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ANYONIC SUPERCONDUCTIVITY WITH FERROMAGNETIC GROUNDSTATE

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ABSTRACT

We study a system of non-relativistic two dimensional spin-1/2 fermions minimally coupled to Chern-Simons and Maxwell gauge fields, at zero temperature. A self-consistent mean field approach leads to a ferromagnetic ground state which nevertheless is also superconducting. We calculate the spin and current correlation functions and show that there is a Meissner effect for weak external magnetic fields, characterised by two penetration lengths. The dispersion relations for the two longitudinal excitations is given. Further, the system supports spin waves which are massless in the absence of electromagnetic fields.

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I. INTRODUCTION

Since the discovery of the high temperature superconductors and the fractional quantum Hall effect, considerable attention has been focussed on physics peculiar to particles confined to two spatial dimensions. In particular, 'anyons', particles with fractional statistics[1], are believed[2,3] to arise as excitations in strongly coupled electron systems. The commonly considered model for anyons is to equivalently look upon them as ordinary particles with flux lines attached to them, dynamically achieved by the interaction with a Chern-Simons (CS) gauge field. While there is some uncertainty regarding this equivalence in the relativistic context[4], such correspondence could be viable in the realm of non-relativistic condensed matter physics.

It is by now generally accepted that a system of anyons exhibits the property of superfluidity and, if charged, is also a superconductor[5-8]. Thus, eg., the analysis of Ref. 6 (CWWH) starts with spinless fermions interacting with the CS field and goes on to establish the existence of a long range (massless) mode, characteristic of superfluidity, by using mean field theory and Random Phase Approximation. Hosotani and Chakravarti[7] (HC) on the other hand, start with spin 1/2 fermions and obtain similar results, by using self-consistent mean field theory including fluctuations. Subsequently Randjbar-Daemi *et al*[9], have performed a finite temperature analysis and they find that the Meissner effect, in fact, persists at all temperatures.

The purpose of this paper is to analyse, in a *consistent* manner, the behaviour of a system of nonrelativistic spin 1/2 anyons. It is our opinion that the treatment of spin in the earlier works is incomplete and rather *ad hoc*: HC couple the spin only to the Maxwell magnetic field, if only for convenience. A variant model of Kapusta. *et al*[10] treats spin as an internal degree of freedom, having little to do with its association with the rotation group. Lykken *et al*[8] have analysed the mean field theory for relativistic fermions (i.e, described by the Dirac lagrangian) interacting with the CS field. The minimally coupled Dirac equation of course incorporates the Zeeman coupling. The difference with the present work is that their Dirac lagrangian describes a *single* spin component, while we adopt a Pauli description involving both the spin components as might be more relevant for the physical case of real fermions confined to a plane. Also, in our non-relativistic approach, as will be seen, a non-zero spin density immediately leads to a non-zero Maxwell magnetic field, a fact that is not very apparent in the relativistic approach.

Admittedly, the coexistence of anyonic nature and superfluidity might be taken to be quite robust, not necessarily affected by the introduction of additional degrees of freedom such as spin. However, the inclusion of spin to anyons is by no means a subject of mere academic interest. Surely, any search for the evidence for anyonic superconductivity should involve these other attributes. Indeed, there is a definite evidence for the Zeeman splitting (and doubling) of Landau levels due to spin in integer quantum Hall effect[11]. Spin has also been successfully incorporated in theories[12] of fractional quantum Hall effect - a phenomenon which is possibly closely related[2,13] to anyonic superconductivity.

Apart from this, as mentioned earlier, the commonly considered model for the anyons are particles with singular flux tubes attached to themselves (see eg. Ref. 14 for an elementary discussion). The Aharanov-Bohm (AB) mechanism then attributes these particles with additional arbitrary phases on interchange.

However, it is known[15] that the inclusion of the Zeeman interaction of the spin with a singular flux tube (as would be natural when considering the AB mechanism for spin-1/2 particles) leads to certain novel features, like the relevance of irregular wave functions - complications which are absent if the fermions are taken to be spinless. Such features have been also found to have a macroscopic significance for a system of spin-1/2 flux carrying particles. In fact, Blum *et. al*[16] have computed the second virial coefficient for such a system in the high temperature low density limit and find substantial differences arising from the spin degree of freedom. In particular, there is an increased compressibility of the system as a direct effect of the irregular states. Recall that quantum effects are more manifest in the low temperature high density limit. We thus propose to study in detail the zero temperature properties, including superconductivity, of a gas of spin 1/2 anyons.

Section II starts with a mean field approach to a system of spin 1/2 fermions interacting with the Chern-Simons as well as the Maxwell gauge fields to which it is minimally coupled via the Pauli equation. The self-consistent ground state at the zeroth order is found to have non-vanishing spin density which makes the system spontaneously magnetised, i.e., a ferromagnet. We then apply the qualitative arguments *à la* CWWH which show that the energy of the system increases when a weak external magnetic field is applied. This suggests the existence of Meissner effect even for spin 1/2 fermions and hence the possibility of the coexistence of ferromagnetism with superconductivity. Indeed, fluctuations around the mean field Hamiltonian do not destroy this magnetisation; however, they yield equations (which substitute the London equations in BCS theory) and which ensure the expulsion of the fluctuating as well as weak external magnetic fields. Further, the Meissner effect is now characterised by two penetration depths, signalling parity violation in the system. In section 3, we set up an effective lagrangian which embodies the expressions for the current density and the spin density and evaluate the response of the currents to an external Maxwell field, by computing the current- current correlators. Significantly, we find that apart from the massless mode in the current- current correlator (which signifies superfluidity), a calculation of the spin correlation function shows that there is also a spin 'wave' which has the dispersion relation quite similar to that of the massless mode. This is very different from what one gets in the usual ferromagnets. The emergence of these exotic magnons could be of interest from an experimental point of view. We conclude with some brief comments in section 4.

II. MEAN FIELD THEORY FOR SPIN 1/2 ANYONS

We follow the approach of HC closely, both in the method and notation. We start with the Pauli lagrangian for a system of spin up and and down fermions interacting minimally with the CS field (a_μ) and the Maxwell field (A_μ),

$$L = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{\mu}{2}\epsilon^{\lambda\nu\rho}a_\lambda\partial_\nu a_\rho + \psi^\dagger iD_0\psi - \frac{1}{2m}|D_k\psi|^2 + \frac{e}{2m}(B+b)\psi^\dagger\sigma_3\psi - eA_0n_e, \qquad (1)$$

with $D_\mu = \partial_\mu - ie(A_\mu + a_\mu)$ and $B = \partial_1 A_2 - \partial_2 A_1$, $b = \partial_1 a_2 - \partial_2 a_1$. It should be noted that in (1) the spin density couples to both the CS and the Maxwell magnetic fields. (The last term in (1) is included to achieve overall neutrality of the system).

The equations of motion that follow from the lagrangian are

$$\frac{\mu}{2}\epsilon^{\lambda\mu\rho}f_{\mu\rho} = j^{\lambda}, \tag{2}$$

$$\partial_\nu F^{\lambda\nu} = j^{\lambda} - en_e\delta^{\lambda 0}, \tag{3}$$

with

$$j^0 = e\psi^\dagger\psi; \quad j^k = -\frac{ie}{2m}[\psi^\dagger(D_k\psi) - (D_k\psi)^\dagger\psi] + \frac{e}{2m}\epsilon^{kj}\partial_j j_\sigma, \tag{4}$$

where

$$j_\sigma = \psi^\dagger\sigma_3\psi \tag{5}$$

is the spin density. The fermion field satisfies

$$i\partial_t\psi = H\psi = \left[-\frac{1}{2m}[D_k^2 + e(B+b)\sigma_3] - e(A_0 + a_0)\right]\psi \tag{6}$$

Henceforth the calligraphic letters $\mathcal{A}_\mu, \mathcal{B}, \mathcal{E}_i$ will denote the quantities $A_\mu + a_\mu$, $B+b$ and $E_i + e_i \equiv F_{0i} + f_{0i}$ respectively.

In the zeroth approximation we assume that the fermions move in a mean field produced by the rest of the system. We take the mean field to consist of uniform CS and Maxwell magnetic fields $b^{(0)}$ and $B^{(0)}$, respectively, which will have to be solved self-consistently at the end of the calculation. Note that we differ with the usual choice[6-9] by not setting $B^{(0)}$ to zero, as in fact required by consistency. Choosing the Landau gauge, $\mathcal{A}_1^{(0)} = -\mathcal{B}^{(0)}x_2$; $\mathcal{A}_2^{(0)} = \mathcal{A}_0^{(0)} = 0$, the Hamiltonian becomes

$$H_0 = -\frac{1}{2m}[(\partial_1 + i\frac{x_2}{l^2})^2 + \partial_2^2] - \frac{1}{2m}e\mathcal{B}^{(0)}\sigma_3, \tag{7}$$

where the magnetic length l is given by

$$l^2 = \frac{1}{|e\mathcal{B}^{(0)}|}. \tag{8}$$

Imposing the periodic boundary condition in the x_1 direction over a length L, one has the well known solutions of the Schrodinger equation,

$$\psi^{(0)}_{nk\sigma}(\vec{x}) = \frac{1}{\sqrt{Ll}}e^{-ikx_1}u_n(\frac{x_2}{l} - kl)v_\sigma, \tag{9}$$

where $n = 0, 1, 2...$; $k = \frac{2\pi p}{L}(p \in Z)$, $u_n(z) = (2^n\sqrt{\pi}n!)^{-1/2}e^{z^2/2}(\frac{d^n}{dz^n}e^{-z^2})$ and $\sigma_3 v_\sigma = \sigma v_\sigma = \pm v_\sigma$. The energy eigenvalues are

$$\epsilon_{n\sigma} = [n + \frac{1}{2}(1-\sigma)]\omega_c; \quad \omega_c = \frac{1}{ml^2}. \tag{10}$$

corresponding to Landau levels. We note that the lowest level, i.e. $n = 0$ supports only spin up state, ($\sigma = +1$). As is clear from Fig.1 all the higher levels in the ground state configuration are degenerate with respect to spin up and spin down. The ground state of the spin 1/2 system is thus ferromagnetic, having a spontaneous spin polarization. It should also be noted that the spin up and spin down particles having the same energy have different spatial wave functions.

We next require that the ground state be non-degenerate and hence to consist of completely filled Landau levels. Given the density of particles to be n_e and the degeneracy of the Landau levels $\rho_l = 1/2\pi l^2$, it is not difficult to see (Fig. 1) that our requirement of completely filled Landau levels implies that

$$\frac{(2\bar{n}+1)}{2\pi l^2} = n_e \tag{11}$$

where $\bar{n}$ is the number of filled Landau levels. This in turn implies, using (8),

$$\frac{e(b^{(0)} + B^{(0)})}{2\pi}(2\bar{n}+1) = n_e. \tag{12}$$

We note that at the ground state we have $\mu b^{(0)} = en_e$. Thus the CS magnetic field gets fixed by the particle density. What fixes $B^{(0)}$? We shall find this once we calculate the expectation values of the current in the ground state. First, (see Fig.1)

$$< j^0 >= e[\sum_{n=0}^{\bar{n}-1}\sum_{k\sigma} \psi^{(0)\dagger}_{nk\sigma}(x)\psi^{(0)}_{nk\sigma}(x) + \sum_k \psi^{(0)\dagger}_{\bar{n}k+}(x)\psi^{(0)}_{\bar{n}k+}(x)]$$

Substituting $\psi^{(0)}_{nk\sigma}(x)$ from (9), one finds $< j^0 >= en_e$. Similarly we have for the spin density $< j_\sigma >= \frac{1}{2\pi l^2}$ and $< j^i >= \frac{e}{2m}\epsilon^{ij}\partial_j < j_\sigma >$. On the other hand from Maxwell's equations we have to zeroth order $\partial_2 B^{(0)} =< j^1 >= \frac{e}{2m}\partial_2 < j_\sigma >$, which has the solution

$$B^{(0)} = \frac{e}{2m} < j_\sigma >= \frac{en_e}{2m(2\bar{n}+1)}. \tag{13}$$

Here it is to be noted that both $B^{(0)}$ and $< j_\sigma >$ have vanishing spatial derivatives (by uniformity) but this fact should not be imposed in the determining equation before solving it since then it would reduce to a trivial identity. We also note that $B^{(0)}$ is determined upto a constant. But since we attribute the magnetic field $B^{(0)}$ to the spin density it should vanish when the spin density vanishes, which determines the constant of integration to be zero.

The non-vanishing of $B^{(0)}$ has a significant impact on the quantization condition on the CS parameter μ. Substituting $B^{(0)}$ in (12) we get

$$\mu\epsilon = \frac{e^2(2\bar{n}+1)}{2\pi}; \quad \epsilon = (1 - \frac{e^2}{4\pi m}) \tag{14}$$

which is qualitatively different from that found in the previous works[6-9]. If we look upon μ as a statistics parameter, then we see that this model does not describe anyons with additional phase (under exchange) of the form $\pi/N, N \in Z$, and in particular. semions which correspond to $N = 2$. This fact could well be of crucial significance in establishing the relevance of a system of spin 1/2 anyons as a model of high T_c superconductivity. However, the factor ϵ in (14) is quantitatively very close to unity since $e^2/4\pi m$ is small. and hence the statistics change can be made arbitrarily close to π/N.

Next we argue qualitatively, *a la* CWWH. that our system despite having a ferromagnetic ground state will exhibit the Meissner effect. To that end we show that a small applied external magnetic field δB tends

to increase the energy of the system in whatever direction it is. First of all when no field is applied, there are $\bar{n}$ Landau levels filled. From the expression for the energy eigenvalue it is seen that the lowest level has zero energy. Then the ground state energy is easily shown to be

$$E = 2\sum_{l=1}^{\bar{n}} \rho_l(l\omega_c) = \frac{\bar{n}(\bar{n}+1)e^2\mathcal{B}^{(0)2}}{2\pi m}. \tag{15}$$

We now calculate the energy of the ground state in the presence of the external magnetic field, for the two possibilities:

(i): δB parallel to $\mathcal{B}^{(0)}$

The degeneracy changes to $\tilde{\rho}_l = \rho_l + \frac{e\delta B}{m}$ and the energy gap increases to $\tilde{\omega}_c = \omega_c + \frac{e\delta B}{2\pi}$. The energy of the lowest level remains at zero. As the degeneracy has increased the top level will be partially filled. The number of particles p in the top level is easily seen to be $p = \frac{2e\mathcal{B}^{(0)}}{2\pi} - \frac{e\delta B}{2\pi}(2\bar{n}-1)$. The modified energy is thus $E_+ = 2\sum_{l=1}^{\bar{n}-1} \tilde{\rho}_l(l\tilde{\omega}_c) + p(\bar{n}\tilde{\omega}_c)$ which gives

$$E_+ = \frac{e^2\bar{n}}{2\pi m}\left[(\bar{n}+1)\mathcal{B}^{(0)2} + \mathcal{B}^{(0)}\delta B - (2\bar{n}-1)\delta B^2\right]. \tag{16}$$

(ii): δB anti-parallel to $\mathcal{B}^{(0)}$

The degeneracy is now $\rho_l' = \rho_l - \frac{e\delta B}{m}$ while the energy gap is $\omega_c' = \omega_c - \frac{e\delta B}{2\pi}$. The energy of the lowest level continues to be zero. As the degeneracy has now decreased particles will make a transition to $(\bar{n}+1)^{th}$ level with the number of particles q in that level being $q = \frac{e\delta B}{2\pi}(2\bar{n}+1)$. Thus, the modified energy is $E_- = 2\sum_{l=1}^{\bar{n}} \rho_l'(l\omega_c') + q(\bar{n}+1)\omega_c'$ which becomes

$$E_- = \frac{e^2(\bar{n}+1)}{2\pi m}\left[\bar{n}\mathcal{B}^{(0)2} + \mathcal{B}^{(0)}\delta B - (2\bar{n}+1)\delta B^2\right]. \tag{17}$$

From the terms linear in δB in (16) and (17) we see that the external field increases the energy as compared to that given by (15). Hence the system while retaining the "frozen" mean magnetic fields would prefer to exclude any weak extra magnetic field.

In the above analysis we confined ourselves to uniform external magnetic field which was at least large enough to allow particles to make transitions. In order to demonstrate the Meissner effect for non-uniform and still smaller perturbations which do not cause transitions among the Landau levels, we carry out the self-consistent calculation including fluctuations about the mean field in the Section III, using standard first order perturbation theory.

III. FIRST ORDER FLUCTUATIONS

We now consider the effect of fluctuations of the gauge fields around the background, $\mathcal{A}_\mu^{(0)} \to \mathcal{A}_\mu = \mathcal{A}_\mu^{(0)} + \mathcal{A}_\mu^{(1)}$. We shall adhere to the gauge choice $\mathcal{A}^\mu = (\mathcal{A}_0, \mathcal{A}_1, 0)$. To simplify the calculation we take $\mathcal{A}_\mu = \mathcal{A}_\mu(x_2)$ which leads to $\mathcal{E}_1 = 0$. We only briefly outline the steps below.

The gauge field fluctuations give rise to a perturbation of H_0 by the potential (neglecting terms quadratic in the fluctuations)

$$V = -\frac{e}{m}\mathcal{A}_1^{(1)}\left(\frac{x_2}{l^2} - i\partial_1\right) - \frac{e}{2m}\mathcal{B}^{(1)}\sigma_3 - e\mathcal{A}_0^{(1)} \tag{18}$$

with its matrix elements with respect to the unperturbed states given by

$$< n'k'\sigma'|V|nk\sigma >= \delta_{kk'}\delta_{\sigma\sigma'}(\frac{-e}{m})\int_{-\infty}^{\infty}\frac{dy}{l}[C(y,k)+\frac{\sigma\mathcal{B}^{(1)}}{2}]u^*_{n'}(\xi)u_n(\xi), \tag{19}$$

with $C(y,k) = \frac{\xi}{l}\mathcal{A}_1^{(1)} + m\mathcal{A}_0^{(1)}$ and $\xi = \frac{y}{l} - kl$. The change in eigenfunctions is given by standard non-degenerate perturbation theory,

$$\psi^{(1)}_{nk\sigma}(x) = \sum_{n'\neq n}\psi^{(0)}_{n'k\sigma}(x)\frac{1}{(\epsilon_{n\sigma}-\epsilon_{n'\sigma})} < n'k\sigma|V|nk\sigma >, \tag{20}$$

$\psi^{(0)}_{n'k\sigma}$ being the eigenfunctions of H_0. Next we define

$$\tilde{K}_n[h(x_2,y,k);x_2] = \frac{2}{\pi}\sum_{n'\neq n}\frac{1}{(n'-n)}\int_{-\infty}^{+\infty}dy\int_{-\infty}^{+\infty}dk h(x_2,y,k)u_n(\xi)u_{n'}(\xi)u_n(\eta)u_{n'}(\eta), \tag{21}$$

where h is any function and $\eta = (\frac{x_2}{l} - kl)$. We also introduce $K[h(x_2,y,k);x_2] = \sum_{n=0}^{\bar{n}-1}\tilde{K}_n[h(x_2,y,k);x_2]$.

With the above defined quantities, the expressions for the expectation values for the current and the spin densities are found, to first order in fluctuations, to be

$$\begin{aligned}
&< j^0(x_2) >=en_e + e^2K[C(y,k);x_2] + \frac{e^2}{2}\tilde{K}_{\bar{n}}[C(y,k)+\frac{\mathcal{B}^{(1)}(y)}{2}],\\
&< j_\sigma(x_2) >=\frac{n_e}{(2\bar{n}+1)} + \frac{e}{2}K[\mathcal{B}^{(1)}(y)] + \frac{e}{2}\tilde{K}_{\bar{n}}[C(y,k)+\frac{\mathcal{B}^{(1)}(y)}{2}],\\
&< j_1(x_2) >= -\frac{(2\bar{n}+1)e^2}{2\pi ml^2}\mathcal{A}_1^{(1)} + \frac{e^2}{ml}K[\eta C] + \frac{e^2}{2ml^2}\tilde{K}_{\bar{n}}[\eta C(y,k)+\frac{\eta\mathcal{B}^{(1)}(y)}{2}]\\
&\qquad + \frac{e}{2m}\partial_2 < j_\sigma >,\\
&< j_2(x_2) >=0.
\end{aligned} \tag{22}$$

Taking the magnetic length l to be small, we can evaluate the functions $\tilde{K}_n$ and K by performing a Taylor expansion of $h(x_2,y,k)$ in the vicinity of $y = x_2$. We then find $\tilde{K}_n[f(y)] = -\frac{l^2}{\pi}\partial_2^2 f$, $\tilde{K}_n[(-\xi,\eta)f(y)] = \frac{l}{\pi}\partial_2 f$, and $\tilde{K}_n[\xi\eta f(y)] = \frac{1}{\pi}[f + l^2(2n+1)\partial_2^2 f]$.

We ultimately get (neglecting terms involving higher derivatives of $\mathcal{B}$ and $\mathcal{E}_i$)

$$\begin{aligned}
&< j_0(x_2) > = en_e + C_1\mathcal{B}^{(1)} + C_2\partial_2\mathcal{E}_2^{(1)}\\
&\frac{e}{2m} < j_\sigma(x_2) > = \frac{en_e}{2m(2\bar{n}+1)} + C_4\mathcal{B}^{(1)} + C_5\partial_2\mathcal{E}_2^{(1)}\\
&< j_1(x_2) > = -C_3\partial_2\mathcal{B}^{(1)} - C_1\mathcal{E}_2^{(1)}\\
&< j_2(x_2) > \equiv 0,
\end{aligned} \tag{23}$$

where

$$\begin{aligned}
&C_1 = \frac{e^2(2\bar{n}+1)}{2\pi};\ C_2 = \frac{e^2m(2\bar{n}+1)^2}{4\pi^2n_e};\ C_3 = \frac{e^2}{4\pi m}[(2\bar{n}+1)^2-1]\\
&C_4 = \frac{e^2}{4\pi m};\ C_5 = \frac{e^2(2\bar{n}+1)}{8\pi^2n_e}.
\end{aligned} \tag{24}$$

Our result for $< j_\sigma >$ is very different from that found in HC, owing to our inclusion of Zeeman coupling of the spin. Further, the spin effects in $< j_\mu(x_2) >$ are seen in the modified coefficients occuring above as compared with Ref. 7. The similarity of the form, however, suggests that Meissner effect and superconductivity would follow in this case as well, in spite of the fact that the ground state is a ferromagnet. Indeed, the Meissner screening operates only on the fluctuating and weak B fields doing little to the spontaneous field produced by the spin density.

Parity violation in this two dimensional system gets manifest in the emergence of two penetration depths, a result already noted by Ref. 9. We shall not pause to show it here, but defer it to the next section where an effective Lagrangian is set up and all these properties are discussed at length.

IV. THE EFFECTIVE LAGRANGIAN -BULK PROPERTIES OF SPIN 1/2 ANYONS

In the previous section we have derived the effect of fluctuations of the gauge fields from the background configuration on the fermionic currents and the spin density. However, the quantum averages indicated in Eqs. (23) are only partial in the sense that the gauge fields were taken to be classical potentials. In other words we have so far only performed an integration over the fermionic variables, so to speak. We next derive the current and spin correlation functions, $K^{\mu\nu}(x,y)$ and $\Sigma(x,y)$ respectively, by completing the quantum average over the gauge fields. These completely summarize the electromagnetic response of the system. We then show that there is a complete Meissner effect for *external* magnetic fields provided they are sufficiently small. This is a remarkable fact in the sense that while the system retains a "frozen" magnetic field within itself, arising from the spin polarized ground state, it prefers to eliminate external magnetic fields from itself. It is in this sense that the spin-1/2 system manages to behave both as a ferromagnet and a perfect diamagnet! The spin correlation functions are also of interest since they reveal the existence of further excitations of the system. While these can be regarded as spin waves, not unexpected in a ferromagnetic ground state, the dispersion relation we find is different. In what follows we present results for both neutral (i.e., excluding the Maxwell field) and charged spin 1/2 systems.

The computation of the correlation functions is most conveniently carried out by the introduction of external sources. For this purpose we include in (1) a minimal coupling of the fermion to an external vector source A^E_μ, along with a scalar source h^E which couples only to the spin density. The sources are taken to be weak enough to be included along with the fluctuations of the potentials. Then the modification of (23) due to the inclusion to these sources can be seen without much difficulty to be (ommitting the superscript previously used to denote fluctuating quantities)

$$\begin{aligned}
&< \tilde{j}_0(x_2) > = en_e + C_1(\mathcal{B} + B^E) + C_2\partial_2(\mathcal{E}_2 + E^E_2) - C_5\partial_2^2 h^E \\
\frac{e}{2m} &< \tilde{j}_\sigma > (x_2) = \frac{en_e}{2m(2\bar{n}+1)} + C_4(\mathcal{B} + B^E) + C_5\partial_2(\mathcal{E}_2 + E^E_2) - C_6\partial_2^2 h^E \\
&< \tilde{j}_1(x_2) > = -C_3\partial_2(\mathcal{B} + B^E) - C_1(\mathcal{E}_2 + E^E_2) + C_4\partial_2 h^E \\
&< \tilde{j}_2(x_2) > \equiv 0
\end{aligned} \tag{25}$$

where the new constant appearing is

$$C_6 = \frac{e^2(2\bar{n}+1)^2}{16\pi^2 m n_e}. \tag{26}$$

It is convenient to introduce a low energy effective lagrangian L_{eff} involving only the gauge fields, which summarizes the results obtained so far. L_{eff} then takes the significance of a Landau-Ginzburg action for superconductivity[6-9] for the spin 1/2 system. We summarize the results of Eqs. (25) by introducing the effective lagrangian,

$$\begin{aligned} L_{eff} &= L_0 + L_1, \\ L_0 &= -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{\mu}{2}\epsilon^{\mu\nu\rho}a_\mu\partial_\nu a_\rho, \\ L_1 &= \frac{C_1}{2}\epsilon^{\mu\nu\rho}(\mathcal{A}+A_E)_\mu\partial_\nu(\mathcal{A}+A_E)_\rho + \frac{C_2}{2}(\mathcal{E}+E_E)^2 - \frac{C_3}{2}(\mathcal{B}+B_E)^2 \\ &\quad + C_4(\mathcal{B}+B_E)h_E + C_5\partial_i(\mathcal{E}+E^E)_i h_E - \frac{C_6}{2}h_e\vec{\nabla}^2 h_E. \end{aligned} \tag{27}$$

The above Lagrangian is valid only at low energies since we have ignored all the higher derivative terms in the expression for $< j_\mu >$. It should be noted we take L_{eff} to be the Lagrangian involving only the fluctuations of the gauge potentials. More explicitly, we have dropped a constant term arising from the background fields, which can be absorbed along with the irrelevant overall normalisation, and also linear terms since these do not affect the calculation of the correlation functions once the external sources are set to zero.

A crucial feature noted in all the previously referred works on anyonic superconductivity is the cancellation of the net Chern-Simons term in L_{eff} for the CS gauge field. This signifies the presence of a massless mode and is really the central result on which the argument for superfluidity of the neutral system (and hence superconductivity of the charged system) rests. This cancellation is not unexpected if one considers the qualitative argument of CWWH who reason that a fully filled Landau level ground state where the background magnetic field is tied to the background density together imply the existence of gapless density perturbations. In the present case there is a crucial difference since the background also gets a contribution from the Maxwell magnetic field arising from a non-zero spin density. For the neutral case, i.e. neglecting the effects of the Maxwell field, the cancellation is easily ascertained from an inspection of L_{eff} and the quantization condition on the CS parameter imposed by our requirement of fully filled Landau levels. However, for the general case this is not exact as one sees from the altered condition on μ, (14). Nevertheless, to the order of approximation for which our calculation is valid, i.e. $O(e^2)$, the necessary cancellation does occur*.

We now turn to the calculation of the response functions. Since we are interested in the correlations of the fluctuations of the various densities about the mean field value, we define the subtracted expectation

* By the Coleman-Hill theorem[17] higher order effects would not alter C_1. Hence, the non-cancellation would appear to persist to all orders in the coupling. We have verified that the residual CS term does not qualitatively affect any of the results obtained here.

values $< \delta\tilde{j}^{\mu} >$ and $< \delta\tilde{j}_{\sigma} >_{l}$ where

$$< \delta\tilde{j}^{0} >=< \tilde{j}^{0} > -en_e; \quad < \delta\tilde{j}^{i} >=< \tilde{j}^{i} >; \quad < \delta\tilde{j}_{\sigma} >=< \tilde{j}_{\sigma} > -\frac{n_e}{(2\bar{n}+1)}.$$

The necessary correlation functions then follow from the definitions

$$K^{\mu\nu}(x,y) = \frac{\delta\langle\delta\tilde{j}^{\mu}(x)\rangle}{\delta A^E_{\nu}(y)}|_{A^E=h^E=0} \tag{28}$$

$$\Sigma(x,y) = \frac{\delta\langle\delta\tilde{j}_{\sigma}(x)\rangle}{\delta h_E(y)}|_{A^E=h^E=0}. \tag{29}$$

It is to be noted that in the above equations the vacuum expectation values are taken to include the vacuum averages of the gauge field variables also.

The computation of the correlation functions is, though straightforward, fairly involved for the general case including both CS and Maxwell fields. We thus outline the steps for the simpler case for neutral spin 1/2 particles. Basically, one has to identitify the independent gauge field variables. From L_{eff} it is easy to derive the equations of motion,

$$C_2\partial_k e_k = Q(x) = -C_2\partial_k E^E_k - C_1 B^E + C_5\vec{\nabla}^2 h^E,$$

$$C_2\dot{e}_l - C_3\epsilon_{kl}\partial_k b = K_l = -C_1\epsilon_{lk}E^E_k - C_2\dot{E}^E_l + C_3\epsilon_{kl}\partial_k B^E - C_4\epsilon_{kl}\partial_k h^E + C_5\partial_l\dot{h}^E \tag{30}$$

The first of these is seen to be the Gauss law constraint. The separation of the independent variables is most easily accomplished by imposing the radiation gauge condition $\vec{\nabla}\cdot\vec{a} = 0$, which in turn allows one to write the spatial components of the gauge field as

$$a_i = \epsilon_{ij}\partial_j\phi$$

where ϕ is a scalar field. Substitution in the Gauss law then leads to the solution

$$a_0 = -\frac{1}{c_2}\int dx'\mathcal{D}(x,x')Q(x'), \tag{31}$$

where $\vec{\nabla}^2\mathcal{D}(x,x') = \delta(x-x')$. Thus a_0 is entirely solved in terms of the sources. Next substituting in the second of the gauge field equations leads to

$$\vec{\nabla}^2(C_2\partial_0^2 - C_3\vec{\nabla}^2)\phi(x) = \epsilon_{lm}\partial_m K_l(x). \tag{32}$$

From this one derives the solution for the vacuum expectation value

$$\langle\phi(x)\rangle = \int dx'\mathcal{G}(x,x')\epsilon_{lm}\partial'_m K_l(x'), \tag{33}$$

(apart from an irrelevant homogeneous term) where $\mathcal{G}(x,x')$ is the Green function satisfying $\vec{\nabla}^2(C_2\partial_0^2 - C_3\vec{\nabla}^2)\mathcal{G}(x,x') = \delta(x-x')$.

Thus one has, for example,

$$K^{00}(x,y) = \frac{\delta\langle\delta\tilde{j}^0(x)\rangle}{\delta A_0^E(y)}|_{A^E=h^E=0} = \frac{\delta\langle C_1 b(x)\rangle}{\delta A_0^E(y)}$$

which is easily computed to be

$$K^{00}(x,y) = -C_1^2\vec{\nabla}^2\vec{\nabla}'^2\mathcal{G}(x,x') \tag{34}$$

The remaining components are similarly easily computed. As expected from gauge invariance and rotational invariance, the components of $K^{\mu\nu}$ can be fitted to the form ($g^{\mu\nu} = + - -$; $q^2 = \omega^2 - \vec{q}^2$)

$$K^{\mu\nu}(q) = K_1(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}) + K_2\delta^{\mu i}\delta^{\nu j}(\delta_{ij} - \frac{q_i q_j}{\vec{q}^2}) + iK_3\epsilon^{\mu\nu\alpha}q_\alpha. \tag{35}$$

The results for the three form factors are as follows (the subscript denotes the neutral case),

$$K_{N1} = \frac{C_1^2 q^2}{[C_2\omega^2 - C_3\vec{q}^2]}; \quad K_{N2} = \frac{C_1^2(C_3 - C_2)\vec{q}^2}{C_2[C_2\omega^2 - C_3\vec{q}^2]}; \quad K_{N3} = C_1. \tag{36}$$

Thus one sees that the density-density correlation function has a pole with the dispersion relation $C_2\omega^2 = C_3\vec{q}^2$ corresponding to a long range sound wave. Somewhat curiously we find the pole exactly cancels out in the parity violating form factor K_3.

Similarly the spin correlation function can be found to be

$$\Sigma_N = -\frac{2m}{e}\frac{\vec{q}^2}{[C_2\omega^2 - C_3\vec{q}^2]}[C_4^2 - (C_6 - \frac{C_5^2}{C_2})(C_2\omega^2 - C_3\vec{q}^2)]. \tag{37}$$

Thus we find the same dispersion relation for the longitudinal spin wave as for the sound wave. This is not surprising since the spin wave is simply produced by the out of phase fluctutations of the two spin species. Nevertheless it signifies a new mode of excitation, which would be absent in the spinless system, and may be expected to affect the bulk properties such as the compressibility of the spin 1/2 system.

For the case of a charged spin 1/2 system the analysis is considerably more cumbersome due to the mixing of the CS and the Maxwell gauge fields. We content ourselves to simply stating the results for the corresponding form factors,

$$\begin{aligned} K_1 &= \frac{C_1^2[C_2(C_2\omega^2 - C_3\vec{q}^2) - (-C_2q^2 + C_1^2)]q^2}{C_2^2(\omega^2 - \omega_+^2)(\omega^2 - \omega_-^2)} \\ K_2 &= \frac{C_1^2[C_3 - C_2]q^2\vec{q}^2}{C_2^2(\omega^2 - \omega_+^2)(\omega^2 - \omega_-^2)} \\ K_3 &= \frac{C_1[C_2\omega^2 - C_3\vec{q}^2]q^2}{C_2(\omega^2 - \omega_+^2)(\omega^2 - \omega_-^2)} \end{aligned} \tag{38}$$

Thus the massless pole gets replaced by two massive poles at the dispersion relations

$$C_2\omega_\pm^2 = \frac{1}{2}\left[C_1^2(C_2+2) + \vec{q}^2(C_2+C_3) \pm \sqrt{[C_1^2C_2 + \vec{q}^2(C_2 - C_3)]^2 + 4C_2C_1^4}\right] \tag{39}$$

From the general linear response relation

$$\langle\tilde{j}^\mu\rangle(\omega.\vec{q}^2) = K^{\mu\nu}(\omega,\vec{q}^2)A_\nu^E(\omega,\vec{q}^2) \tag{40}$$

the principal susceptibilities of interest are easily computed. Perhaps the most interesting of them is the static magnetic susceptibility $\chi(0,\vec{q}^2)$, which follows upon specializing (40) to the case of a static external magnetic field. The screening current leads to a magnetisation $M = \chi B^E$ with,

$$\chi(0,\vec{q}^2) = \frac{[-K_1 + K_2]}{\vec{q}^2}|_{\omega=0} = -\frac{C_1^2[C_1^2 + \vec{q}^2(C_2 + C_2C_3)]}{[\vec{q}^4C_2C_3 + \vec{q}^2C_1^2(C_2 + C_3 + C_2C_3) + C_1^4]} \tag{41}$$

In particular $\chi(0,0) = -1$! Thus we see that the spin 1/2 system is a perfect diamagnet as far as external magnetic fields are concerned. The general Meissner effect is given by the relation $B = (1+\chi)B^E$ giving the net field magnetic field in the system,

$$[\vec{q}^4C_2C_3 + \vec{q}^2C_1^2(C_2 + C_3 + C_2C_3) + C_1^4]B = (-C_1^4 + \vec{q}^2C_3C_1^2 + \vec{q}^4C_2C_3)B^E \tag{42}$$

from which it follows that there are two penetration depths,

$$\lambda_{1,2}^{-2} = \frac{C_1^2}{2C_2C_3}\left[C_2 + C_3 + C_2C_3 \pm \sqrt{(C_2 - C_3)^2 + C_2^2C_3^2 + 2C_2C_3(C_2 + C_3)}\right]. \tag{43}$$

From the numerical estimates made by HC, it follows that $\frac{e^2}{4\pi m}$ is a small dimensionless quantity. We have also made the assumption that magnetic length l^2 is small, i.e. $\frac{(2\bar{n}+1)}{2\pi n_e} << 1$. It can then be seen that C_3 is numerically much smaller than C_1 and C_2. If we neglect C_3 in (42) the penetration depths reduce to $\lambda^2 = \frac{C_2}{C_1^2} = \frac{m}{e^2 n_e}$, which equals the BCS value, as noted by HC.

The dielectric constant $\epsilon(\omega,\vec{q}^2)$ follows from the response to an external electric field to be

$$\epsilon(\omega,\vec{q}^2) = [1 + \frac{K_1}{q^2}]^{-1} = \frac{C_2^2(\omega^2 - \omega_+^2)(\omega^2 - \omega_-^2)}{(-C_2q^2 + C_1^2)(-C_2\omega^2 + C_3\vec{q}^2)} \tag{44}$$

and is seen to have zeroes signifying longitudinal modes at the two dispersion relations. Further, it also follows that external electric fields are also screened out by polarization effects.

The conductivity tensor $\sigma_{ij}(\omega,\vec{q}^2)$ is computed to be

$$\sigma_{ij}(\omega,\vec{q}^2) = \epsilon_{ij}K_3(\omega,\vec{q}^2). \tag{45}$$

Thus the supercurrent has a transverse, "Hall"-like, conductivity.

Finally, we give a simplified expression for the spin correlation function

$$\Sigma(q) = \frac{C_1^2(C_4 - \frac{C_5C_1}{C_2})^2\vec{q}^2}{C_2(\omega^2 - \omega_+^2)(\omega^2 - \omega_-^2)}, \tag{46}$$

where we have left out terms involving numerically small constants, including C_6.

V. CONCLUSIONS

The correlation functions we have derived summarize the bulk properties of the spin 1/2 system and in principle the constants introduced introduced in the effective lagrangian are measureable. There is an ongoing search, as yet inconclusive[18], for violation of parity and time reversal invariance in the high T_c superconductors, as predicted[19] by anyonic superconductivity. Hence it is too early to speculate about experimental significance of the spin effects that we find here. On the theoretical side the idea of fractional statistics continues to be popular[19] in the context of both high T_c superconductivity and the fractional quantum Hall effect. The work presented here serves to emphasize, as does Ref. 16, that unless phenomenology dictates otherwise, the spin degree of freedom cannot be ignored in systems of flux carrying particles.

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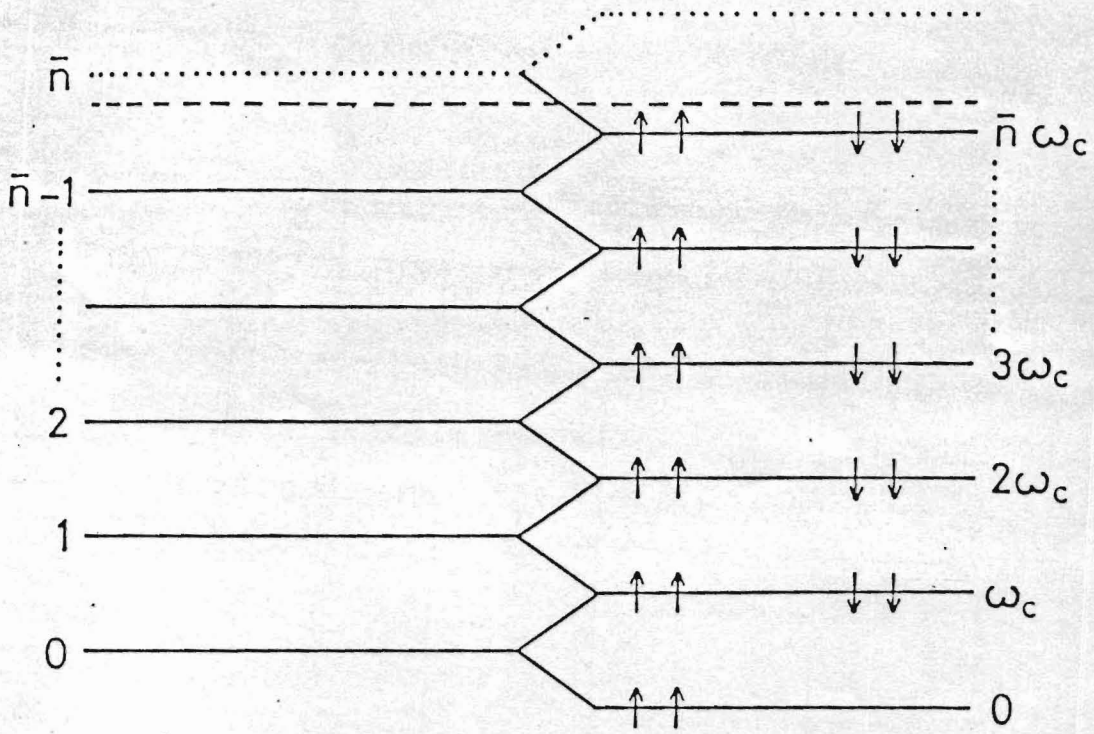

FIGURE CAPTION

Fig. 1: The ferromagnetic ground state configuration of the spin 1/2 fermions with $\bar{n}$ completely filled Landau levels. The arrows indicate the spin and the energies are given at the right. The dashed line represents the Fermi level.