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Hidezumi Terazawa

Institute for Nuclear Study, University of Tokyo

Midori-cho, Tanashi, Tokyo 188, Japan

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The Meaning of Dirac's Large Number Hypothesis

Hidezumi Terazawa

Institute for Nuclear Study, University of Tokyo

Midori-cho, Tanashi, Tokyo 188, Japan

Abstract

Dirac's large number hypothesis (LNH) states that the Eddington large numbers N_1 ($\equiv \alpha/Gm_e m_p \cong 10^{39}$), N_2 ($\equiv m_e/\alpha H \cong 10^{40}$) and N_3 ($\equiv 4\pi\rho/3m_p H^3 \cong 10^{80}$) are not independent but related with each other. By reconsidering the meaning of the LNH, it is shown that not only the "dynamical" LNH relation of $N_3 \sim N_1 N_2$ but also the "geometrical" LNH relation of $N_3 \sim (N_2)^2$ holds so that the LNH may not be taken as a hypothesis but become the large number rule (LNR).

The large number hypothesis (LNH) proposed by Dirac in 1937[1] is one of the most profound ideas not only in physics but also in natural science in general. It states that some large (dimensionless) numbers in nature are not independent but related with each other. The electron and proton masses $m_e\left(\cong 0.511\ \mathrm{MeV}/c^2\right)$ and $m_p\left(\cong 0.938\ \mathrm{GeV}/c^2\right)$ representing "atomic" mass (or energy) scales and the fine structure constant α ($\equiv e^2/4\pi\hbar c \cong 1/137$ where e is the proton electric charge) representing an "atomic" force strength, together with the Hubble parameter (or constant) H ($\cong 100\ \mathrm{km\ s^{-1}Mpc^{-1}}$) and the energy (or mass) density of the universe $\rho\left(\cong 10^{-26}\mathrm{kg\ m^{-3}}\right)$ representing "cosmological" scales and the Newtonian gravitational constant G ($\cong 10^{-10}\mathrm{m^3kg^{-1}s^{-2}}$) representing a "cosmological" force strength make the following Eddington (dimensionless) large numbers:[2]

$$N_1 \equiv \alpha/Gm_e m_p, \quad N_2 \equiv m_e/\alpha H, \quad N_3 \equiv 4\pi\rho/3m_p H^3. \tag{1}$$

Here and hereafter, use of the natural unit where $\hbar = c = 1$ should be understood. The magnitudes of these numbers are of order of 10^{39}, 10^{40} and 10^{80}, respectively. More precisely, by using the currently available most precise experimental data[3] of $G = 6.67259\,(85)\times 10^{-11}\mathrm{m^3kg^{-1}s^{-2}}$, $m_p = 938.27231\,(28)$ MeV, $H = 100h$ km $\mathrm{s^{-1}Mpc^{-1}}$ $(0.5 < h < 0.85)$ and $\rho \equiv \Omega\rho_c$ $(0.1 < \Omega < 2)$ with $\rho_c \equiv 3H^2/8\pi G = 1.87882\,(24)$ $\times 10^{-29}h^2\mathrm{g\ cm^{-3}}$, they are given by

$$N_1 \cong 2.269\times 10^{39}, \quad N_2 \cong 3.28\times 10^{40}/h, \quad N_3 \cong 0.744\times 10^{80}\Omega/2h. \tag{2}$$

The LNH, more concretely, states that these large numbers are not independent but related with each other as

$$N_2 = c_2 N_1 \ \text{ and } \ N_3 = c_3 (N_1)^2, \tag{3}$$

where c_2 and c_3 are constants of order close to unity. Note that these constants are related with

each other as

$$c_3/c_2 = \Omega/2. \tag{4}$$

It is well known and must be one of the motivations taken by Dirac that the LNH is remarkably well satisfied experimentally as

$$hc_2 \equiv (2h/\Omega)\, c_3 \cong 14.4. \tag{5}$$

How to explain this coincidence or to find a more fundamental theory in which the hypothesis turns out to be a consequence seems to be one of the most challenging problems in theoretical physics.

In 1971 Hoyle and Narlikar[4] proposed the model of "mass field", and in 1977 Canuto, Adams, Hsieh and Tsiang[5] proposed the model of scale-covariant gravity in order to accommodate the LNH, especially the famous Dirac's prediction of varying Newtonian constant as a consequence, with the standard Friedmann cosmology. In 1981, the present author[6] proposed the model of gravity with spontaneous breakdown of conformal invariance, which is a combination of the Hoyle-Narlikar model and the Canuto-Adams-Hsieh-Tsiang model, showed that spontaneous breakdown of conformal invariance due to the very existence of our Universe may originate all the mass (or length) scales ranging from the Planck mass $G^{-1/2}(\cong 10^{19}$ GeV) to the Hubble parameter ($H \cong 10^{-42}$ GeV), suggested that the photon may have a curvature-dependent mass which is as small as 10^{-42}GeV, demonstrated that an approximate LNH relation of

$$N_3/N_1N_2\ (\equiv 4\pi G\rho/3H^2 \equiv \Omega/2) \cong 1/2 \quad \text{or} \quad c_3/c_2 \cong 1/2 \tag{6}$$

holds in the standard Friedmann cosmology in the usual Einstein theory of gravity with the non-varying Newtonian constant unless there occurs a certain accidental coincidence and

further speculated that another approximate LNH relation of

$$N_2/N_1\ (\equiv Gm_e^2m_p/\alpha^2H) \sim 1 \quad \text{or} \quad c_2 \sim 1 \tag{7}$$

holds if the photon mass is induced by charge non-conservation occurring at the Planck length $G^{1/2}(\cong 10^{-33}$cm) and if it is the minimum non-vanishing mass of order of the Hubble parameter that is allowed in our Universe. The purpose of this letter is to show that not only the "dynamical" LNH relation (6) but also the "geometrical" LNH relation of

$$N_3/(N_2)^2 \sim 1 \quad \text{or} \quad c_3/(c_2)^2 \sim 1 \tag{8}$$

holds so that the LNH may not be taken as a hypothesis but become the large number rule (LNR), i.e. $c_2 \sim 1$ and $c_3 \sim 1$.

Let us first remember the meanings of the large numbers. They are all simple: N_1 is the ratio of the Coulombic electric force strength ($e^2/4\pi r^2$) to the Newtonian gravitational one (Gm_em_p/r^2) between an electron and a proton at a distance (r), N_2 is the ratio of the present "age" (or "radius") of our Universe ($H^{-1} \cong 10^{10}$ yr) to the classical radius of the electron ($\alpha/m_e \cong 10^{-13}$cm) and N_3 is the ratio of the "total mass (or energy)" of our Universe ($4\pi\rho/3H^3 \cong 10^{53}$kg) to the mass of the proton ($m_p \cong 1$ GeV) which can be approximately taken as the baryon number of our present Universe. Let us also note that some combinations of the large numbers have simple physical meanings. For example, the product of m_p/N_1N_2 ($\equiv Gm_p^2H$) can be taken as the magnitude of the Newtonian gravitational (potential) energy of two protons at a distance of H^{-1}. Furthermore, the total internal (gravitational potential) energy (or gravitational self-mass) of the Universe has an order of magnitude of $(N_3)^2m_p/N_1N_2$. This should be compared with the total mass (or energy) of the Universe, which is approximately given by N_3m_p. Therefore, <u>the "dynamical" LNH relation (6) means that the gravitational self-mass of the Universe has the same order of magnitude as the total mass of the Universe</u>. It is well known that this is always the case in the standard Friedmann

cosmology in the usual Einstein theory of gravity.

More precisely, let us recall the "initial-value equation" of Einstein field equations given by

$$H\left(\equiv \frac{\dot{a}}{a}\right) = \left(\frac{8\pi G\rho}{3} - \frac{\kappa}{a^2}\right)^{1/2}, \tag{9}$$

where κ=0, +1 or -1 for the spacially flat, closed or open universe and a is the scale factor defined in the Friedmann-Robertson-Walker space-time metric of

$$ds^2 = -dt^2 + a^2(t)\,[dr^2/(1-\kappa r^2) + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)]. \tag{10}$$

Eq. (9) can be re-written in terms of the large numbers as

$$\frac{2\,N_3}{N_1N_2} - \frac{\kappa}{a^2H^2} = 1. \tag{11}$$

Therefore, the exact LNH relation of $N_3/N_1N_2 = 1/2$ (or $c_3/c_2 = 1/2$) holds for κ=0 (or the spacially flat universe). For κ=1 (or the closed universe), the approximate LNH relation (6) must hold unless $2N_3/N_1N_2 \cong 1/a^2H^2$. The latter approximate relation can be transformed into another approximate LNH relation of

$$N_3'/N_1N_2' \;(\equiv 4\pi\, G\rho\, a^2/3) \cong 1/2 \tag{12}$$

where N_2' and N_3' are the modified large numbers defined by

$$N_2' \equiv m_c a/\alpha, \quad N_3' \equiv 4\pi\rho\, a^3/3m_p. \tag{13}$$

Even for κ=-1 (or the open universe), Eq. (11) indicates that the approximate LNH relation (6) still holds unless $a^2H^2 \cong 1$ to a very good accuracy. The latter relation can be transformed into

$\dot{a} \cong 1$ which would lead to a very special case where $a \cong t$ and, therefore, the deceleration parameter q ($\equiv -\ddot{a}a/\dot{a}^2$) almost vanishes. In any case, to sum up, the approximate LNH relation (6) must hold in the standard Friedmann cosmology in the usual Einstein theory of gravity unless there occurs a certain accidental coincidence.

A point to remember here is that the "dynamical" LNH relation (6) holds due to the Einstein-Friedmann dynamics in which the internal gravitational energy (or self-mass) of the universe reduces (and exactly cancels in a special case) the total mass of the universe. The special case where the former energy exactly cancels the latter would remind us of the possible cancellation of the divergent electromagnetic and weak self-masses of quarks and leptons in the unified gauge theory of electroweak interactions.[7] It also reminds us of the exotic possibility in composite models of quarks and leptons[8] that subquarks, the constituents of quarks and leptons, whose masses are as large as the Planck mass are bound tightly at a distance as small as the Planck length so that quarks and leptons may be very light due to the cancellation of the large gravitational energy and the large subquark masses. It seems amazing that our vast and complicated Universe may have such similarity to an "elementary" particle!

Let us now move on to derivation of the "geometrical" LNH relation (8). Since the "initial-value equation" (9) has already been used in deriving the "dynamical" LNH relation (6), the "acceleration (or dynamic) equation" of

$$q\left(\equiv -\frac{\ddot{a}a}{\dot{a}^2}\right) = \frac{1}{2} + \frac{\kappa}{2a^2H^2} + \frac{4\pi Gp}{H^2} \tag{14}$$

is the only one independent equation left among the Einstein field equations. However, the latter equation relates the deceleration parameter q with the pressure p but does not directly relate the Hubble parameter H with the mass-energy density ρ as the "initial-value equation" (8). Although ρ and p are related with each other by the "law of conservation of energy" as

$$(\rho a^3)_{,t} = -p(a^3)_{,t}, \tag{15}$$

these three Eqs. (9), (14) and (15) are not independent with each other. Therefore, we cannot

expect to derive any other LNH relations from the usual Einstein field equations in the standard Friedmann cosmological model but can hope to do from some unknown "initial conditions". What kind of "initial condition" may lead to such LNH relation as (8)?

As the product of m_p/N_1N_2 ($\equiv Gm_p^2H$) has been taken as the magnitude of the Newtonian gravitational (potential) energy of two protons at a distance of H^{-1} in deriving the "dynamical" LNH relation (6), the product of $\pi/(N_2)^2$ ($\equiv \pi\alpha^2H^2/m_e^2$) can be taken as the solid angle of a particle whose radius is r_e ($\equiv \alpha/m_e \cong 2.8\times10^{-13}$cm), the classical radius of the electron, seen from a distance of H^{-1} ($\cong 10^{10}$yr). More precisely, let r_p be the charge radius of the proton. Then, the product of $\pi r_p^2H^2$ can be approximately taken as the solid angle of a proton seen from a distance of H^{-1}. Therefore, the product of $\pi N_3 r_p^2 H^2\left[\equiv \pi(r_p/r_e)^2 N_3/(N_2)^2\right]$ can be approximately taken as the sum of solid angles of all the protons (or baryons) seen from any point in the Universe since the average distance between two baryons in the Universe is of order of H^{-1}. Furthermore, since the experimental value of r_p ($\cong 0.81\times10^{-13}$cm) has the same order of magnitude as that of r_e ($r_p/r_e \cong 0.29$), <u>the "geometrical" LNH relation (8), if it holds, means that the sum of solid angles of all the baryons seen from any point in the Universe is of order close to 4π</u>.

This remarkable observation would lead us to either one of the following assumptions for the "initial condition" of the Universe: 1) In the present Universe (and probably at any time in the history of our Universe), the amount of matter in the Universe and the size of the Universe are related with each other in such a way as any light (or observer) may be geometrically blocked (or blinded) once (and probably only once) by a baryon before it travels for a period of order of H^{-1} (or a distance of order of H^{-1}) to any direction from any point in the Universe. This possibility may be closely related to the fact that all the possible measurements of the cosmological parameters of the Universe such as H^{-1} are naturally affected by the attenuation of light by the inter-galactic matter in the Universe. 2) In the history of our Universe, there must be the era in which the space-time was of three dimensions and all the baryons were most packed in the two-dimensional space in the Universe. After the three-dimensional era, there occurred the phase transition of the space-time to the present four-

dimensional Universe, which has naturally left the miraculous situation described in 1). This possibility seems much more exotic than the first possibility but does not yet seem impossible. In fact, the present author has suggested that such dimensional phase transitions of the space-time (or the universe) may occur at the minimum action near the Planck scale[9].

If we make either one of these assumptions, we can immediately derive the "geometrical" LNH relation (8) by tracing back along the deductive way leading to the assumptions. Therefore, we have almost completed the purpose of this letter to show that <u>not only the "dynamical" LNH relation (6) but also the "geometrical" LNH relation (8) holds so that the LNH may not be taken as a hypothesis but become a rule, the LNR</u>. What is left as an important subject for future investigations is to find (or formulate) a theory in which these assumptions can be accommodated. To this end, we must at least extend the Einstein field equations in the standard Friedmann cosmology by taking into account the finite non-vanishing sizes of matter particles such as the baryons as van der Waals extended the equation of state for ideal gas (or liquid) by taking into account those of atoms and molecules. Before conclusion, it may go without saying that the new estimate of $H = 87\pm7$ km $s^{-1}Mpc^{-1}$ given by Pierce and colleagues[10] would not change any part of discussions in this letter.

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