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H. Niewodniczański
INSTITUTE OF NUCLEAR PHYSICS,
Kraków, Poland.

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HIGH DENSITY BEHAVIOUR OF NUCLEAR SYMMETRY ENERGY

Marek Kutschera

Henryk Niewodniczański Institute of Nuclear Physics
Department of Theoretical Physics
ul. Radzikowskiego 152, 31-342 Kraków, Poland

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Marek Kutschera

H. Niewodniczański Institute of Nuclear Physics

ul. Radzikowskiego 152, 31-342 Kraków, Poland

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Abstract

There exists profound discrepancy in the high density behaviour of the nuclear symmetry energy obtained in realistic variational many-body (VMB) calculations and in relativistic mean-field (RMF) calculations. While the symmetry energy decreases to negative values in the former approach it increases monotonically in the latter one. The origin of this discrepancy is discussed and it is argued that VMB prediction is more reliable. It is shown that vanishing of the symmetry energy implies proton-neutron separation instability in dense matter.

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The nuclear symmetry energy $E_s(n)$, as measured at saturation density, is $E_s(n_0) = 34 \pm 4MeV$ [1]. Its behaviour at higher densities, however, is not uniquely determined by model equations of state. In fact, this quantity is subject to the worst uncertainty among all properties of dense matter. Realistic VMB calculations predict that $E_s(n)$ after saturating at a few times n_0 decreases reaching negative values at high densities [2]. RMF calculations predict $E_s(n)$ to increase monotonically with density [3]. This discrepancy has profound consequences for neutron star matter as the behaviour of the symmetry energy is reflected by the composition of the neutron star matter. In the former case the neutron star

matter becomes pure neutron matter at high densities [2,4] whereas it becomes proton-rich nuclear matter in the latter case [3]. Hyperons are also suppressed at high densities in the former approach [4] while there are plenty of them in the latter one [3].

The aim of this paper is to trace back origin of the discrepancy. We analyse contributions to the symmetry energy in both cases. We conclude that VMB prediction is more reliable as based on phenomenological interactions and treating properly the tensor force contribution. The RMF approach is too simplified and its prediction of high density $E_s(n)$ is not to be trusted. We also discuss consequences of VMB result which indicate that there exits a separation instability in dense matter.

The energy per particle of nucleon matter with proton fraction x obtained in VMB calculations can be parametrized as [5]

$$E(n,x) = T(n,x) + V_0(n) + (1-2x)^2 V_2(n), \tag{1}$$

where $T(n,x)$ is the kinetic energy contribution; V_0 and V_2 are interaction contributions. This is a very good approximation as far as the x dependence is concerned [5].

In simple relativistic mean-field models of nucleon matter, which include π, σ, ω and ρ mesons [3], the energy per particle is

$$E(n,x) = T_R(n,x) + \frac{U(\sigma)}{n} + \frac{1}{2}\frac{g_\omega^2}{m_\omega^2}n + \frac{1}{8}\frac{g_\rho^2}{m_\rho^2}(1-2x)^2 n. \tag{2}$$

Here $T_R(n,x)$ is the relativistic kinetic term, which involves the σ-field and $U(\sigma)$ is the potential term of the scalar field. If we neglect the weak x-dependence of the scalar field, eq.(2) has the same x-dependence as eq.(1) with the function $V_2(n)$

$$V_2(n) = \frac{1}{8}\frac{g_\rho^2}{m_\rho^2}n. \tag{3}$$

In the following we shall use the eq.(1) as adequately describing $E(n,x)$ in both cases and we shall focus on the function $V_2(n)$.

The nuclear symmetry energy is

$$E_s(n) = E_{kin}(n) + V_2(n), \tag{4}$$

where E_{kin} is the kinetic contribution and the function $V_2(n)$ measures the difference of the interaction energy of neutron matter and symmetric nuclear matter. Discrepancy mentioned above is due to a different behaviour of $V_2(n)$ at high densities in the two approaches.

The behaviour of $E_s(n)$ is generally similar for all realistic nucleon interactions [2,4]: VMB calculations reproduce the empirical value $E_s(n_0)$ with some accuracy. At higher densities $E_s(n)$ increases near n_0 then saturates at a few times n_0 and finally decreases to negative values at high densities. From eq.(4) one can see that this is the interaction term $V_2(n)$ which becomes negative at high densities.

In RMF models the symmetry energy is accounted for by introducing a mean ρ-meson field. This contribution is required as the kinetic term gives only about one half [3] of the empirical value $E_s(n_0)$. The function $V_2(n_0)$, eq.(3), with properly chosen g_ρ provides the missing part. The function $V_2(n)$ increases then linearly with density for higher n.

In Fig.1 the function $V_2(n)$ is shown in both cases for nonrelativistic E_{kin} in eq.(4). The VMB function $V_2(n)$ corresponds to the $UV14 + TNI$ interactions and is calculated in Ref.[2].

In order to understand why the two approaches give contradictory predictions let us briefly analyse contributions to V_2 in both approaches. It is easy to recognize these contributions for VMB calculations at high densities where only repulsive central interactions matter. Negative values of $V_2(n)$ indicate that interaction energy in neutron matter is smaller than in symmetric matter at high densities. Pandharipande and Garde [4] showed that this is a simple consequence of the nuclear interactions: The central part of the Reid potential [6] they considered for the $T = 0$, $^3S_1$ N-N pair is more repulsive than that for $T = 1$, $^1S_0$ N-N pair. The same effect occurs for modern two-body potentials, $UV14$

[7] and $AV14$ [8], used by [2]. Thus, at high densities an energetically favorable state corresponds to pure neutron matter and protons should disappear from nuclear matter.

At lower densities $V_2 > 0$ and the interaction energy in symmetric nuclear matter is lower than in pure neutron matter. This is because at low densities the attractive tensor force contribution is important in $T = 0$ state [4]. Tensor force contribution is fully through correlations and hence it vanishes at high densities. At low densities the tensor force attraction wins over the short range repulsion for $T = 0$ pairs. Thus, in realistic nuclear matter calculations, the tensor force contribution is responsible for the positive value of $V_2(n_0)$ which gives the empirical value of the symmetry energy $E_s(n_0)$.

This physically convincing mechanism can not work in RMF models of nuclear matter since tensor forces do not contribute in the mean field approximation. Instead, to reproduce the empirical value of the symmetry energy $E_s(n_0)$ which requires making the interaction energy of symmetric nuclear matter lower than that of neutron matter, a mean ρ-meson field which couples to isospin density $t_3 = n_P - n_N$, is added. One should, however, notice that this mechanism works just in an opposite way compared with the tensor force in VMB calculations. It adds repulsion in neutron matter, $\Delta E_{neut} \sim n$, whereas it does not affect the symmetric nuclear matter where $t_3 = 0$. In this way a proper splitting of interaction energies in symmetric nuclear matter and in neutron matter at n_0 is achieved by a very different mechanism from that provided by tensor forces.

The fact, that the two mechanisms differ drastically is apparent at high densities. The tensor force contribution, in VMB calculations, vanishes at high densities making interaction energy in symmetric matter higher than in neutron matter and hence $V_2(n) < 0$. The ρ-meson contribution in RMF calculations adds repulsion in neutron matter at any density, making symmetric nuclear matter energetically favorable and giving $V_2(n) \sim n > 0$. It is clear from the above analysis that this RMF result should not be trusted since two main features determining the behaviour of symmetry energy, namely tensor forces and different repulsive cores for isospin-singlet and isospin-triplet nucleon pairs, are not

accounted for by RMF models.

The fact, that the function $V_2(n)$ decreases to negative values at high densities, as predicted by VMB calculations, has important consequences for dense matter in neutron stars. It implies that the proton fraction x of β-stable nuclear matter, which ,neglecting muons, is given by the equation

$$(3\pi^2 nx)^{1/3} = \mu_N(n,x) - \mu_P(n,x), \tag{5}$$

where

$$\mu_N(n,x) - \mu_P(n,x) = \frac{1}{2m}(3\pi^2 n)^{2/3}[(1-x)^{2/3} - x^{2/3}] + 4(1-2x)V_2(n). \tag{6}$$

disappears at high density. This occurs at a density n_v such that right-hand side of eq.(6) vanishes for $x = 0$. In Fig.2 we show $x(n)$ corresponding to the $UV14 + TNI$ interactions [2] with V_2 parametrization from Ref.[9].

For $V_2(n) \sim n$ the proton fraction increases monotonically with density and neutron star matter becomes proton-rich at high densities. This is the case for RMF models, Fig.2.

The behaviour of the VMB symmetry energy also suggests that there exists an instability with respect to separation of protons and neutrons at lower densities, where the protons are still present. By performing a standard analysis [10] of small density fluctuations one can obtain the instability condition

$$w_0 = \frac{\partial \mu_P}{\partial n_P} - \frac{(\partial \mu_P/\partial n_N)^2}{\partial \mu_N/\partial n_N} < 0. \tag{7}$$

Here w_0 is the effective proton-proton potential in the long-wavelength limit [10]. In Fig.3 we plot w_0 for a few values of x. As one can see, w_0 becomes negative at sufficiently high a density in all cases. The instability corresponds to separation of protons and neutrons. This is due to positive values of $\partial \mu_P/\partial n_N$ at high densities.

There exist a possibility of a different separation mechanism which results in localization of individual protons in neutron matter. This is discussed in Refs.[10,11].

In conclusion, we have shown that the discrepancy in the predicted high density behaviour of the nuclear symmetry energy should be resolved in favour of the VMB result, which shows that $E_s(n)$ goes negative at high n. Such a behaviour implies that protons are a dilute component of the neutron star matter and tend to be separated from neutrons.

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FIGURE CAPTIONS

Fig.1

V_2 as a function of baryon density for RMF and VMB calculations. Both functions fit the empirical value $E_s(n_0)$ for nonrelativistic $E_{kin}(n_0)$. The VMB curve corresponds to the $UV14 + TNI$ interaction, as parametrized in Ref.[9].

Fig.2

The proton fraction of the β-stable nuclear matter for the two functions V_2 from Fig.1.

Fig.3

The effective proton-proton potential, eq.(7), for different functions V_0 and V_2 and for indicated proton fractions. Soft and hard V_0 correspond to $UV14+TNI$ and $AV14+UVII$ interactions of Ref.[2], respectively. The function V_2 for curves A and B vanishes at $n_x = 0.5fm^{-3}$ and $n_x = 0.7fm^{-3}$, respectively. The latter case corresponds to the $UV14+TNI$ interactions.

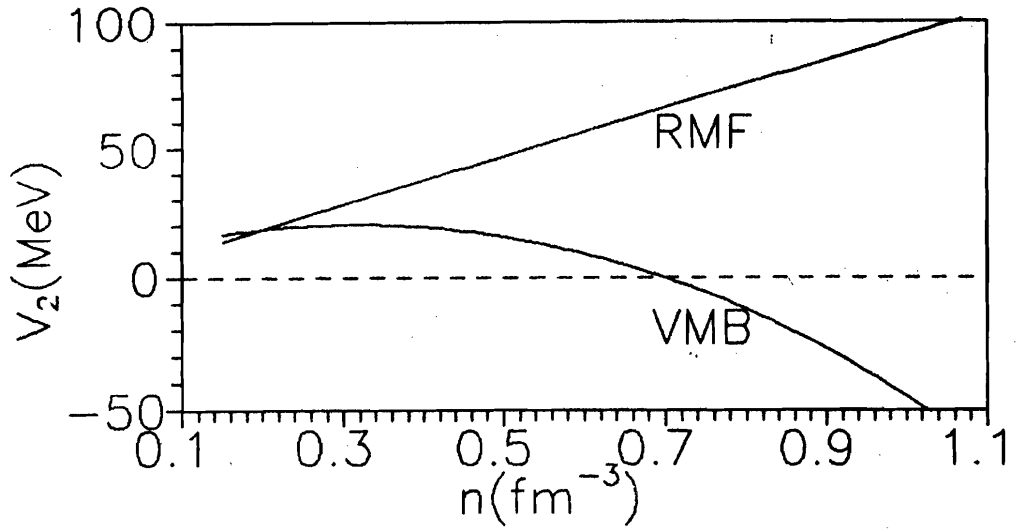
100
50
0
−50
V_2(MeV)
RMF
VMB
0.1
0.3
0.5
0.7
0.9
1.1
n(fm^{-3})

Fig. 1

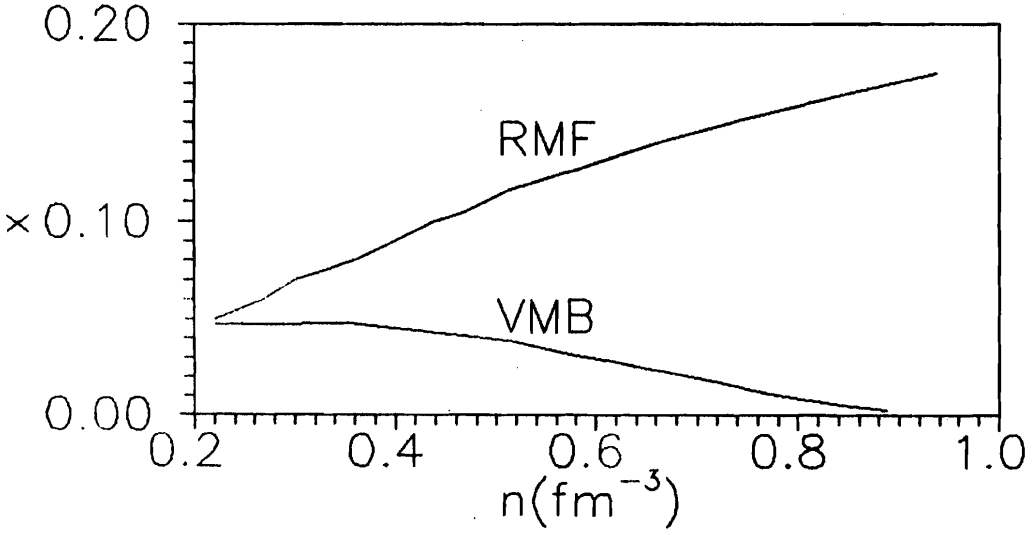
0.20
×0.10
0.00
RMF
VMB
0.2
0.4
0.6
0.8
1.0
n(fm^{-3})

Fig. 2

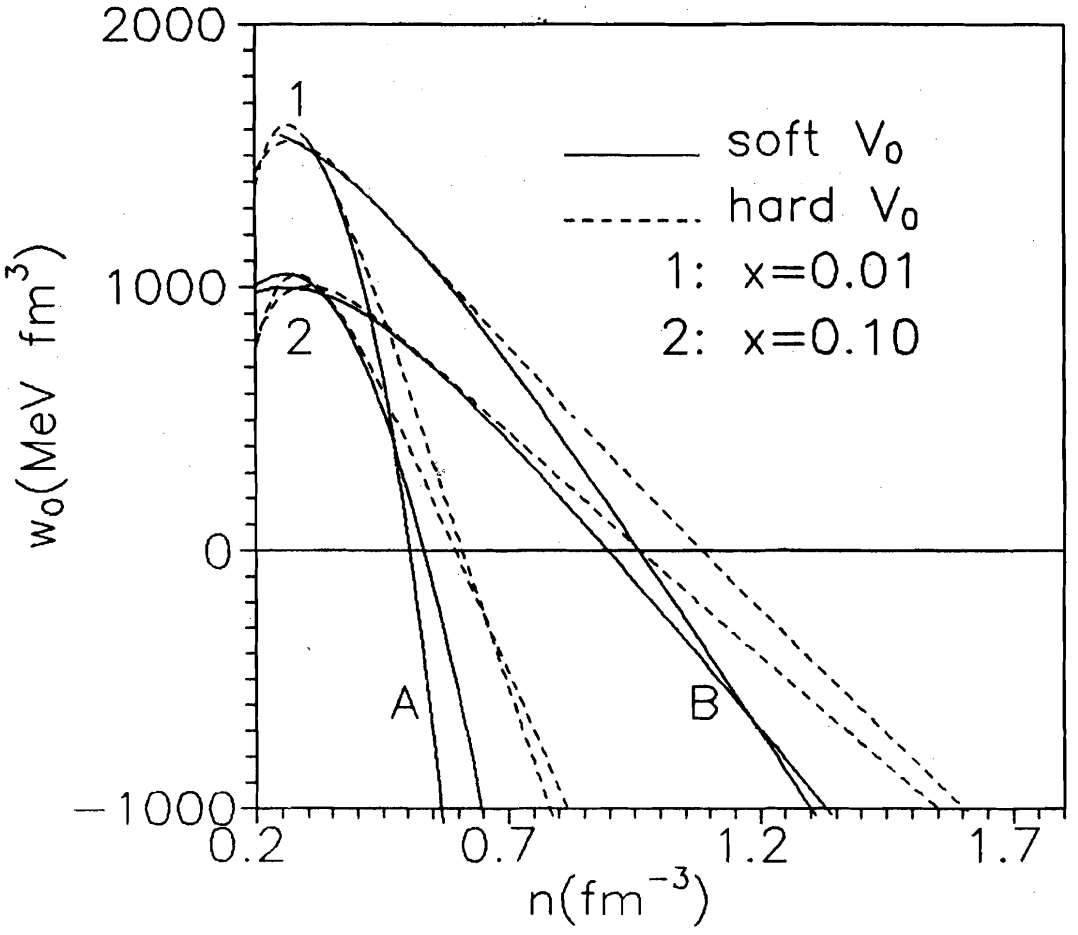
2000
1000
0
−1000
0.2
0.7
1.2
1.7
w_0(MeV fm^3)
n(fm^{-3})
1
2
A
B
soft V_0
hard V_0
1: x=0.01
2: x=0.10

Fig. 3