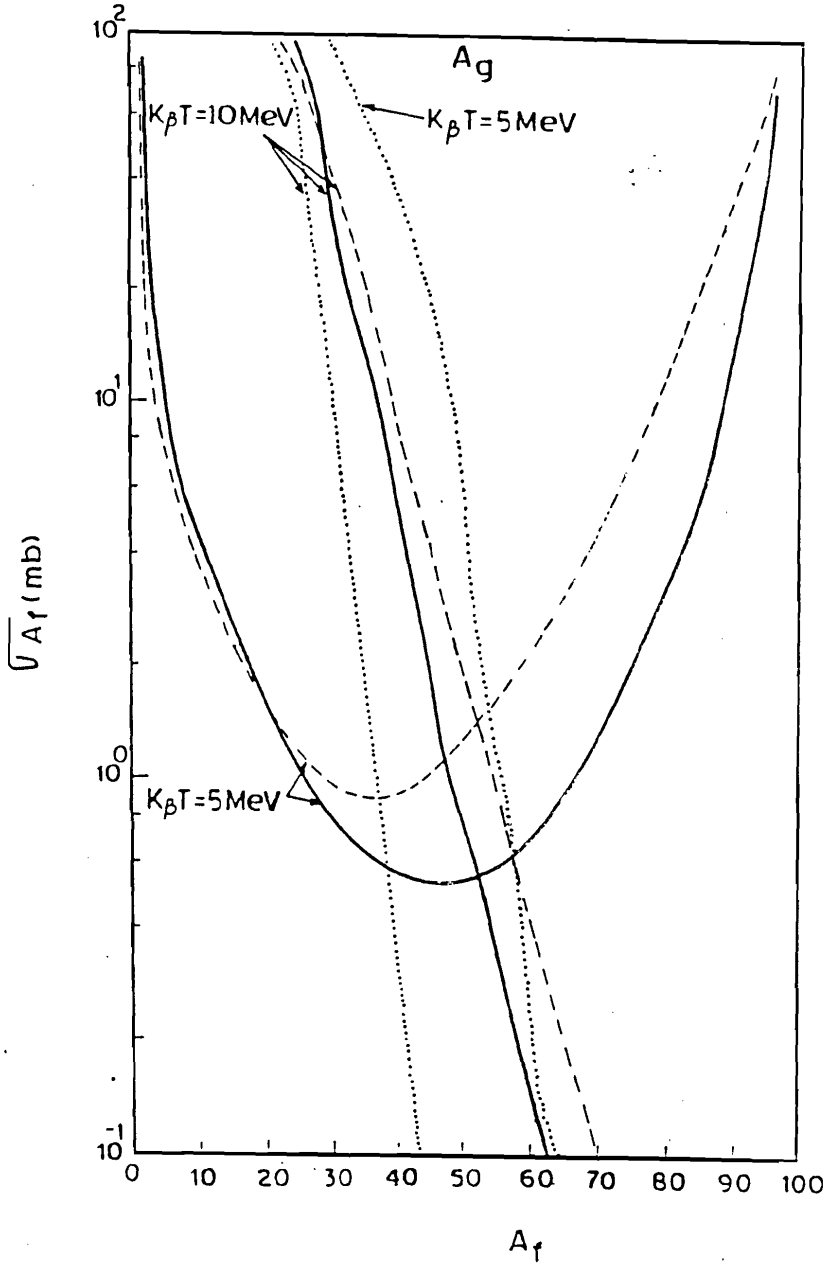
Ag
$K_\beta T = 10$ MeV
$K_\beta T = 5$ MeV
$K_\beta T = 5$ MeV
$\sqrt{A_f}$ (mb)
A_f

Fig.17

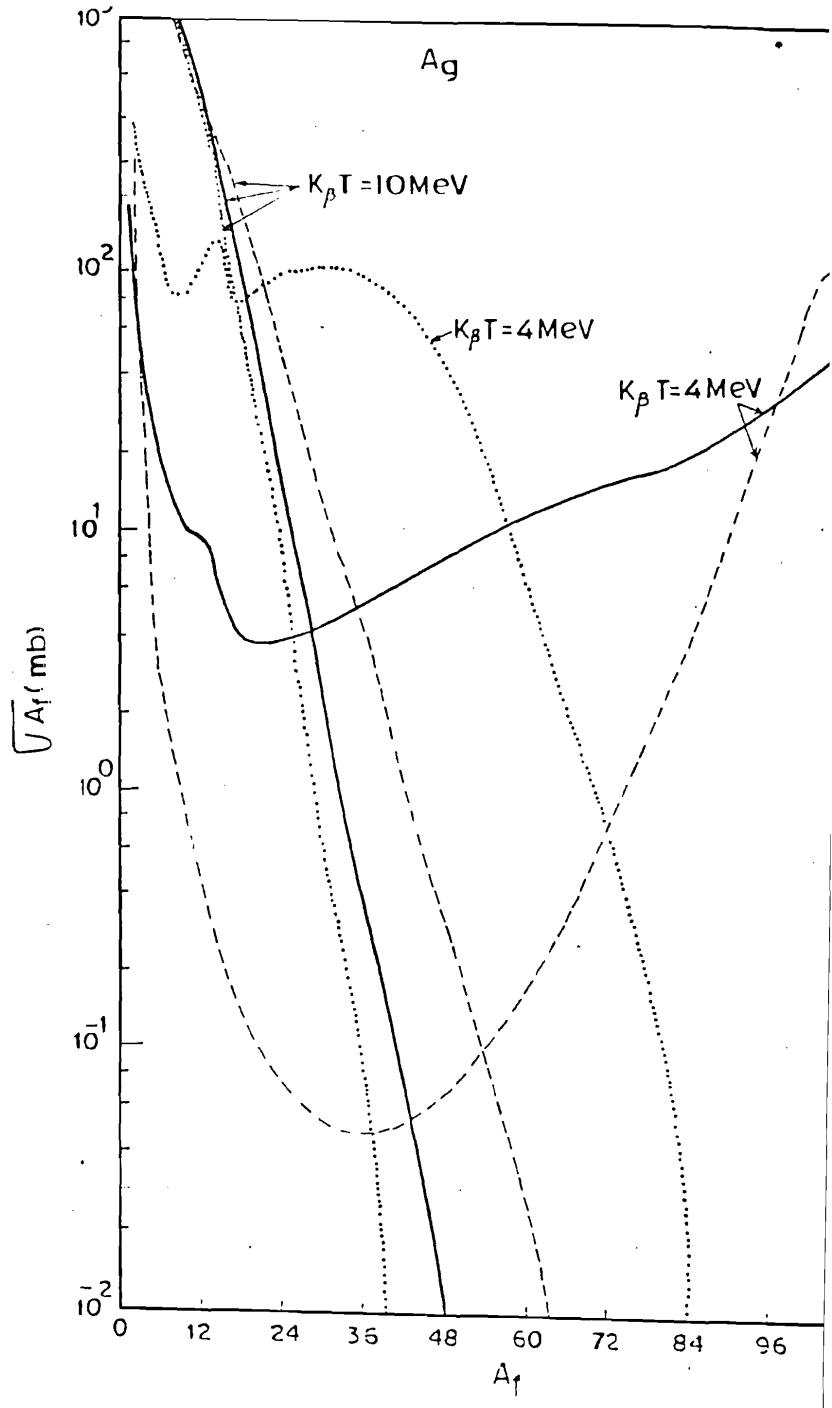
Ag
$K_\beta T = 10$ MeV
$K_\beta T = 4$ MeV
$K_\beta T = 4$ MeV
$\sqrt{A_f}$ (mb)
A_f

Fig.18

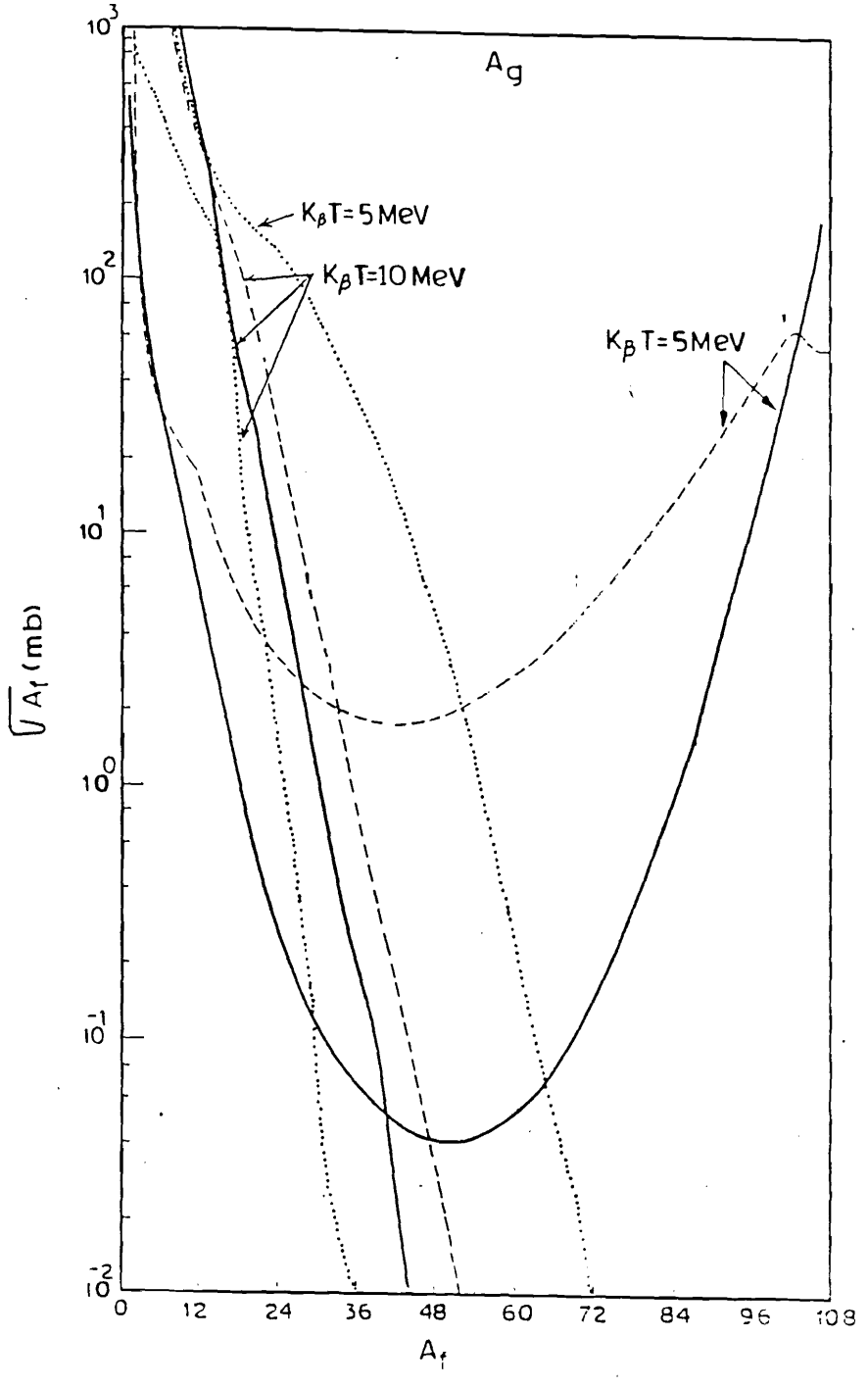
Ag
$K_\beta T = 5$ MeV
$K_\beta T = 10$ MeV
$K_\beta T = 5$ MeV
$\sqrt{A_f}$ (mb)
A_f
10^3
10^2
10^1
10^0
10^{-1}
10^{-2}
0
12
24
36
48
60
72
84
96
108

Fig 19