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Comment on the paper

"Magnetic Monopoles, Galilean Invariance and Maxwell's Equations"

by F.S. Crawford, [Am.J.Phys. 60(1992)109-114]

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Recently, F. Crawford [1] wrote an interesting paper on Galilean electrodynamics. There is however one point which needs clarification. In Ref.[1] the transformation rules for the electromagnetic fields $\vec{E}$ and $\vec{B}$ in the Galilean limit were found to have the following form

$$\vec{E}\,' = \vec{E} + \vec{v} \times \vec{B} \tag{1}$$

$$\vec{B}\,' = \vec{B} - \varepsilon_0 \mu_0\, \vec{v} \times \vec{E} \tag{2}$$

where $\vec{v}$ is the relative velocity of the reference frames S and S' in which the fields $(\vec{E}, \vec{B})$ and $(\vec{E}\,', \vec{B}\,')$ are defined. As pointed out in Ref.[2], these transformation rules are inconsistent with the Galilean transformation of the velocities

$$\vec{u}\,' = \vec{u} - \vec{v} \tag{3}$$

and therefore cannot be accepted as correct transformation rules.

Ref.[2] shows that Galilean covariant electrodynamics is internally consistent in two cases only. The first, called the low velocity electric limit of electrodynamics, has

$$\vec{E}\,'_e = \vec{E}_e \tag{4}$$

$$\vec{B}\,'_e = \vec{B}_e - \varepsilon_0 \mu_0\, \vec{v} \times \vec{E}_e \tag{5}$$

The second case, called the low velocity magnetic limit of electrodynamics, has

$$\vec{E}\,'_m = \vec{E}_m + \vec{v} \times \vec{B}_m \tag{6}$$

$$\vec{B}\,'_m = \vec{B}_m \tag{7}$$

One cannot have (1) and (2) simultaneously. While Ref.[1] imposes an important limitation that the transformation rules (1) and (2) are valid only up to terms of first order in $\frac{v}{c}$, the limitation neither follows from the derivation nor helps to save the transformation rules. The transformation rules were derived from that of the Lorentz force, which in the Newtonian case is

$$\vec{F}' = \vec{F} \tag{8}$$

But in a Galilean covariant physics (8) is valid for all velocities. Of course, the physical domain of applicability of (8) is restricted to small velocities but this fact is not built into the formalism and cannot be built in the framework of Galilean physics. On the other hand, the magnetic field $\vec{B}$ in (1) may be (and in Ref.[1] it is; see eqs.(7) and (22) of Ref.[1]) of the order of $\frac{v}{c}$ and according to the limitation made it should be omitted from (1). We arrive therefore to (4) and (5).

The source of the inconsistency apparently lies in the assumptions of Ref.[1]. Rule (1) was derived from the usual assumption of electrodynamics. Repeating this rule twice with two different velocities we can see that we must add to it the rule (7) and not (2). On the other hand the rule (2) followed from the assumption that in addition to electric charges q_e there exist magnetic charges q_m which at rest exert the "magnetic Lorentz force"

$$\vec{F} = q\vec{B} \tag{9}$$

In spite of the wide use of this expression it is an incorrect assumption. The electromagnetic fields $\vec{E}$ and $\vec{B}$ are operationally defined through the Lorentz force

$$\vec{F} = q_e\left(\vec{E} + \vec{v}_e \times \vec{B}\right) \tag{10}$$

and have only to do with electric charges. They describe the action of the electromagnetic fields solely on the electric charges, not on magnetic ones. The use of (9) is a far going extrapolation and the Crawford paper shows that it is indeed inconsistent with other parts of electrodynamics.

References:

[1]. F.S. Crawford, Am.J.Phys.60(1992)109-114.

[2]. M.Le Bellac and J-M. Levy-Leblond, Il Nuovo Cim. 14B(1973),217-234.