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Upper Bound on the Top Quark Mass for Supersymmetric SU(5) Model

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From the requirement of the absence of the Landau pole singularity for the effective top quark Yukawa coupling constant $\bar{h}_t$ up to the Planck scale in SU(5) supersymmetric model we find for the top quark mass an upper bound $m_t \leqslant 187$ Gev. For the SU(5) fixed point solution, which can be interpreted as the case of composite superhiggses we find $m_t \approx 183$ Gev. From the requirement that the m_b/m_τ prediction of the supersymmetric SU(5) has been succesfull we find that 145 Gev $\leqslant m_t \leqslant$ 190 Gev.

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Верхнее ограничение на массу Т-кварка
в суперсимметричной SU(5)-модели

Из требования отсутствия полюса Ландау для эффективной топ кварковой юкавской константы h_t вплоть до масштаба Планка в суперсимметричной SU(5)-модели мы находим верхнее ограничение на массу топ кварка $m_t \leq$ $\leq$ 187 GeV. Для SU(5) фиксированного решения, интерпретируемого как составной суперхиг с, мы получаем m_t = 183 GtV. Из требования успешности предсказания m_b/m_τ в суперсимметричной SU(5) мы получаем 145 GeV $\leq$ $\leq m_t \leq$ 190 GeV.

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At present one of the most urgent problems in experimental high energy physics is the search for the top quark. The existing lower bound on the top quark mass m_t>91GeV [1] means that the top quark Yukawa coupling constant h_t is not small and probably the top quark plays an essential role in electroweak symmetry breaking.The precision tests of the standard model with the electroweak readiative corrections allow to extract the value of the t-quark mass. Recent analysis [2] gives $m_t = (132\ ^{+24}_{-26})$Gev. for the Higgs mass $m_H = m_Z$. In refs.[3,4,] from the requirement of the absence of the Landau pole singularity up to some scale of about $0(10^{16})$ Gev for the top quark effective Yukawa coupling constant an upper bound on the top quark mass of the order of O(200GeV) has been derived in supersymmetric extension of the Weinberg-Salam model.

In this paper we obtain an upper bound on the t-quark mass in SU(5) grand unified supersymmetric model from the requirement of the absence of the Landau pole singularity up to the scale $M_{Pl}=10^{19}$GeV. For the fixed point solution of the renormalization group equations in SUSY SU(5) we find that $m_t \approx 183$ GeV. We also estimate that if at GUT-scale $\bar{h}_t^2(M_{GUT}) = 2^{\pm 1}\bar{g}_5^2(M_{GUT})$ then the top quark mass is in the range of (184 - 162) Gev. Further from the requirement of the right prediction of m_τ/m_b in SU(5) we find similar bound on t-quark mass 145 Gev $\leq m_t \leq$ 190 GeV.

Consider the supersymmetric SU(5) model [5] with standard kinetic terms and with the superpotential

$$W = 2^{0.5}\lambda_1 T_{\alpha\beta}(\bar{F}^\alpha\bar{H}^\beta - \bar{F}^\beta\bar{H}^\alpha) + (1/4)\lambda_2\varepsilon^{\alpha\beta\gamma\delta\varepsilon}T_{\alpha\beta}T_{\gamma\delta}H_\varepsilon + \lambda_3 Tr\Phi^3 + \lambda_4\bar{H}\Phi H + M_1 Tr\Phi^2 + M_2\bar{H}H \quad (1)$$

Here we consider only the third (top quark) supermatter generation and neglect the Yukawa couplings of the first and the second generations. The model contains the matter supermultiplets $\bar{F}^\alpha(\bar{5})$, $T_{\alpha\beta}(10)$ which correspond to the third generation and the superhiggses $\Phi(24)$, $\bar{H}^\alpha(\bar{5})$, $H_\alpha(5)$. The nonzero vacuum solution $<\Phi> = Diag(2,2,2,-3,-3)$ breaks the SU(5) gauge group to the electroweak $SU(3)\otimes SU(2)\otimes U(1)$ gauge group. For energies higher then the grand unified scale M_{GUT} we have effective restoration of the SU(5) gauge group. We shall neglect all Yukawa coupling constants in the effective potential except that one for the top quark Yukawa coupling constant λ_2. The corresponding renormalization group equations in one loop approximation have the form (for the case of three generations) [6]

$$d\bar{h}_t^2/dt = A(\bar{h}_t^2)^2 - B\,\bar{h}_t^2\,\bar{g}_5^2, \quad (2)$$

$$d\bar{g}_5^2/dt = -b(\bar{g}_5^2)^2, \quad (3)$$

$$b = 3/8\pi^2,\ A = 9/8\pi^2,\ B = 192/80\pi^2$$

Here g_5 is the SU(5) gauge coupling, $t = \ln(M/\mu)$ and $h_t \equiv \lambda_2$. The solution of the eqn.(2,3) has the form

$$\bar{g}_5^2 = g_5^2/(1+g_5^2 bt), \quad (4a)$$

$$1/\bar{h}^2 = A(g_5^2(B-b))^{-1}(1+g_5^2 bt) + (1/\bar{h}^2 - A((B-b)g_5^2)^{-1})(1+g_5^2 bt)^{B/b}, \quad (4b)$$

We shall use the numerical values $\alpha_{GUT} \equiv g_{5GUT}^2/4\pi = 1/24$ for the

SU(5) gauge coupling constant at the unification scale $M_{GUT} = 10^{16}$ Gev. From the requirement of the absence of the Landau pole singularity up to the Planck scale $M_{PL} = 1.2\ 10^{19}$ Gev from the equation (4b) we find that the t-quark Yukawa coupling constant $\bar{h}_t(M_{GUT}) \leq 1.4$. For the renormalization group equations (4a,4b) the infrared fixed point solution [7] is

$$\bar{h}_t^2 = k\bar{g}_5^2,\ k = (B - b)/A = 1.8 \tag{5}$$

For the infrared fixed point solution (5) we find that $\bar{h}_5^2(M_{GUT}) = 0.94$. It should be noted that in our analysis we neglected all Yukawa coupling constants in the superpotential (1) except that one for the top quark constant. However the neglected Yukawa coupling constants give positive contibutions to the β-functions that can only make the bound $\bar{h}_t(M_{GUT}) \leq 1.4$ more stringent.

For the special fixed point solution (5) the ultraviolet asymptotics for the propagator of the superfield $H_\alpha(5)$ is (for the scalar component of the superfield)

$$D_H(p^2) \sim -(i/p^2)[\ln(-p^2/\mu^2)]^{0.2} \tag{6}$$

From the equal-time commutation relation

$$[\partial_0 H_\alpha\ ,\ H_\beta^*]\big|_{x^o=y^o} = \delta_{\alpha\beta}(1/iZ_H)\delta^3(x-y) \tag{7}$$

for the scalar component of the superfield $H_\alpha(5)$ and the Kallen-Lehmann representation for the propagator $D_H(p^2)$ we find that the ultraviolet asymptotics for the $D_H(p^2)$ propagator is $-i/p^2Z_H$ (here Z_H is the wave function renormalization). Thus we find from the relation (6) that the wave function renormalization Z_H is equal to zero in the limit of the regularization removing. Therefore we can treat the superfield $H_\alpha(5)$ as composite, namely:

the probability $Z_H = |\langle\varphi|\phi\rangle|^2$ (where φ is the "bare" state and Φ is the "renormalized" state) of the physical state being at the "bare" state is equal to zero and besides the kinetic term for the renormalized superfield $H_\alpha^{ren} = Z_H^{1/2} H_\alpha$ vanishes in the limit of the regularization removing [8]. So the "compositeness" condition allows to determine the top quark Yukawa coupling constant and hence to predict the top quark mass.

The renormalization group equations for the SU(3)⊗SU(2)⊗U(1) electroweak supersymmetric gauge theory allow to connect the top quark Yukawa coupling constant at grand unified scale with the Yukawa coupling constant at the scale of the supersymmetry breaking M_{SUSY} ; to relate also the Yukawa coupling constant at the M_{SUSY} scale with the observable Yukawas coupling constant at the electroweak scale M_W we have to use again the renormalization group equations for the standard Weinberg-Salam theory. The renormalization group equations have the form [9,10,]

$$d\,\bar{h}_t^2/dt = A_1(\bar{h}_t^2)^2 - h_t^2(B_1\bar{g}_1^2 + B_2\bar{g}_2^2 + B_3\bar{g}_3^2), \tag{8}$$

$$d\bar{g}_i^2 = -b_i\bar{g}_i^2, \tag{9}$$

with $b_1 = -22/16\pi^2$, $b_2 = -2/16\pi^2$,$b_3 = 6/16\pi^2$, $A_1 = 12/16\pi^2$, $B_1=(26/9)1/16\pi^2$, $B_2=6/16\pi^2$, $B_3=(32/3)1/16\pi^2$ for the supersymmetric SU(3)⊗SU(2)⊗U(1) and with $b_1 = -(82/6)/16\pi^2$, $b_2 = (19/3)/16\pi^2$, $b_3 = 14/16\pi^2$, $A_1 = 9/16\pi^2$, $B_1 = (17/6)/16\pi^2$, $B_2 = (9/2)/16\pi^2$, $B_3 = -16/16\pi^2$ for the standard case. The solution of the equation (9) can be represented in the form

$$1/\bar{h}^2(M) = \prod^{3} (1 + g_i^2 b_i t)^{B_i/b_i} \{1/\bar{h}^2(\mu) - A_1 K(t)\}, \tag{10}$$

$$K(t) = \int_0^t [\prod_{i=1}^{3} (1+g_i^2 b_i t')^{-B_i/b_i}]dt', \quad t = \ln(M/\mu)$$

Numerically we find

$$(\bar{h}_t^2(0.1\text{Tev})^{-1} = \frac{0.088}{\bar{h}_t^2(M_{GUT})} + 0.815 \text{ (for } M_{SUSY} = 0.1\text{Tev}), \tag{11}$$

Using our previously derived bound $\bar{h}_t(M_{GUT}) \leqslant 1.4$ we find that $\bar{h}_t(0.1\text{Tev}) \leqslant 1.08$ for $M_{SUSY} = 0.1$Tev. The top quark mass is equal to $m_t = \bar{h}_t(m_t)\langle H\rangle$. From the experiment we know that $\langle H\rangle^2 + \langle\bar{H}\rangle^2 = (174 \text{ Gev})^2$. In the supersymmetric Weinberg-Salam model with soft supersymmetry breaking we have the following inequality [11] for the Higgs boson mass:

$$m_H \leqslant m_Z \cos(2\beta), \quad tg(\beta) = \langle\bar{H}\rangle/\langle H\rangle \tag{12}$$

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Using this inequality (12) and the experimental bound $m_H \geqslant 59$ Gev [12] we find that $\langle H\rangle \geqslant 160$ Gev. Moreover for big values of the top quark Yukawa coupling constant $\langle\bar{H}\rangle \ll \langle H\rangle$, in the rest of this paper we shall assume that $\langle H\rangle \approx 174$ Gev. The corresponding bound for the top quark mass is $m_t \leqslant 187$ Gev. Note that from the requirement of the absence of the Landau pole for the effective top-quark Yukawa coupling constant up to the GUT scale we find that $m_t \leqslant 193$ Gev.

For the fixed point solution (5) we find that $\bar{h}_t(0.1\text{Tev}) = 1.05$ and m_t=183 Gev for $\langle H\rangle \approx 174$ Gev. It appears that the top quark mass depends rather weakly on the Yukawa coupling constant at grand unified scale provided it has the order of the grand unified coupling constant. For instance, for $0.5g_{5GUT}^2 \leqslant \bar{h}_t^2(M_{GUT})$

$\leq 2\bar{g}^2_{5GUT}$ we find that 162 Gev $\leq m_t \leq$ 184 Gev and for $g^2_{5GUT}/3 \leq \bar{h}^2_t(M_{GUT}) \leq 3g^2_{5GUT}$ we find that 151 Gev $\leq m_t \leq$ 186 Gev. To obtain the top quark mass predictions for the M_{SUSY} = 1 Tev we have to solve in addition the renormalization group equations in the region $M_{SUSY} \leq E \leq m_w$. We have found that the corresponding values of the top quark mass are increased approximately by 3% compared to the case when M_{SUSY} = 0.1 Tev.

It should be noted that a relatively large top quark Yukawa coupling constant $\bar{h}_t(M_w)$ is also required if we want to preserve the succesfull prediction of the standard SU(5) in SUSY SU(5) for m_b/m_τ. Namely, in SUSY SU(5) with $M_{SUSY} \sim M_w$ one can find [6] that

$$m_b(M_w)/m_\tau = [\bar{\alpha}_3(M_w)/\alpha_{GUT}]^{8/9}[\bar{\alpha}_1(M_W)]^{10/99}\Delta, \tag{13}$$

where

$$\Delta = (1 + 12\bar{h}^2_t(M_{GUT})F)^{-1/12}, \quad F = (4\pi)^{-2}\int_{M_{GUT}}^{m_w} f(t)dt,$$

$$f(t) = (\alpha_{GUT}/\alpha_3(t))^{-16/9}(\alpha_{GUT}/\alpha_2(t))^{3}(\alpha_{GUT}/\alpha_1(t))^{13/99}$$

$$t = \ln(M/\mu).$$

For $m_b(m_b)$ = (4.25 ± 0.15) Gev [13] the "experimental" value of $m_b(m_w)/m_\tau$ is 1.8 ± 0.05. The main uncertainties come from the not well known threshould effects and from the nonexact knowledge of strong coupling constant which according to recent mesurements is [14] $\bar{\alpha}_3^{-1}(M_z)$ = 9.26 ± 0.43. As far as the overall uncertainty is around 10% we shall require that the theoretical and "experimental" values have to coincide also only up to 10%. From this requirement we find that $0.14 \leq \bar{h}^2_t(M_{GUT}) \leq$ 3.6. For the top quark mass we find that 145 Gev $\leq m_t \leq$ 190 Gev.

To conclude we have found that the most probable values for top quark mass are in the range 150 - 190 Gev. If we combine our bounds with the bound $m_t = (132 \, ^{+24}_{-26})$ Gev obtained in ref.[2] we find that the prefered value of m_t is between (150 -160) Gev.
It should be noted that similar bounds have also been established for the case of general two Higgs doublet model [15] and for the case of SUSY broken at high scale [16].

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