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Order in Large and Chaos in Small Components of Nuclear Wave Functions

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Abstract:

An investigation of the order and chaos of the nuclear excited states has shown that there is order in the large and chaos in the small quasiparticle or phonon components of the nuclear wave functions. The order-to-chaos transition is treated as a transition from the large to the small components of the nuclear wave function. The analysis has shown that relatively large many-quasiparticle components of the wave function at an excitation energy (4÷8)MeV may exist. The large many-quasiparticle components of the wave functions of the neutron resonances are responsible for enhanced E1-, M1- and E2-transition probabilities from neutron resonances to levels lying (1÷2)MeV below them.

The purpose of this paper is to descuss the order-to-chaos transition in terms of properties of nuclear wave functions and to analyze how the structure of nuclear states change with increasing excitation energy. This problem was formulated in[1,2] in 1972 as "are there relatively large many-quasiparticle components in the wave function of neutron resonances?"

To simplify the problem as much as possible, we confine ourselves to the low-spin bound and quasi-bound stationary non-rotation states of rigid nuclei. We treat all non-rotational states as small-amplitude collective or weakly collective vibrational states or quasiparticle states.

As clearly demonstrated by calculation for a few-body system the ground, low-lying and high-lying nuclear states are very complex. We find a representation in which several states are described in the simplest way, though the wave functions of other states are very complex. If the density matrix is diagonal in a Hartree-Fock-Bogoliubov approximation, then the average nuclear field and superconducting pairing interactions can be separated[3]. A representation is usually used in which the density matrix is diagonal for the ground states of the doubly-closed shell or well-deformed nuclei. This representation has physical meaning, since the transitions from the ground to excited states are usually experimentally observed. We can choose another representation in which the density matrix is diagonal for a given excited state. In this case, though the wave function of the given state has a simple form whereas the wave functions of the other states are very complex.

If the density matrix is diagonal for the ground state of a doubly-closed shell or a well-deformed nucleus, then the wave function of an excited state can be represented as an expansion of a number of many-quasiparticles states or many-phonon states. In this representation there is a hierarchy of the components of wave functions with different numbers of quasi-particles. According to [2,4] the wave function of an excited state with a fixed angular momentum and the parity of an odd-mass nucleus has the following form:

$$\Psi_n(J^\pi) = \{\sum_1 b^n_1 \alpha^+_1 + \sum_{123} b^n_{123}\alpha^+_1\alpha^+_2\alpha^+_3 + \sum_{1a} b^n_{1,a}\alpha^+_1 Q^+_a$$
$$+ \sum_{12345} b^n_{12345}\alpha^+_1\alpha^+_2\alpha^+_3\alpha^+_4\alpha^+_5 + \sum_{123a} b^n_{123,a}\alpha^+_1\alpha^+_2\alpha^+_3 Q^+_a$$
$$+ \sum_{1aa'} b^n_{1,aa'}\alpha^+_1 Q^+_a Q^+_{a'} + \cdots\}\Psi_0 \ . \qquad (1)$$

Here, α^+ denotes a quasiparticle creation operator and Ψ_0 is the ground state wave function of an A-1 doubly even nucleus which is determined as a phonon or quasiparticle vacuum. Also, $|b^n|^2$ defines the contribution of the corresponding quasiparticle component to the normalization of the wave function (1). To fulfill the orthogonality of the wave function (1) product $\alpha^+_1\alpha^+_{\bar{1}}$ is replaced by the 0^+ phonon operator Q^+_a [4]. The operator form of the wave function (1) can be generalized by replacing a two-quasi-particle operator $\alpha^+_1\alpha^+_2$ by a phonon operator $Q^+_{\lambda\mu i}$.

The wave function (1) of a highly excited state has many thousands terms. The correlations of two processes occurring through one state exist if both processes go through the same component b^n of the wave function (1). The state can be excited through one term b^n of the wave function (1) and can decay through another term. In this case, the excitation of the state is independent on its decay.

The nuclear mean field is responsible for the order. The residual interaction plays a two-fold role:

1) The superconducting pairing interaction stabilizes the regularity of the nuclear mean field. The coherent interaction between quasi-particles leads to the formation of low-lying vibrational states and giant resonances, which generate regularity in the nuclei.
2) The quasiparticle-phonon interaction leads to fragmentation of the quasiparticle and phonon states. It generates the chaos in the nuclei.

The wave function of the one-phonon state comprises a coherent sum of many two-quasiparticle components, and presents the regularity of the collective states in the nuclei. The common description of collective, weakly collective and quasipar-ticle states should be used for investigation of the complication of the state structure with increasing excitation energy.

There is experimental information that the wave functions of the low-lying states have one dominating one-quasiparticle or one-phonon component. They demonstrate the regularity in nuclei. A reasonably good description for the low-lying states has been obtained by means of dominant component alone. The low-lying states show individuality.

With increasing excitation energy, the structure of the states becomes more complex and the wave function (1) has several relatively large components; the domination of single component decreases. The complication of the nuclear states with excitation energy is a result of the coupling between the collective and non-collective degrees of freedom. This complication may be partly caused by a quasiparticle-phonon interaction.

The fragmentation (strength distribution) of the one-quasiparticle component increases as excitation energy increases. Experimental investigations on the fragmentation of the one-quasiparticle states in spherical nuclei have shown[5] that pronounced maxima of the strength distribution take place up to an excitation energy 10 MeV. This means that one-quasiparticle states with a large angular momentum lying in a region rather far from the Fermi surface are not fully fragmented. The fragmentation of one-quasi-particle states in spherical nuclei has been described[5-8] using the Quasiparticle-Phonon Nuclear Model (QPNM) by the wave functions, which contains quasiparticle, quasiparticle $\otimes$ phonon and quasiparticle $\otimes$ two-phonon components.

The role of the quasiparticle-phonon interaction increases with excitation energy. The structure of the nuclear states becomes more complex and the contribution of a few-quasiparticle components to the wave function strongly decreases with the excitation energy. The wave functions of the states with energies greater than (3-4)MeV are superpositions of many terms with different numbers of quasiparticles and phonons. A strong

fragmentation of the one-phonon states takes place at excitation energies above 4 MeV, resulting in a difficulty to make experimental observations.

The wave functions of the excited states of heavy nuclei with energies below 3 MeV have, as a rule, a dominating one-quasiparticle or two-quasiparticle or one-phonon component. At an excitation energy (3-4)MeV the wave functions have large five-quasiparticle components or three-phonon or four- or six quasiparticle components. These large components characterize the individuality of the nuclear states.

The number of terms in the wave function (1) becomes large with increasing excitation energy. In an energy region close to the neutron binding energy the wave function (1) contains many thousands components.

The partial radiative and reduced neutron widths were expressed in [2] in terms of the coefficients b^n of the wave function (1). The experimental values of the reduced neutron and partial radiative widths were used in [2] to estimate the average values of the one- and two-quasiparticle components of the wave functions of the neutron resonances. For nuclei with $50 < A < 250$ they were found to be $|\bar{b}|^2 = 10^{-6}$-10^{-8}, and somewhat large for nuclei around closed shells.

The neutron strength functions and the giant resonances were described in QPNM in terms of the strength function method[8]. Descriptions of the s-, p- and d-wave neutron strength functions have been obtained by calculating the fragmentation of the one- or two-quasiparticle states. The calculated neutron strength functions in many spherical and deformed nuclei are in sufficiently good agreement with the experimental data, even through they are defined by the tails of the strength distributions of subshells $s_{1/2}$ or $p_{1/2}$, $p_{3/2}$ or $d_{3/2}$, $d_{5/2}$. A good description of the widths of the giant resonances has been obtained in QPNM. The experimental information concerning the neutron strength functions and the giant resonances is limited to few-quasiparticle components of the excited states wave function.

Much attention has recently been paid on an interplay between order and chaos in nuclei[9]. Studies concerning the level spacing distribution in nuclei have usually identified chaos in nuclei via an agreement with Gaussian Orthogonal Ensemble (GOE) statistics. The GOE distribution of the level density cannot prove that the nuclear structure is chaotic, since the behavior of the level density gives only restricted information about the nuclear wave functions. The nuclear wave function of the excited state with energy more than (2+4)MeV usually has many components with different quasiparticle numbers, and with different isospin quantum numbers T_0 and T_0+1, and with different K quantum numbers. Such wave functions are superpositions of several interacting GOE spectra. In [10] this was demonstrated using a simple soluble model in which the appearance of a GOE-type distribution function for the nearest-neighbor level spacing does not directly correspond to a dissolution of the quantum numbers associated with the model. An analysis[11] of the level spacing in ^{26}Al has shown a good agreement with the GOE statistics, even in the low-lying region. Extension and complete

nuclear schemes were analyzed in [12,13]. The level spacing for nuclei with A $\leq$ 50 is close to GOE statistics, while the data for A > 230 are close to Poisson statistics. From the GOE statistics of the level spacing, it cannot be concluded whether or not the nuclear wave functions have an essentially chaotic character.

If we consider the order and chaos separately with respect to large and small quasiparticles as well as the phonons components of the excited states wave function, there are many experimental data available concerning the large components of the wave functions of low-lying and isobaric analog states. They have demonstrated a regularity in nuclei. The high-spin many-quasiparticle isomers have demonstrated a regularity of the model of independent quasiparticles. It is possible to conclude that the regularity is governed by the large components of the wave functions of the nuclear excited states.

Practically there are no experimental data which tells information on the small components of the wave functions of the low-lying states. The small components of a wave function manifests itself in the distribution function of partial widths for the transition from a neutron resonance to the few-quasiparticle components of the wave function of the low-lying state. The distribution of the partial radiative widths of the neutron resonances are in good agreement with GOE statistics. This shows that the one- or two-quasiparticle components of the wave functions of neutron resonance have a chaotic character. This distribution, however, does not contain any information concerning the entire wave function. Suppose we measure only the components with isospin T_0 in the region of an isobaric analog resonance with isospin T_0+1. In this case, we get data indicating a chaotic character of these components. However we cannot conclude that an isobaric analog state does not exist in this region.

In [14] using the thermal neutron capture data and comparing the intensities of gamma transitions to low-lying states with the same spin and parity, but the different K values suggest that no complete K mixing exists in the domain of the neutron resonances. This conclusion is valid only for the tails of the two- and four-quasiparticle components of the wave function of the neutron resonances. For investigating of K mixing, it is necessary, as proposed in [2], to investigate the gamma transition from the neutron resonance states, to low-lying state with the same spin and parity, but different K values, instead of the thermal neutron. In this case, we can obtain information concerning the K mixing of the two- and four-quasiparticle components of the wave function of the neutron resonances, which contribute to the normalization their wave function by a value of equal 10^{-6}.

The investigation of the K mixing in the neutron resonance region carried out in [15] involved the same method as in the investigation of the isospin mixing in [16] with practically whole wave functions of the excited states in ^{26}Al. The K mixing in neutron resonance region was compared with the K mixing of the low-lying states. It is difficult to understand how it is possible to carry out such an investigation of K mixing and to make a comparison with the low-lying states using the experimental data involving the 10^{-6} part of the neutron resonances wave functions.

The dependence of the s- or p-wave neutron strength function on the position of the $s_{1/2}$ or $p_{1/2}$, $p_{3/2}$ subshell relative to the neutron binding energy reflects the regularities of the nuclear mean field. The giant resonances demonstrate that the regularities of the average values of the particle-hole components of the overlapping nuclear states. This means that an effect of the regularity of the average values takes place and that the chaos characterizes the small components of the wave function.

We now consider the transition from order to chaos as a transition from large to small components of the nuclear wave function. For this consideration is necessary to clarify the degree of fragmentation of the many-quasiparticle states.

In spherical and deformed nuclei there are many high-spin isomers whose main component are described by the many-quasiparticle configuration. They have long lifetimes because there are no high-spin states whose main components are described by few quasipartice configurations. The existence of isomers with many-quasiparticle components indicates a small fragmentation of such many-quasiparticle high-spin configurations. The small fragmentation of the high-spin many-quasiparticle configurations is due to the very low density of high-spin states, especially the density of the high-spin many-quasiparticle configurations.

An amplitude of the many-quasiparticle configurations in the neutron resonances states was analyzed in [1]. The many-quasiparticle low-spin configurations should be more strongly fragmented in comparison with high-spin states, because the density of the low-spin state is much larger than the density of the high-spin states at intermediate excitation energies. The density of the low-spin states with many-quasiparticle configuration are small in comparison with the total density of the low-spin states. We can therefore expect that the fragmentation of the low-spin many-quasiparticle configurations is much weaker than the fragmentation of the low-spin few-quasiparticle configurations.

The one-quasiparticle configuration with relatively large angular momentum of a spherical nuclei at an excitation energy 6-8 MeV is not fully fragmented. The three- and five-quasiparticle configurations should be less fragmented than the one-quasiparticle configurations in this energy region. The seven-quasiparticle configurations appear just at this energy and, therefore, cannot be strongly fragmented. From this consideration it is possible to conclude that a low-spin states with large many-quasiparticle configuration with energy centroids (4-8)MeV is not fully fragmented. It should therefore be low-spin states at an energy (4 8)MeV with a relatively large many-quasiparticle components in their wave functions.

Practically, no experimental data exist concerning the many-quasiparticle components of the wave function of the highly excited low-spin states. What experiments should to be performed

to answer the question concerning the existence of the many-quasiparticle components of the wave function of the highly excited states? It seems that the most straight-forward way to detect the large many-quasiparticle components is to analyze the many-nucleon transfer reactions. However, this method involves great difficulties. In [1] it has been suggested that the most favourable way to observe the many-quasiparticle components of the wave functions is to study the gamma-transition from the neutron resonance states to the states lying 1-2 MeV below them.

The one- and three- or two and four-quasiparticle configurations at excitation energies close to the neutron binding energy are strongly fragmented. At these energies, the five- and seven- or six- and eight-quasiparticle configurations start to fragment. We can expect that the wave function of the neutron resonance states contain a large components of many-quasiparticle configurations.

Some information concerning the values of the many-quasiparticle components can be obtained by studying the E1, M1 and E2 transition probabilities from the neutron resonance states to the levels with energies lower than them by 1-2 MeV. A large contribution of the many-quasiparticle configuration to the normalization of the neuron resonance wave function would enhance in E1, M1 and E2 transitions.

If the wave function of a neutron resonance states has a large eight-quaisparticle component. The enhanced E1, or M1 or E2 transitions to the states which contain specific six-quasiparticle components should be observed. This gamma-transition should be followed by the large cascade gamma-transitions. The reduced probabilities of these cascades must be much greater than the reduced probabilities of the gamma-transitions from a neutron resonance state to the low-lying states.

If the contribution of the many-quasiparticle component to the normalization of the neutron resonance wave function is equal to 20%, the corresponding reduced gamma-transition probabilities are 3-4 orders of magnitude larger than the reduced gamma-transition probabilities from the neutron resonance states to the low-lying states. It may be possible to say that the state, whose largest components is described by a single configuration more than (10-20)%, has its own individual characteristic feature.

If the neutron resonance state is very close to the neutron binding energy it may be possible to use the (n_{th},γ) reaction for detecting the many-quasiparticle components.

The above consideration allowed us to derive the following conclusions:

1) The order is governed by the large components of the wave function of the excited states.
2) Chaos takes place in the small components of the wave function of the nuclear excited states. The excited state is chaotic if its wave function is composed by only small components of many-quasiparticle or many-phonon configurations.
3) The widths of the giant resonance and the dependence of the neutron strength function on the mass number demonstrate the remnant of the regularity of the average values.
4) The nuclear mean field is responsible for the order. The superconducting pairing interaction stabilizes the regularity of the nuclear mean field.
5) The coherent interaction between quasiparticles forms the low-lying collective vibrational states as well as the giant resonances, leading to order in nuclei.
6) The quasiparticle-phonon interaction fragments the quasiparticles and phonon configuration and is responsible for the order-to-chaos transition. With increasing excitation energy the role of the quasiparticle phonon interaction, and therefore the fragmentation of the quasiparticles and/or phonons configuration, increase.
7) An experimental investigation of the many-quasiparticle components of the wave function of the neutron resonance states may be carried out by using the new generation gamma-ray detector, which could observe the enhanced gamma-transition from neutron resonances to the levels lying $(1\div2)$MeV below them.

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References

1) V. G. Soloviev, Phys. Lett. 42B (1972) 409.

2) V. G. Soloviev, Sov. J. Part. Nucl. 3 (1973) 390.

3) V. G. Soloviev, Theory of Complex Nuclei (Pergamon Press, Oxford, 1976).

4) V. G. Soloviev, Sov. Journ. Nucl. Phys. 13 (1971) 27.

5) S. Gales, Ch. Stoyanov and A. I. Vdovin, Phys. Rep. 166 (1988) 125.

6) V. G. Soloviev, Ch. Stoyanov and A. I. Vdovin, Nucl. Phys. A342 (1980) 261.

7) V. G. Soloviev, Ch. Stoyanov and V. V. Voronov, Nucl. Phys. A399 (1983) 141.

8) V. G. Soloviev, Theory of Atomic Nuclei: Quasiparticles and Phonons (Moscow, Energoatomizdat, 1988).

9) O. Bohigas and H. Weidenmüller, Ann. Rev. Nucl. Part. Sci. 38 (1988) 421.

10) F. Sakata and T. Marumori, Direction in Chaos (World Scientific, Singapore, 1992).

11) G. E. Mitchell, E. G. Bulpuch, P. M. Endt and J. R. Shriner, Phys. Rev. Lett. 61 (1988) 1473.

12) T. von Egidy, H. H. Schmidt and A. N. Behkami, Nucl. Phys. A481 (1988) 189.

13) J. F. Shriner, G. E. Mitchell and T. von Egidy, Phys. Rev. C. (to be pubslished)

14) J. Rekstad, T. S. Tveter and M. Guttormsen, Phys. Rev. Lett. 65 (1990) 2122.

15) B. R. Barrett, R. F. Casten, J. N. Ginocchio, T. Seligman and