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CLASSICAL TIME MACHINE

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Abstract:

Generalizing concepts of the Einstein radiolocation method and, as a consequence, special relativity transformation rules we get that the time flow in the moving system depends on the direction of motion.

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Recently, L.Vaidman [1] has described the quantum "time machine" which changes the rate of time flow for a closed system. In his paper he has also mentioned the classical way of slowing down the time flow in fast travelling system. According to the common belief it is impossible to have in the framework of classical physics a "time machine" which speeds up the time flow for the evolution of the system.

The aim of our paper is to show that the standard conclusions on the relativistic time dilation are based on a particular version of special relativity and that in more general version it is possible to realize the classical "time machine" which speeds up the time flow.

We shall perform our discussion using the radiolocation method of measuring the space-time coordinates [2]. Its main feature consists in the fact that in each reference frame we measure two instants of time: t_1 and t_2 which denote respectively the time of emission of the light signal and the time of detection t_2 of the same light signal reflected at the position of given event. In the reference frame in which we measure t_1 and t_2 the space and time coordinates are determined by standard relations

$$x = \frac{c}{2}(t_2 - t_1) \tag{1}$$

$$t = \frac{1}{2}(t_1 + t_2) \tag{2}$$

In any other reference frame in which we get for the same space-time event the times t_1' and t_2' we have the coordinates

$$x' = \frac{c}{2}(t_2' - t_1') \tag{3}$$

$$t' = \frac{1}{2}(t_1' + t_2') \tag{4}$$

The essence of relativity theory is contained in the relation between the pair (t_1, t_2) and the pair (t_1', t_2'). In the standard formulation of special relativity we consider relations of the type

$$t_1' = \lambda t_1 \tag{5}$$

$$t_2' = \lambda^{-1} t_2 \tag{6}$$

where λ is a dimensionless parameter related to the relative velocity of two reference frames taken into consideration. Substituting (5) and (6) into (3) and (4) and using (1) and (2) we get

$$x' = \frac{x(\lambda^2+1) - ct(\lambda^2-1)}{2\lambda} \tag{7}$$

$$t' = \frac{t(\lambda^2+1) - c^{-1}x(\lambda^2-1)}{2\lambda} \tag{8}$$

which are the usual Lorentz transformations written down in a slightly unusual form. In fact, it follows from (7) that the relative velocity of the reference frames is equal to

$$v = c\frac{\lambda^2-1}{\lambda^2+1} \tag{9}$$

and expressing λ in terms of $\frac{v}{c}$ we may replace (7) and (8) by the textbook form of the Lorentz transformation:

$$x' = \frac{x - vt}{\sqrt{1-\frac{v^2}{c^2}}} \tag{10}$$

$$t' = \frac{t - \frac{v}{c^2}x}{\sqrt{1-\frac{v^2}{c^2}}} \tag{11}$$

The change of the time flow in moving reference system, called the time dilation, is one of the consequences of the Lorentz transformation and is given by the formula

$$\Delta t' = \Delta t\sqrt{1-\frac{v^2}{c^2}} \tag{12}$$

which means that the time is always going slower in the moving reference frame.

We begin our considerations from the following observation. It is known that formulas (5) and (6) are based on the particular assumption concerning the time flow determined by periodic processes seen from two different reference frames [3]. Therefore, these formulas already contain some presuppositions on the time flow in two reference frames. Theoretically it is possible to consider more general situation in which, instead of the relations (5) and (6), we assume that

$$t' = \lambda_1 t_1 \tag{13}$$

$$t_2' = \lambda_2 t_2 \tag{14}$$

where both dimensionless parameters λ_1 and λ_2 are *a priori* independent from each other. The relations (13) and (14) are the most general linear relations admitted by intuitive understanding of the causality principle. Without respecting the usual causality we could replace (13) and (14) even by more general relations

$$t_1' = \lambda_1 t_1 + \mu_1 t_2 \tag{15}$$

$$t_2' = \lambda_2 t_1 + \mu_2 t_2 \tag{16}$$

but it would lead us too far from the real world.

Let us see what kind of consequences follows from the replacement of (5) and (6) by (13) and (14). The transformation rules have now the form

$$x' = \frac{x(\lambda_1 + \lambda_2) + ct(\lambda_2 - \lambda_1)}{2} \tag{17}$$

$$t' = \frac{t(\lambda_1 + \lambda_2) + c^{-1}x(\lambda_2 - \lambda_1)}{2} \tag{18}$$

instead of (7) and (8) and it follows from (17) that the relative velocity v of the two reference frames is equal to

$$v = c\,\frac{\lambda_1 - \lambda_2}{\lambda_1 + \lambda_2} \tag{19}$$

Introducing the notation $\lambda = \lambda_1$ and using the formula (19) we may rewrite (17) and (18) in the following way

$$x' = \lambda\,\frac{x - vt}{1 + \frac{v}{c}} \tag{20}$$

$$t' = \lambda\,\frac{t - \frac{v}{c^2}x}{1 + \frac{v}{c}} \tag{21}$$

The transformation rules (20) and (21) are Lorentz transformations because the composition law of the velocities v_1 and v_2 defining two subsequent transformations of this type is given exactly by the formula

$$v_{12} = \frac{v_1 + v_2}{1 + \frac{v_1 v_2}{2}} \tag{22}$$

known from the Lorentz group. The transformations (20) and (21) contain however an arbitrary change of scale of the space-time coordinates used in two different reference frames. To eliminate this ambiguity we shall assume that the parameter

λ is a function of the relative velocity only. The transformations (20) and (21) will obey the group property if the function $\lambda(v)$ will satisfy the following functional equation

$$\lambda(v_1)\lambda(v_2) = \lambda\left(\frac{v_1 + v_2}{1 + \frac{v_1 v_2}{c^2}}\right) \tag{23}$$

Using the standard trick of solving such equations it may be shown that the general solution of this equation is given by

$$\lambda(v) = \left(\frac{1 + \frac{v}{c}}{1 - \frac{v}{c}}\right)^{\mu} \tag{24}$$

where μ is an arbitrary real number. Substituting (24) into (20) and (21) we get

$$x' = \left(\frac{1 + \frac{v}{c}}{1 - \frac{v}{c}}\right)^{\mu - \frac{1}{2}} \frac{(x - vt)}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{25}$$

$$t' = \left(\frac{1 + \frac{v}{c}}{1 - \frac{v}{c}}\right)^{\mu - \frac{1}{2}} \frac{(t - \frac{v}{c^2}x)}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{26}$$

which obviously mean that the value $\mu = \frac{1}{2}$ realizes the usual Lorentz transformations. Using standard procedure of calculating time intervals in two reference frames moving with relative velocity v we obtain from (25) and (26) the modified formula for the change of time flow

$$\Delta t' = \Delta t\sqrt{1 - \frac{v^2}{c^2}}\left(\frac{1 + \frac{v}{c}}{1 - \frac{v}{c}}\right)^{\mu - \frac{1}{2}} \tag{27}$$

describing, in contradistinction to the standard time dilation (12), not only slowing down the time flow but also its speeding up for suitable values of v and μ. Detailed analysis of the properties of (27) with respect to v and μ is easier if we distinguish between two cases: $\mu \in \{(-\infty, 0] \cup [1, \infty)\}$ and $\mu \in (0, 1)$.

For the first case we have $\Delta t' > \Delta t$ for $\{ v > 0$ and $\mu \geq 1 \}$ or $\{ v < 0$ and $\mu \leq 0\}$ and $\Delta t' < \Delta t$ for $\{ v < 0$ and $\mu \geq 1 \}$ or $\{ v > 0$ and $\mu \leq 0\}$. It may be written in the more compact form

$\Delta t' > \Delta t$ if $\mathrm{sgn}(\mu v) = 1$ or $v > 0$ while $\mu = 0$,

$\Delta t' < \Delta t$ if $\mathrm{sgn}(\mu v) = -1$ or $v < 0$ while $\mu = 0$,

which shows that speeding up the time flow is possible as well as its slowing down and that the effect depends explicitly on the direction of motion.

The case when $\mu \in (0, 1)$ is a little bit more complicated. We know that for the Lorentzian case i.e. for $\mu = \frac{1}{2}$ the time dilation takes place for any v. If $\mu \neq \frac{1}{2}$ we obtain

$\Delta t' > \Delta t$ for $\{ v \in (0, v_0^+)$ if $\frac{1}{2} < \mu < 1 \}$ or $\{ v \in (v_0^-, 0)$ if $0 < \mu < \frac{1}{2}\}$

$\Delta t' < \Delta t$ for $\{ v \notin [0, v_0^+]$ if $\frac{1}{2} < \mu < 1 \}$ or $\{ v \notin [v_0^-, 0]$ if $0 < \mu < \frac{1}{2}\}$

where v_0^+ and v_0^- denote some positive and negative, respectively, values of the velocity for which the formula (27) gives no change in the time flow, i.e. $\Delta t' = \Delta t$. This shows that speeding up as well as slowing down of the time flow may occur during the motion in the same direction and that the magnitude and the "sign" of the effect depend on the relation between the velocity of motion and velocities $v_0^{\pm}$ which are defined by the parameter μ.

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