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A NEW TIME CONTOUR FOR THERMAL FIELD THEORIES*

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ABSTRACT

A simpler new time contour for path-ordered approaches to real-time thermal field theories is presented. In doing so the use of a so called 'asymptotic' condition as seen in existing derivations is seen to be incorrectly applied but luckily unnecessary.

In this talk, I shall be concerned with path ordered approaches to equilibrium thermal field theory. These include both PORTF (path-ordered real-time formalisms) of the various types[1,2,3,4,5] and the ITF (imaginary-time formalism or Matsubara method)[4,5,6]. The path-ordered approach to a real-time formalism is to be distinguished from the Thermo Field Dynamics type approaches to real-time thermal field theories. The latter includes the approach due to Umezawa and collaborators[4,7] as well as the axiomatic field theory version involving C^*-algebra methods[5], since the two approaches are identical[8]. Thermo Field Dynamics and PORTF are related but they have important differences[5].

The starting point for path-ordered approaches to thermal field theory is the thermal generating functional, $Z[j]$,

$$Z[j] := \mathrm{Tr}\{e^{-\beta H}\exp\{T_C\int_C d\tau\int d^3\vec{x}\ j(\tau,\vec{x})\phi(\tau,\vec{x})\}\} \tag{1}$$

The sources j are coupled to the fields, here generically denoted by ϕ. In principle there is a source for every field but for simplicity this will be represented by a single $j\phi$ term. These sources are unphysical and are set to zero at the end of the calculation. The T_c indicates that the fields are path ordered with respect to the relative order of their time arguments along a directed path, C, in the complex time plane[2,5]. Then, by using one's favourite method, such as the path-integral[2,4,5] or operator methods[3], one can obtain Feynman rules, the effective action or whatever else is required.

In order for path-ordered methods to work, the path C starts at some arbitrary time, say τ_{in}, and then must end at a time $\tau_{out} = \tau_{in} - i\beta$. It has been suggested on formal grounds that the path must also always have a decreasing imaginary part but this limitation is not seen in the Feynman rules.‡ Here we stay also within this limitation. Any C satisfying these conditions may be choosen. Physical results are therefore independent of C. One of the great advantages of the path-ordered approach to thermal field theory is that the different FTFT formalisms simply correspond to different choices for C.

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‡For instance the general curve I described elsewhere[1] can slope upwards yet the Feynman rules are independent of this factor.

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The Green functions which are generated from $Z[j]$ are path ordered expectation values fields where the fields are ordered according to the position of their time arguments on C. Th the physics is encoded in different ways for different choices of C and thus for different FTF formalisms[9]. All the thermodynamic information can be obtained from calculating the partiti function $Z = Z[j=0]$.

The ITF approach uses the curve $C = C_I$ of figure 1. ITF is good for static quantities whi

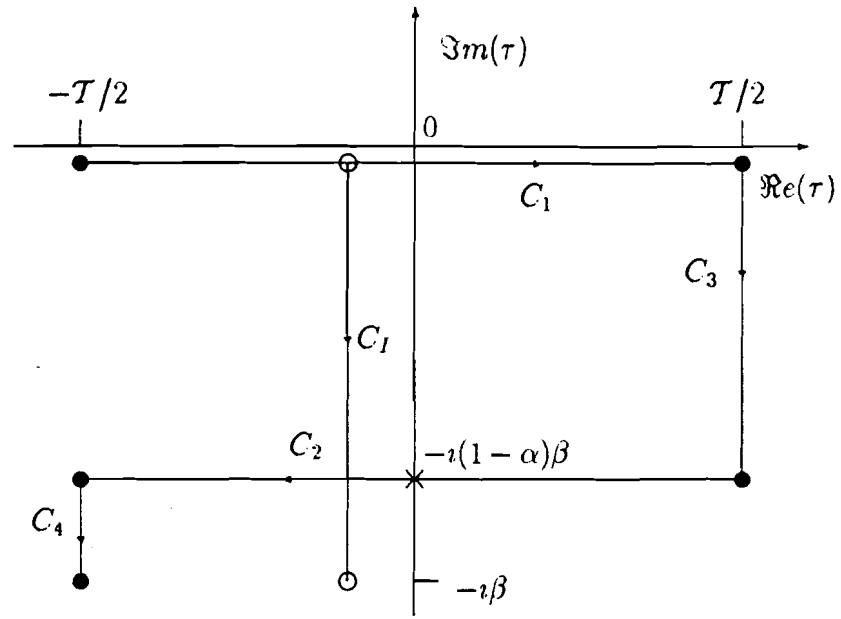

Figure 1: The curves for imaginary-time and old real-time approaches

include all the bulk macroscopic information contained in the partition function. It is hard to extract dynamical information from ITF where real-times are required[9] and thus difficult extend the formalism to non-equilibrium. For these reasons a different curve C was sought whi would lead to a real-time formalism, similar to that obtained in the Thermo Field Dynami approaches.

For a real-time formalism, one part of C must run along the whole of the real time axis. T question is then how one completes C which must finish $-i\beta$ below its starting point.

The traditional curve for PORTF is $C = C_{old} = C_1 \oplus C_2 \oplus C_3 \oplus C_4$ shown in figure 1. T limit $\mathcal{T} \to \infty$ is taken and the figure suggests that $0 \le \alpha \le 1$. Again no physical result depen on α suggesting any value can be taken provided we make use of the periodic or anti-period boundary conditions when redrawing the curve.

The sections running parallel with the real time axis, C_1, C_2, are not too bad to deal with they can be parameterised by a real time. Thus a formalism close to familiar Minkowskii fie theory is obtained from these sections. It is the vertical portions that are unfamiliar and whi cause problems. In Thermo Field Dynamics approaches there are only two real fields in th formalisms, and so there is nothing which corresponds to the vertical sections C_3, C_4 of C_{old}. Tl suggests that only the C_1 and C_2 sections should be kept in PORTF. This is achieved in t

literature through a process called factorisation, namely

$$Z[j] \xrightarrow{?} Z_{12}[j].Z_{34}[j] \tag{2}$$

where in Z_{ab} all the fields and sources are limited to lie on C_{ab},

$$C_{ab} = C_a \oplus C_b. \tag{3}$$

We can follow the usual derivation of this result[2,5] in the path integral approach to PORTF. In this case

$$\begin{aligned} Z[j] &= exp\{i\int_C d\tau\, V[-i\frac{\partial}{\partial j}]\} \;.\; Z_0 \\ &= exp\{i\int_{C_{12}} d\tau\, V[-i\frac{\partial}{\partial j}]\}exp\{i\int_{C_{34}} d\tau\, V[-i\frac{\partial}{\partial j}]\} \;.\; Z_0, \end{aligned} \tag{4}$$

$$Z_0[j] = exp\{-\frac{i}{2}\int_c d\tau d\tau'\, j(\tau)\Delta_c(\tau-\tau')j(\tau'). \tag{5}$$

Here the interaction terms in the Lagrangian are represented by the functional V and it is easy to see that the interaction part factorises in Eq. (4).

The free part is expressed in terms of the free propagator $\Delta_C(\tau-\tau')$. For many systems of interest the propagator, Δ_C, tends to zero as the real part of the time difference tends to infinity

$$\lim_{Re\{\tau-\tau'\}\to\infty} \Delta_c(\tau-\tau') = 0 \tag{6}$$

This is essential in the usual derivations of PORTF as the C_3 and C_4 sections are at infinity whereas most of the C_1 and C_2 are not. For that perennial example, the relativistic scalar field, this condition is satisfied provided the solution of the Klein-Gordon equation is suitably regularised[2,5,10,11]. The Feynman ϵ regularisation achieves this. Strictly, ϵ must be left finite till the end of the calculation.

However, Z_0 in Eq. (5) includes non-zero contributions from regions where one integral is near an end of C_1 or C_2 and the other integral is running along C_3 or C_4. In these situations Eq. (6) can not be used but such contributions to Z_0 must be zero if factorisation is to be true. The usual solution, termed an "asymptotic condition", is to say that the sources tend to zero for times lying at the ends of C_1 and C_2[2,5]. Unfortunately this turns out to be unacceptable. The whole point of a generating function and of the separation in Eq. (4) is that the sources j must not be fixed. In particular infinitesimal variations are needed for the derivatives in the interaction terms in Eq. (4). It makes the expression Eq. (4) meaningless if j is set to zero in Eq. (5) in some regions.

One might try to get round this by switching off all interactions at the ends of C_1 and C_2, a proper asymptotic condition c.f.[12]. However this means we have a time dependent Hamiltonian which invalidates the fundamental assumption of an equilibrium situation. It also means that the partition function, which is also a normalisation factor for the connected Green functions, is being disturbed. Finally, since the asymptotic condition is not used in ITF, it seems strange that PORTF would need this extra boundary condition.

In fact, however factorisation of Eq. (2) is enforced, it leads to a series of inconsistencies[11]. The inescapable conclusion is that the generating function of PORTF does *not* factorise. However this does not alter the fact that if we do drop the C_3 and C_4 parts of the old PORTF curve, one obtains the same Feynman rules as one finds in the Thermo Field Dynamics approaches are obtained[2,5].

There are two ways out of this dilemma. One is to keep working with the whole of the old PORTF curve of figure 1 and to just use Eq. (6) to get the usual answers. For instance if this is done problems encountered with the normalisation of Green functions and with the partition function[11] are avoided. However it is a bit cumbersome.

The alternative way out is to use a different curve for the PORTF. Such a curve must run along the whole real axis to get all real times accessible within the formalism. There must be only two sections, each parameterised by a real time parameter that runs between $\pm\infty$, by analogy with Thermo Field Dynamics methods and the existing successful Feynman rules for real-time formalisms. Likewise there should be no dropping of any sections. One curve, $C_{new} = C_1 \oplus C_{n2}$ which satisfies these criteria is where the curve is run straight back to the end point as shown in figure 2. This is a special case of the curves presented elsewhere[1]. While we could of course

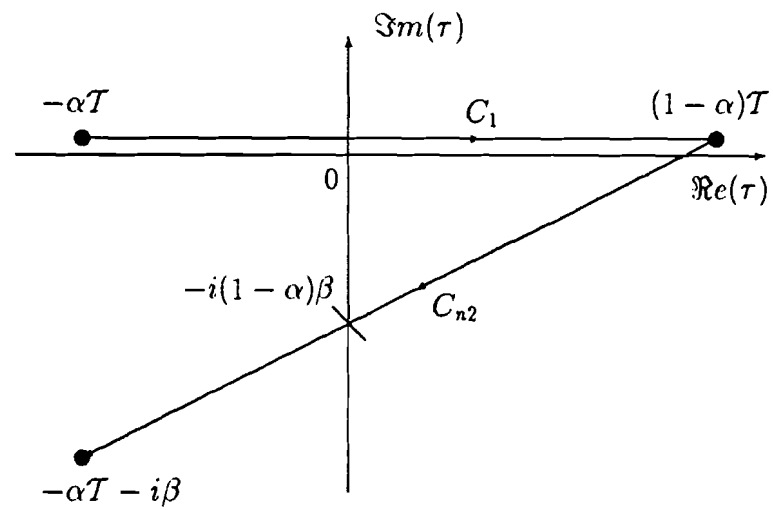

Figure 2: A new curve for real-time formalisms.

parameterise C_{n2} in terms of a real parameter, the gradient would surely hit you somewhere. In this case one must remember that the ends of the curve are going to be taken to infinity, $\mathcal{T}\to\infty$, so that the gradient is going to become zero. The Feynman rules can be derived in time coordinates, with $\mathcal{T}$ kept finite if required. The usual real-time Feynman rules in four-momentum space are obtained on taking $\mathcal{T}\to\infty$ and then doing the Fourier transform[1].

It has been suggested that the vertical sections C_3, C_4 of the traditional PORTF curve are essential to ensure the correct energy-momentum Feynman rules[13]. That is they are supposed to ensure that $n(\omega)\delta(k_0^2-\omega^2)$ terms in propagators (n is the equilibrium number density) are replaced by the correct $n(|k_0|)\delta(k_0^2-\omega^2)$ terms. However the $n(\omega)$ form is simply incorrect because then the propagator does not satisfy the boundary conditions of equilibrium Green functions[5]. It is therefore not a problem that the new curve does not have these vertical pieces present in the old curve, the problem is one of how to correctly take the Fourier transform of a free propagator.

The gradient of C_{n2} of the new curve can be ignored in calculating thermal Green functions provided Eq. (6) holds[11]. It can not, however, be ignored when calculating vacuum diagrams e.g. in a diagrammatic expansion of the partition function or Free energy[11]. In this case one can use the trick[14] in which a vacuum diagram is treated as a time integral multiplied by a tadpole type Green function diagram.

It turns out that the C_3, C_4 vertical sections of the old PORTF curve can be ignored in exactly the same circumstances that the gradient of the new curve can be neglected. Likewise when the gradient must be included, the effect of the vertical pieces is important. Thus the same problems are hiding in both methods. The new curve does however allow a much simpler and cleaner derivation of the Feynman rules. It also emphasises that there is no need for any sort of "asymptotic condition". This is true whatever sort of curve is used in path-ordered methods despite what is found in the literature on PORTF using the old curve. In particular parts of the standard PORTF derivations are clearly wrong yet the answers the resulting formalism gives is correct.

Finally when the vertical sections are needed, it not easy to see how to include their effects in the old PORTF whereas it is straightforward to keep the gradient terms in the new approach when they are needed. This is most important when long time correlations are not zero, i.e. Eq. (6) no longer holds for some of the fields in the problem. This occurs in certain models such as the Anderson model,[5], and in certain physical situations e.g. near critical points. The new approach to PORTF using the curve of figure 2 is therefore likely to be of practical benefit.

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