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# CLASSICAL GRAVITATIONAL FIELD THEORY AND MACH PRINCIPLE


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## Abstract

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Equations for the massive gravitational field have been derived in the framework of the special theory of gravitation, based on the geometrization principle. Graviton mass has been shown to be crucial for the elaboration of the relativistic theory of gravitation. According to this theory, homogeneous and isotropic Universe develops in a series of alternating cycles, from high to minimal density etc., and can not be other than flat. The theory predicts the presence of a large latent mass of matter in the Universe and prohibits the existence of 'black holes'. Also, the theory explains all observable events so far known to occur in the Solar system.



## Аннотация

Логунов А.А. Теория классического гравитационного поля и принцип Маха: Препринт ИФВЭ 95-53. – Протвино, 1995. – 29 с., библиогр.: 7.

В работе в рамках специальной теории относительности на основе принципа геометризации находятся уравнения для массивного гравитационного поля. Наличие массы гравитона имеет принципиальное значение для построения теории. Согласно этой теории гравитации, однородная и изотропная Вселенная развивается циклически, от большой плотности до минимальной и т.д., и может быть только плоской. Теория предсказывает существование во Вселенной значительной скрытой массы вещества. Существование во Вселенной "черных" дыр полностью исключается. Теория объясняет все известные наблюдательные факты в Солнечной системе.


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# Introduction

Einstein's General Relativity Theory (GRT), whose basic equations were constructed by Hilbert and Einstein in 1915, opened a new stage in the investigation of gravitational phenomena. But though quite successful this theory from the very first moment of its existence met with principal difficulties in determining physical characteristics of the gravitational field, and as a consequence, in formulating energy-momentum conservation laws.

Einstein clearly understood the fundamental importance of the energy-momentum conservation laws, moreover, he considered, that a total tensor of matter and gravitational field taken together should be the source of the gravitational field. Hence in 1913 he wrote that *" the tensor of the gravitational field $\theta_{\mu\nu}$ is a source of the field alongside with the tensor of the material systems $\Theta_{\mu\nu}$. An exclusive position of the gravitational field energy as compared with other forms of energy should lead to inadmissible consequences."* In the same work Einstein came to a conclusion, that *"in a general case the gravitational field is characterized by ten space-time functions,"* components of metric tensor of Riemannian space $g_{\mu\nu}$. However sticking to this of constructing the theory Einstein did not manage to make the tensor of matter and gravitational field the field source, as instead of the gravitational field tensor in GRT there arose a pseudotensor in the Riemannian space.

In 1918 Schrödinger showed that under a proper choice of the coordinate system all the components of the energy-momentum pseudotensor of the gravitational field outside the spherically-symmetric source may be turned into zero. In this connection Einstein wrote: *"As for Schrödinger considerations, they are very convincing due to their analogy with the electrodynamics, where the stresses and density of energy of any field are different from zero. However I cannot find the reason, why we are to have the same state of things for the gravitational fields. The gravitational fields may be given, without introducing stresses and energy density."*

As we see Einstein gave up the concept of classical Faraday-Maxwell type field, that possessed energy-momentum density as far as the gravitational field was concerned, though he made an important step forward having related the gravitational field with

a tensor quantity. Einstein took a metric tensor of the Riemannian space $g_{\mu\nu}$ as such a quantity. This trend of thought seemed to Einstein quite natural, since his point of view on the gravitational field formed under the influence of the equivalence principle for the inertia and gravitation forces, introduced by himself: *"...for an infinitesimal domain one can always choose the coordinates in such a way, that the gravitational field would be absent from it."*

He stressed this idea several times, for instance in 1923 he wrote: *"For any infinitesimal neighborhood of a point in an arbitrary gravitational field we can always point out a local coordinate system in such a state of motion, that there would be no gravitational field w.r.t. this local system (local inertial system)"*. In this way a notion arose, that the gravitational field could not be localized. The presence of the energy-momentum pseudotensor is, in Einstein's opinion, in complete correspondence with the equivalence principle.

However Einstein's previous statement is not, in fact, fulfilled in GRT since the curvature tensor of the Riemannian space is to be considered here as physical characteristic of the field. It is Synge to whom we are obliged for clear realization of this fact. He wrote: *"If accept the idea that space-time is Riemannian four-dimensional space ( and if we are "relativists", we are to do it) then, our first task will obviously consist in feeling this four-dimensionality, similar to the ancient sailors were to feel sphericity of the ocean. And the first thing we are to realize is the Riemannian tensor, since this tensor is nothing else but the gravitational field: if it turns into zero ( and only in this case), the field does not exist. However, and which is very strange, such an important fact was withdrawn to the background."* Then he added: *"In Einstein's theory depending on the fact, whether the Riemannian tensor is different from zero or equals zero, the gravitational field is either present or absent. This property is absolute, it is connected in no way with the world line of any observer."*

Hence according to GRT, the matter (all substance fields, except gravitational) is characterized by the energy-momentum tensor, and the gravitational field is characterized by the Riemannian curvature tensor. In this if the first one possesses the second rank, the second one has the fourth rank, i.e., in fact there appeared a principal difference between the characteristics of matter and gravitational field in GRT.

The introduction of the energy-momentum pseudotensor of the gravitational field did not help Einstein to conserve the energy-momentum conservation laws in his theory. This fact was clearly understood by Hilbert. In connection with this he wrote in 1917: "*...I state that for the General Relativity Theory, i.e. in the case of general invariance of the Hamiltonian function, no energy equations, that.... correspond to the energy equations in orthogonal-invariant theories, exist, I could also stress this circumstance as a characteristic feature of GRT.*"

In virtue of the absence of ten-parameter space-time group of motions in GRT, in principle, one cannot introduce in it the energy-momentum and angular momentum conservation laws like those taking place in any other physical theory. These laws are the fundamental ones in nature as just they introduce the universal physical characteristics for all forms of matter which allow one to consider quantitatively the transformation of one forms of matter into others. In this connection it is expedient to construct such gra-

vitation theory, where all energy-momentum and angular momentum conservation laws would be fulfilled and gravitational field would possess the energy-momentum density as it is the case for the electromagnetic field of Faraday-Maxwell.

In GRT as different from all other physical theories, the scalar Lagrangian density of the gravitational field contains second-order derivatives.

About 50 years ago Rosen showed in his work [1] that if, in addition to the Riemannian metric $g_{\mu\nu}$ one introduced the Minkowski metric $\gamma_{\mu\nu}$ then one might always construct a scalar Lagrangian density of the gravitational field with respect to arbitrary coordinate transformations, that would contain derivatives of the order not higher than one. In particular, he constructed the Lagrangian density that led to the Hilbert-Einstein equations. That was the way the bimetric formalism came into existence. However, this approach immediately complicated the construction of the gravitational theory, because using the tensors $\gamma_{\mu\nu}$ and $g_{\mu\nu}$ one could write quite a large number of scalar densities with respect to arbitrary coordinate transformations, making it completely unclear which scalar density should be chosen as the Lagrangian density when constructing the gravitation theory.

Following this direction Nathan Rosen chose different scalar densities for the Lagrangian density and constructed on their basis various gravitation theories, which, generally speaking, yielded quite naturally different predictions for these or those gravitational effects. Below we shall see that within the SRT (Special Relativity Theory) frames, that describes phenomena in inertial and noninertial reference frames, with the help of the geometrization principle reflecting the universality of the gravitational interaction with matter, and with introduction of the graviton mass, we shall manage to unify the Poincaré idea of the gravitational field [2] as a physical one in Faraday-Maxwell spirit with Einstein's idea of the Riemannian space-time geometry. It is this geometrization principle that will help us to find an infinite dimensional noncommuting gauge group that will allow us to construct the Lagrangian density of the proper gravitational field. All this has brought us to the Relativistic Theory of Gravitation (RTG) [3] that possesses all conservation laws, as it takes place in all physical theories.

In this theory, owing to geometrization, the conservable total energy-momentum tensor of matter and gravitational field is the source of the gravitational field, right what Einstein wanted when constructing the gravitation theory. In what follows we shall see that quite general physical requirements bring us to unambiguous construction of the complete system of equations for a massive gravitational field. Equations (66) and (67) in this theory considerably differ from Hilbert-Einstein equations, since it conserves the notion of inertial coordinate system, and gravitational forces differ, in principle, from the inertia ones, since they are caused by physical field. It should particularly be stressed that the rest mass of the gravitational field, as we shall see further on, is of principal importance. This article once again presents the basic principles and equations of the theory with certain additions and clarifications.

The Relativistic Theory of Gravitation with graviton mass is a field theory to the same extent, as the classical electrodynamics, therefore it might be called classical gravidynamics.

# 1. Basic Postulates of the RTG

Passing to the construction of the gravitational field theory we will proceed from the following basic postulates.

**Postulate 1.**

The Relativistic Theory of Gravitation is based on the Special Relativity Theory. This means that the Minkowski space, i.e. the pseudo-Euclidean geometry of space-time is a fundamental space for all physical fields, including the gravitational one. This statement is necessary and sufficient for both the energy-momentum and angular momentum conservation laws to hold true for matter and gravitational field taken together. In other words the Minkowski space reflects the dynamic properties common for all forms of matter. Thus they are provided with the existence of common physical characteristics, which allow one to quantitatively describe the transformation of one forms of matter into others.

The Minkowski space cannot be considered as the a priori existing one because it reflects the properties of matter and, hence, is indispensable from it. Though formally, just due to the independence of space from the forms of matter, it is sometimes treated abstractly, neglecting matter.

The Minkowski space admits the description in both inertial coordinate system (e.g. Galilean coordinates) and noninertial one (accelerated). From the mathematical point of view it is quite obvious, because a wide class of admissible coordinate systems (curvilinear including) can be introduced into the Minkowski space. However this quite simple fact was not clear for the long period even to some great physicists. This can be explained by the fact that the Minkowski space was considered by many scientists as a formal geometric interpretation of the Special Relativity Theory. Such understanding reduces the SRT framework considerably. To construct the RTG one was to proceed from the most general formulation of the SRT, which reads: all physical processes (gravitational one including) go on in the four-dimensional world, i.e., in space and time with pseudo–Euclidean geometry. This understanding of the SRT removes the clock synchronization, the principle of the light velocity constancy to the background, since they are of a rather limited, particular character, because only the interval carries physical sense.

At the beginning of the century H.Poincaré wrote in his book "Science and Hypothesis" that though *"...experience plays a necessary role in the geometry origin, but it would be a mistake to conclude that geometry – at least partially – is an experimental science. If it were experimental, it would have only temporary, approximate — and very roughly approximate — meaning.* Then he went on: *"Geometry studies only particular <<group>> of displacements, but the general group notion first exists in our minds, at least in the form of a possibility".*

"*Under this choice the experiment gives us a direction, however not making it obligatory; it shows which geometry is more* <u>*convenient*</u> *for us rather than which geometry is the most correct one.*" If we follow this stream of Poincaré's thoughts, then guided by the fundamental physical principles, such as matter energy-momentum and angular momentum conservation laws, we are to use just the pseudo–Euclidean space-time geometry as the foundation. This choice is not only *convenient*, it is actually the *unique* till the

moment the conservation laws hold. In 1921 A. Einstein wrote in his book "Geometry and Experiment": *"The question whether this continuum has Euclidean, Riemannian or any other structure is a physical question, and the answer can be given by an experiment, rather than by an agreement on choosing on the basis of pure expediency".*

In principle it is true, however, there arises a question – what experimental facts are needed, so that we would be able to unambiguously characterize the geometry? In our opinion, the fundamental energy-momentum and angular momentum conservation laws may be taken as such facts, since namely they reflect general dynamical properties of matter. This brings us to the pseudo–Euclidean space-time geometry, as the simplest one.

Hence when establishing the structure of space-time geometry, one should naturally proceed from fundamental physical principles obtained through generalization of numerous experimental data, related to different forms of matter, rather than the particular experimental facts (e.g. light and test bodies motion).

The Minkowski space has a profound physical meaning as it defines the universal properties of matter, such as energy, momentum, angular momentum.

The gravitational field is described by the second-rank symmetric tensor $\phi^{\mu\nu}$ and is a true physical field possessing an energy-momentum density, rest mass $m$ and polarization states corresponding to spins 2 and 0. The representations corresponding to spins 1 and 0' are eliminated from the states of field $\phi^{\mu\nu}$ by making the components of $\phi^{\mu\nu}$ obey the field equation

$$D_\mu \phi^{\mu\nu} = 0, \tag{1}$$

where $D_\mu$ is a covariant derivative in the Minkowski space.

In addition to exclude the nonphysical field states, Eq.(1) introduces into the theory a Minkowski metric $\gamma_{\mu\nu}$ thus allowing one to separate the inertia forces from the action of the gravitational field. Choosing the Galilean metric $\gamma_{\mu\nu}$ one can eliminate the action of the inertia forces completely. The Minkowski metric makes it possible to introduce the notion of the standard length and time interval in the absence of the gravitational field. Below we shall see that the interaction of tensor gravitational field with matter can be introduced in such a way, that it would cause the one deforming the Minkowski space by varying the metric properties, but without violating causality.

**Postulate 2. Geometrization Principle**

Since the gravitational field is described by the symmetric second-rank tensor $\phi^{\mu\nu}$ and its interaction with other fields may be considered as universal one, a unique opportunity is opened up "to join" this field directly to the tensor $\gamma^{\mu\nu}$ in the matter Lagrangian density according to the rule

$$L_M(\tilde{\gamma}^{\mu\nu}, \phi_A) \to L_M(\tilde{g}^{\mu\nu}, \phi_A), \tag{2}$$

where

$$\tilde{g}^{\mu\nu} = \tilde{\gamma}^{\mu\nu} + \tilde{\phi}^{\mu\nu},\ \tilde{g}^{\mu\nu} = \sqrt{-g}\, g^{\mu\nu},\ \tilde{\gamma}^{\mu\nu} = \sqrt{-\gamma}\, \gamma^{\mu\nu},\ \tilde{\phi}^{\mu\nu} = \sqrt{-\gamma}\, \phi^{\mu\nu}; \tag{3}$$

$\phi_A$ are matter fields; $g = \det g_{\mu\nu}$; $\gamma = \det \gamma_{\mu\nu}$; $\tilde{g}^{\mu\nu}\tilde{g}_{\nu\sigma} = \delta^\mu_\sigma$. The tensor $g_{\nu\sigma}$ is found from the last equality. The field indices are lifted and lowered with the help of $\gamma_{\mu\nu}$ and those

of tensor $g^{\mu\nu}$ with the help of the Riemannian metric tensor. Speaking about matter we understand all forms of substance excluding the gravitational field.

This form of interaction between the gravitational field and matter introduces the notion of the effective Riemannian space, where the motion of matter takes place, and is termed the geometrization principle. According to this principle, the matter motion under the action of the gravitational field $\phi^{\mu\nu}$ in the Minkowski space with the metric $\gamma_{\mu\nu}$ is identical to its motion in the effective Riemannian space with the metric $g_{\mu\nu}$. The effective Riemannian space has literally the field origin due to the presence of the gravitational field $\phi^{\mu\nu}$.

Since the metric properties are determined in the presence of the gravitational field by the effective Riemannian tensor or by the Minkowski tensor $\gamma_{\mu\nu}$ in its absence, this theory can answer the question about the variation in the body dimensions and in the advance of a clock under the action of the gravitational field. And if the theory does not contain the tensor $\gamma_{\mu\nu}$ in the field equations, basically it cannot answer such questions. In the GRT the gravitational field is characterized by the metric tensor $g_{\mu\nu}$, whereas in our theory it is defined by the tensor quantity $\phi^{\mu\nu}$ and the effective Riemannian space is constructed with the help of the field $\phi^{\mu\nu}$ and also the Minkowski metric tensor $\gamma^{\mu\nu}$ fixing a certain choice of the coordinate system.

Since there are Galilean (inertial) coordinate systems in our theory, acceleration has the absolute nature. The motion of a test body in the effective Riemannian space follows the geodesic line of this space, but it is not free as it is caused by the action of the gravitational field. If a test body was charged it would irradiate electromagnetic waves because it would move in the field with acceleration.

As the effective Riemannian space is produced by the gravitational field $\phi^{\mu\nu}$ present in the Minkowski space, it can always be specified, which is a very important point, in one coordinate system. This means that we will deal only with such Riemannian spaces which are specified in one chart. In our viewpoint, the Riemannian spaces possessing a complicated topology are eliminated completely as they are not of the field origin. It should be noted that as matter moves in the effective Riemannian space, the Minkowski metric tensor $\gamma_{\mu\nu}$ will not be contained in the equations of matter motion. The Minkowski space effects the matter motion only through the Riemannian metric tensor $g_{\mu\nu}$ found from the equations containing the metric tensor $\gamma_{\mu\nu}$.

Hence, though the geometrization principle allows one to go over to the description of motion in the effective Riemannian space, nevertheless the metric of the initial Minkowski space has not been excluded. As we will see in what follows it remains in the gravitational field equations, thus conserving the notion of the inertial system, where the inertia forces are identically equal to zero.

## 2. Gauge Transformation Group

Since the Lagrangian density of matter has the form

$$L_M(\tilde{g}^{\mu\nu}, \phi_A), \tag{4}$$

then one can easily find the gauge transformation group, under which the Lagrangian density of matter changes only by a divergence. For this purpose let us use the invariance of the action

$$S_M = \int L_M(\tilde{g}^{\mu\nu},\, \phi_A)\, d^4x \tag{5}$$

at an arbitrary infinitesimal variation of the coordinates

$$x'^{\alpha} = x^{\alpha} + \xi^{\alpha}(x), \tag{6}$$

where $\xi^{\alpha}$ is an infinitesimal displacement 4-vector. Under these coordinate transformations the field functions $\tilde{g}^{\mu\nu}$, $\phi_A$ vary in the following way

$$\begin{aligned} \tilde{g}'^{\mu\nu}(x') &= \tilde{g}^{\mu\nu}(x) + \delta_\xi \tilde{g}^{\mu\nu}(x) + \xi^{\alpha}(x) D_\alpha \tilde{g}^{\mu\nu}(x), \\ \phi'_A(x') &= \phi_A(x) + \delta_\xi \phi_A(x) + \xi^{\alpha}(x) D_\alpha \phi_A(x), \end{aligned} \tag{7}$$

where the expressions

$$\delta_\xi \tilde{g}^{\mu\nu}(x) = \tilde{g}^{\mu\alpha} D_\alpha \xi^{\nu}(x) + \tilde{g}^{\nu\alpha} D_\alpha \xi^{\mu}(x) - D_\alpha(\xi^{\alpha} \tilde{g}^{\mu\nu}),$$

$$\delta_\xi \phi_A(x) = -\xi^{\alpha}(x) D_\alpha \phi_A(x) + F^{B;\alpha}_{A;\beta}\, \phi_B(x) D_\alpha \xi^{\beta}(x) \tag{8}$$

are Lie variations.

The operators $\delta_\xi$ satisfy the conditions of the Lie algebra, i.e. the commutation relation

$$[\delta_{\xi_1},\, \delta_{\xi_2}](\cdot) = \delta_{\xi_3}(\cdot) \tag{9}$$

and Jacobi identity

$$[\delta_{\xi_1},\, [\delta_{\xi_2},\, \delta_{\xi_3}]] + [\delta_{\xi_3},\, [\delta_{\xi_1},\, \delta_{\xi_2}]] + [\delta_{\xi_2},\, [\delta_{\xi_3},\, \delta_{\xi_1}]] = 0,$$

where

$$\xi_3^{\nu} = \xi_1^{\mu} D_\mu \xi_2^{\nu} - \xi_2^{\mu} D_\mu \xi_1^{\nu} = \xi_1^{\mu} \partial_\mu \xi_2^{\nu} - \xi_2^{\mu} \partial_\mu \xi_1^{\nu}. \tag{10}$$

So that (9) take place, it is necessary that the following conditions should be fulfilled

$$F^{B;\mu}_{A;\nu}\, F^{C;\alpha}_{B;\beta} - F^{B;\alpha}_{A;\beta}\, F^{C;\mu}_{B;\nu} = f^{\mu\alpha;\tau}_{\nu\beta;\sigma}\, F^{C;\sigma}_{A;\tau}, \tag{11}$$

where the structure constants $f$ are equal to

$$f^{\mu\alpha;\tau}_{\nu\beta;\sigma} = \delta^{\mu}_{\beta} \delta^{\alpha}_{\sigma} \delta^{\tau}_{\nu} - \delta^{\alpha}_{\nu} \delta^{\mu}_{\sigma} \delta^{\tau}_{\beta}. \tag{12}$$

One can easily get convinced that they satisfy the Jacobi identity

$$f^{\alpha\nu;\sigma}_{\beta\mu;\tau}\, f^{\tau\rho;\omega}_{\sigma\varepsilon;\delta} + f^{\nu\rho;\sigma}_{\mu\varepsilon;\tau}\, f^{\tau\alpha;\omega}_{\sigma\beta;\delta} + f^{\rho\alpha;\sigma}_{\varepsilon\beta;\tau}\, f^{\tau\nu;\omega}_{\sigma\mu;\delta} = 0 \tag{13}$$

and possess antisymmetric properties

$$f^{\alpha\nu;\rho}_{\beta\mu;\sigma} = -f^{\nu\alpha;\rho}_{\mu\beta;\sigma}.$$

Under coordinate transformation (6) the variation of the action is equal to zero

$$\delta_c S_M = \int\limits_{\Omega'} L'_M(x')\, d^4x' - \int\limits_{\Omega} L_M(x)\, d^4x = 0. \tag{14}$$

The first integral in (14) may be presented in the form

$$\int\limits_{\Omega'} L'_M(x')\, d^4x' = \int\limits_{\Omega} J\, L'_M(x')\, d^4x,$$

where

$$J = det\left(\frac{\partial x'^{\alpha}}{\partial x^{\beta}}\right).$$

In the first order over $\xi^\alpha$ the determinant $J$ is equal to

$$J = 1 + \partial_\alpha \xi^\alpha(x). \tag{15}$$

With an account of the expansion

$$L'_M(x') = L'_M(x) + \xi^\alpha(x)\frac{\partial L_M}{\partial x^\alpha},$$

as well as of (15) one may present the expression for the variation in the form

$$\delta_c S_M = \int\limits_{\Omega} [\delta L_M(x) + \partial_\alpha(\xi^\alpha L_M(x))]\, d^4x = 0.$$

Owing to the arbitrariness of the integration volume $\Omega$ we have an identity

$$\delta\, L_M(x) = -\partial_\alpha(\xi^\alpha(x) L_M(x)), \tag{16}$$

where Lie variation $\delta L_M$ is equal to

$$\delta L_M(x) = \frac{\partial L_M}{\partial \tilde{g}^{\mu\nu}}\,\delta\tilde{g}^{\mu\nu} + \frac{\partial L_M}{\partial(\partial_\alpha \tilde{g}^{\mu\nu})}\,\delta(\partial_\alpha \tilde{g}^{\mu\nu}) + \frac{\partial L_M}{\partial \phi_A}\,\delta\phi_A + \frac{\partial L_M}{\partial(\partial_\alpha \phi_A)}\,\delta(\partial_\alpha \phi_A). \tag{17}$$

Whereof, in particular, it follows that if the scalar density depends only on $\tilde{g}^{\mu\nu}$ and its derivatives then under transformation (8) it will also change only by a divergence

$$\delta L(\tilde{g}^{\mu\nu}(x)) = -\partial_\alpha(\xi^\alpha(x)\, L(\tilde{g}^{\mu\nu}(x))), \tag{16a}$$

where Lie variation $\delta L$ can be equal to

$$\delta L(\tilde{g}^{\mu\nu}(x)) = \frac{\partial L}{\partial \tilde{g}^{\mu\nu}}\,\delta\tilde{g}^{\mu\nu} + \frac{\partial L}{\partial(\partial_\alpha \tilde{g}^{\mu\nu})}\,\delta(\partial_\alpha \tilde{g}^{\mu\nu}) + \frac{\partial L}{\partial(\partial_\alpha \partial_\beta \tilde{g}^{\mu\nu})}\delta(\partial_\alpha \partial_\beta \tilde{g}^{\mu\nu}). \tag{17a}$$

Lie variations (8) were found in the context of coordinate transformations (6). However one can stick to another point of view, and then in agreement with it one can consider

transformations (8) as gauge transformations. In this case the arbitrary infinitesimal 4-vector $\xi^\alpha(x)$ will already be a gauge vector rather than a coordinate displacement vector. In what follows in order to stress the difference between the gauge group and coordinate transformation group, we shall denote the group parameter as $\varepsilon^\alpha(x)$, and the transformations of the field functions

$$\tilde{g}^{\mu\nu}(x) \to \tilde{g}^{\mu\nu}(x) + \delta\tilde{g}^{\mu\nu}(x), \qquad \phi_A(x) \to \phi_A(x) + \delta\phi_A(x) \tag{18}$$

with increments

$$\begin{aligned} \delta_\varepsilon \tilde{g}^{\mu\nu}(x) &= \tilde{g}^{\mu\alpha} D_\alpha \varepsilon^\nu(x) + \tilde{g}^{\nu\alpha} D_\alpha \varepsilon^\mu(x) - D_\alpha(\varepsilon^\alpha \tilde{g}^{\mu\nu}), \\ \delta_\varepsilon \phi_A(x) &= -\varepsilon^\alpha(x) D_\alpha \phi_A(x) + F^{B;\alpha}_{A;\beta}\, \phi_B(x)\, D_\alpha\, \varepsilon^\beta(x) \end{aligned} \tag{19}$$

will be called gauge transformations.

In complete agreement with formulas (9) and (10) the operators satisfy the same Lie algebra, i.e. the commutation relation

$$[\delta_{\varepsilon_1},\ \delta_{\varepsilon_2}](\cdot) = \delta_{\varepsilon_3}(\cdot) \tag{20}$$

and Jacobi identity

$$[\delta_{\varepsilon_1},\ [\delta_{\varepsilon_2},\ \delta_{\varepsilon_3}]] + [\delta_{\varepsilon_3},\ [\delta_{\varepsilon_1},\ \delta_{\varepsilon_2}]] + [\delta_{\varepsilon_2},\ [\delta_{\varepsilon_3},\ \delta_{\varepsilon_1}]] = 0. \tag{21}$$

Similar to the above we have

$$\varepsilon_3^\nu = \varepsilon_1^\mu D_\mu \varepsilon_2^\nu - \varepsilon_2^\mu D_\mu \varepsilon_1^\nu = \varepsilon_1^\mu \partial_\mu \varepsilon_2^\nu - \varepsilon_2^\mu \partial_\mu \varepsilon_1^\nu.$$

The gauge group sprung out from the geometrized structure of the scalar Lagrangian density of matter $L_M(\tilde{g}^{\mu\nu}, \phi_A)$, which owing to identity (16) changes only by a divergence under gauge transformations (19). Hence, the geometrization principle, that has determined the universal character of the interaction between the matter and gravitational field, gives us a possibility to formulate non-commuting infinite dimensional gauge group (19).

An essential difference between the gauge and coordinate transformations will manifest itself at a decisive point of the theory when constructing the scalar Lagrangian density of the proper gravitational field. The difference appears due to the fact, that under gauge transformations the metric tensor $\gamma_{\mu\nu}$ does not change, and consequently in virtue of (3) we have

$$\delta_\varepsilon \tilde{g}^{\mu\nu}(x) = \delta_\varepsilon \tilde{\phi}^{\mu\nu}(x).$$

Basing on (19) there follows the transformation for the field

$$\delta_\varepsilon \tilde{\phi}^{\mu\nu}(x) = \tilde{g}^{\mu\alpha}\, D_\alpha\, \varepsilon^\nu(x) + \tilde{g}^{\nu\alpha}\, D_\alpha\, \varepsilon^\mu(x) - D_\alpha(\varepsilon^\alpha\, \tilde{g}^{\mu\nu}).$$

However this transformation differs greatly from the one under coordinate displacement

$$\delta_\xi\, \tilde{\phi}^{\mu\nu}(x) = \tilde{\phi}^{\mu\alpha}\, D_\alpha \xi^\nu(x) + \tilde{\phi}^{\nu\alpha}\, D_\alpha \xi^\mu(x) - D_\alpha(\xi^\alpha\, \tilde{\phi}^{\mu\nu}).$$

Under gauge transformations (19) the equations of motion for the matter do not change since under any such transformations the Lagrangian density of the matter changes only by a divergence.

## 3. Lagrangian Density and Equations of Motion for the Proper Gravitational Field

As is known, if using only the tensor $g_{\mu\nu}$, one cannot construct the scalar density of the Lagrangian for the proper gravitational field with respect to the arbitrary coordinate transformations as quadratic form of the derivatives not higher than the first order. Therefore the metric $\gamma_{\mu\nu}$ will necessarily enter the Lagrangian density alongside with the metric $g_{\mu\nu}$. Since the metric $\gamma_{\mu\nu}$ does not change under gauge transformations (19), then it follows that in order to make the Lagrangian density of the proper gravitational field change under this transformation only by a divergence, strong limitations for its structure should arise. This is right the point where principal difference between gauge and coordinate transformations appears.

While the coordinate transformations impose almost no limitations on the structure of the scalar Lagrangian density of the proper gravitational field, the gauge transformations allow us to find the Lagrangian density. A straightforward general method to construct the Lagrangian is given in [3].

Here we shall choose a simpler technique to construct the Lagrangian. Basing on (16a) we conclude that the simplest scalar densities $\sqrt{-g}$ and $\tilde{R} = \sqrt{-g}R$, where R is the scalar curvature of the effective Riemannian space, vary in the following way

$$\sqrt{-g} \to \sqrt{-g} - D_\nu(\varepsilon^\nu \sqrt{-g}), \tag{22}$$

$$\tilde{R} \to \tilde{R} - D_\nu(\varepsilon^\nu \tilde{R}), \tag{23}$$

under gauge transformation (19). The scalar density $\tilde{R}$ is expressed through Christoffel symbols

$$\Gamma^\lambda_{\mu\nu} = \frac{1}{2} g^{\lambda\sigma}(\partial_\mu g_{\sigma\nu} + \partial_\nu g_{\sigma\mu} - \partial_\sigma g_{\mu\nu}) \tag{24}$$

in the following way

$$\tilde{R} = -\tilde{g}^{\mu\nu}(\Gamma^\lambda_{\mu\nu}\Gamma^\sigma_{\lambda\sigma} - \Gamma^\lambda_{\mu\sigma}\Gamma^\sigma_{\nu\lambda}) - \partial_\nu(\tilde{g}^{\mu\nu}\Gamma^\sigma_{\mu\sigma} - \tilde{g}^{\mu\sigma}\Gamma^\nu_{\mu\sigma}). \tag{25}$$

Since the Christoffel symbols are not tensor quantities, each term in (25) is not a scalar density. However if tensor quantities $G^\lambda_{\mu\nu}$,

$$G^\lambda_{\mu\nu} = \frac{1}{2} g^{\lambda\sigma}(D_\mu g_{\sigma\nu} + D_\nu g_{\sigma\mu} - D_\sigma g_{\mu\nu}), \tag{26}$$

be introduced, then the scalar density may identically be presented in the form

$$\tilde{R} = -\tilde{g}^{\mu\nu}(G^\lambda_{\mu\nu} G^\sigma_{\lambda\sigma} - G^\lambda_{\mu\sigma} G^\sigma_{\nu\lambda}) - D_\nu(\tilde{g}^{\mu\nu} G^\sigma_{\mu\sigma} - \tilde{g}^{\mu\sigma} G^\nu_{\mu\sigma}). \tag{27}$$

Note, that in (27) each group of terms separately behaves as a scalar density under arbitrary coordinate transformations. With account of (22) and (23) the expression

$$\lambda_1(\tilde{R} + D_\nu Q^\nu) + \lambda_2\sqrt{-g} \tag{28}$$

changes only by a divergence under arbitrary gauge transformations. Choosing the vector density $Q^\nu$ equal to

$$Q^\nu = \tilde{g}^{\mu\nu}\, G^\sigma_{\mu\sigma} - \tilde{g}^{\mu\sigma}\, G^\nu_{\mu\sigma},$$

we exclude the terms with derivatives higher than the first order from the previous expression, and hence obtain the following Lagrangian density

$$-\lambda_1 \tilde{g}^{\mu\nu}(G^\lambda_{\mu\nu}\, G^\sigma_{\lambda\sigma} - G^\lambda_{\mu\sigma}\, G^\sigma_{\nu\lambda}) + \lambda_2\sqrt{-g}. \tag{29}$$

Hence we see that the requirement that the Lagrangian density of the proper gravitational field changes only by a divergence under gauge transformations (19) unambiguously determines the structure of Lagrangian density (29). But if one restricts himself only with this density, then the gravitational field equations will be gauge invariant, and the metric of the Minkowski space $\gamma_{\mu\nu}$ would not enter the system of equations, determined by Lagrangian density (29). Since in this approach the metric of the Minkowski space disappears, then a possibility to present the gravitational field as a physical field of Faraday-Maxwell type in the Minkowski space is excluded.

With the Lagrangian density (29) the introduction of the metric $\gamma_{\mu\nu}$ with the help of equations (1) does not save the situation, since the physical quantities – the interval and curvature tensor of the Riemannian space, as well as the tensor $t_g^{\mu\nu}$ of the gravitational field – will depend on the choice of the gauge, which is inadmissible from the point of view of physics. In order to conserve the notion of the field in the Minkowski space and to avoid such ambiguity, the Lagrangian density of the gravitational field should be enriched with term, violating the gauge group. At first sight, it may seem, that there arises a great arbitrariness in choosing the Lagrangian density of the gravitational field, since the group may be violated in many different ways. However it turns out not to be true, since our physical requirement of the polarization properties of the gravitational field as a field with 2 – and 0 – spins, imposed by equation (1), brings us to the necessity to choose the term violating group (19) in a way so that equations (1) would be consequences of the system of equations for the gravitational field and fields of matter, because only in this case no over–determined system of differential equations would arise. For this purpose we shall introduce a term of the form

$$\gamma_{\mu\nu}\tilde{g}^{\mu\nu} \tag{30}$$

into the scalar Lagrangian density of the gravitational field. In the presence of (1) and under transformations (19) this term also changes by a divergence, however only on the class of vectors, satisfying the condition

$$g^{\mu\nu} D_\mu D_\nu \varepsilon^\sigma(x) = 0. \tag{31}$$

Almost an analogous situation takes place in electrodynamics with the photon rest mass different from zero. With an account of (28)–(30) the total scalar density of the Lagrangian has the form

$$L_g = -\lambda_1 \tilde{g}^{\mu\nu}(G^\lambda_{\mu\nu}\, G^\sigma_{\lambda\sigma} - G^\lambda_{\mu\sigma}\, G^\sigma_{\nu\lambda}) + \lambda_2\sqrt{-g} + \lambda_3\, \gamma_{\mu\nu}\, \tilde{g}^{\mu\nu} + \lambda_4\sqrt{-\gamma}. \tag{32}$$

The last constant term in (32) was introduced in order to turn the Lagrangian density into zero in the absence of the gravitational field. The reduction of the class of gauge vectors owing to the introduction of term (30) brings us automatically to the fact that equations (1) will be the consequences of the gravitational field equations. We will get convinced in this fact in what follows.

According to the principle of the least action the equations for the proper gravitational field have the form

$$\frac{\delta L_g}{\delta \tilde{g}^{\mu\nu}} = \lambda_1 R_{\mu\nu} + \frac{1}{2}\lambda_2\, g_{\mu\nu} + \lambda_3\, \gamma_{\mu\nu} = 0, \tag{33}$$

where

$$\frac{\delta L_g}{\delta \tilde{g}^{\mu\nu}} = \frac{\partial L_g}{\partial \tilde{g}^{\mu\nu}} - \partial_\sigma\left(\frac{\partial L_g}{\partial(\partial_\sigma \tilde{g}^{\mu\nu})}\right)$$

and the Ricci tensor $R_{\mu\nu}$ will be given in the form

$$R_{\mu\nu} = D_\lambda\, G^\lambda_{\mu\nu} - D_\mu\, G^\lambda_{\nu\lambda} + G^\sigma_{\mu\nu}\, G^\lambda_{\sigma\lambda} - G^\sigma_{\mu\lambda}\, G^\lambda_{\nu\sigma}. \tag{34}$$

Since in the case of the absence of the gravitational field equations (33) are to be identically fulfilled, it follows that

$$\lambda_2 = -2\,\lambda_3. \tag{35}$$

Now let us find the density of energy–momentum tensor of the gravitational field in the Minkowski space

$$t_g^{\mu\nu} = -2\frac{\delta L_g}{\delta\, \gamma_{\mu\nu}} = 2\sqrt{-\gamma}(\gamma^{\mu\alpha}\gamma^{\nu\beta} - \frac{1}{2}\gamma^{\mu\nu}\gamma^{\alpha\beta})\, \frac{\delta L_g}{\delta \tilde{g}^{\alpha\beta}} + \lambda_1\, J^{\mu\nu} - 2\lambda_3\, \tilde{g}^{\mu\nu} - \lambda_4\, \tilde{\gamma}^{\mu\nu}, \tag{36}$$

where

$$J^{\mu\nu} = D_\alpha D_\beta(\gamma^{\alpha\mu}\tilde{g}^{\beta\nu} + \gamma^{\alpha\nu}\tilde{g}^{\beta\mu} - \gamma^{\alpha\beta}\, \tilde{g}^{\mu\nu} - \gamma^{\mu\nu}\, \tilde{g}^{\alpha\beta}). \tag{37}$$

If one takes into account dynamic equations (33) in expression (36), then we will get an equation for the proper gravitational field, that will have the following form

$$\lambda_1\, J^{\mu\nu} - 2\,\lambda_3 \tilde{g}^{\mu\nu} - \lambda_4\, \tilde{\gamma}^{\mu\nu} = t_g^{\mu\nu}. \tag{38}$$

In order this equation be identically fulfilled in the absence of the gravitational field, one should put

$$\lambda_4 = -2\,\lambda_3. \tag{39}$$

As the equality

$$D_\mu\, t_g^{\mu\nu} = 0 \tag{40}$$

always takes place for the proper gravitational field, then from equation (38) it follows

$$D_\mu\, \tilde{g}^{\mu\nu} = 0. \tag{41}$$

Hence equations (1) that determine the field polarization properties, follow straightforwardly from equation (38). With account of (41) field equations (38) may be presented in the form

$$\gamma^{\alpha\beta}\, D_\alpha\, D_\beta\, \tilde{\phi}^{\mu\nu} - \frac{\lambda_4}{\lambda_1}\, \tilde{\phi}^{\mu\nu} = -\frac{1}{\lambda_1}\, t_g^{\mu\nu}. \tag{42}$$

In Galilean coordinates this equation has a simple form

$$\Box\,\tilde{\phi}^{\mu\nu} - \frac{\lambda_4}{\lambda_1}\,\tilde{\phi}^{\mu\nu} = -\frac{1}{\lambda_1}\,t_g^{\mu\nu}. \tag{43}$$

It is reasonable to give the meaning of graviton mass squared to the numeric factor $-\lambda_4/\lambda_1 = m^2$, and according to the correspondence principle the value for $-1/\lambda_1$ should be taken equal to $16\pi$. Hence all the unknown constants entering the Lagrangian density have been determined:

$$\lambda_1 = -\frac{1}{16\pi}, \quad \lambda_2 = \lambda_4 = -2\,\lambda_3 = \frac{m^2}{16\,\pi}. \tag{44}$$

The constructed scalar density of the Lagrangian of the proper gravitational field will have the form

$$L_g = \frac{1}{16\,\pi}\,\tilde{g}^{\mu\nu}(G^{\lambda}_{\mu\nu}\,G^{\sigma}_{\lambda\sigma} - G^{\lambda}_{\mu\sigma}\,G^{\sigma}_{\nu\lambda}) - \frac{m^2}{16\,\pi}\left(\frac{1}{2}\,\gamma_{\mu\nu}\tilde{g}^{\mu\nu} - \sqrt{-g} - \sqrt{-\gamma}\right). \tag{45}$$

Its relevant dynamic equation for the proper gravitational field may be given in the form

$$J^{\mu\nu} - m^2\,\tilde{\phi}^{\mu\nu} = -16\pi\,t_g^{\mu\nu}, \tag{46}$$

or

$$R^{\mu\nu} - \frac{m^2}{2}(g^{\mu\nu} - g^{\mu\alpha}g^{\nu\beta}\gamma_{\alpha\beta}) = 0. \tag{47}$$

These equations considerably restrict the class of gauge transformations, leaving only trivial ones, satisfying Killing conditions. Such transformations follow from the Lorentz invariance and take place in any theory.

The Lagrangian density constructed above, brings us to equations (47), from which it follows that equations (41) are their consequences, therefore outside the matter we will have ten equations for ten unknown field functions. With the help of equation (41) the unknown field functions $\phi^{o\alpha}$ can easily be expressed through the field functions $\phi^{ik}$, where $i$ and $k$ run the values 1,2,3. Thus, in the Lagrangian density of the proper gravitational field the structure of the mass term, that violates the gauge group, is unambiguously determined by the polarization properties of the gravitational field.

## 4. Equations of Motion for the Gravitational Field and Matter

The total Lagrangian density of matter and gravitational field is equal to

$$L = L_g + L_M(\tilde{g}^{\mu\nu},\ \phi_A), \tag{48}$$

where $L_g$ is defined by expression (45).

Basing on (48) and with the help of the least action principle we obtain a complete system of equations for matter and gravitational field

$$\frac{\delta L}{\delta\tilde{g}^{\mu\nu}} = 0, \tag{49}$$

$$\frac{\delta L_M}{\delta \phi_A} = 0. \tag{50}$$

Since under arbitrary infinitesimal changes of the coordinates the variation of the action $\delta_c S_M$ is equal to zero,

$$\delta_c S_M = \delta_c \int L_M(\tilde{g}^{\mu\nu}, \phi_A)\, d^4x = 0,$$

then one can obtain an identity [3] in the form

$$g_{\mu\nu}\nabla_\lambda T^{\lambda\nu} = -D_\nu\left(\frac{\delta L_M}{\delta \phi_A} F^{B;\nu}_{A;\mu}\, \phi_B(x)\right) - \frac{\delta L_M}{\delta \phi_A} D_\mu\, \phi_A(x). \tag{51}$$

Here $T^{\lambda\nu} = -2\delta L_M/\delta g_{\lambda\nu}$ is the energy-momentum tensor density of matter in the Riemannian space; $\nabla_\lambda$ is a covariant derivative in this space with the metric $g_{\lambda\nu}$. From identity (51) it follows that if equations of matter motion (50) are fulfilled, then the following equation takes place

$$\nabla_\lambda T^{\lambda\nu} = 0. \tag{52}$$

In that case if the number of equations (50) for matter is equal to four, one can use their equivalent equations (52) instead of them. Since in what follows we will deal only with such equations for matter let us always use the equations for matter in form (52). Hence the complete system of equations for matter and gravitational field will have the form

$$\frac{\delta L}{\delta \tilde{g}^{\mu\nu}} = 0, \tag{53}$$

$$\nabla_\lambda T^{\lambda\nu} = 0. \tag{54}$$

The matter will be described by the velocity $\vec{v}$, matter density $\rho$ and pressure $p$. The gravitational field is determined by ten components of the tensor $\phi^{\mu\nu}$. Hence we have fifteen unknown quantities. In order to determine them we are to add the equation of the matter state to fourteen equations (53)–(54). If one takes into consideration the relations

$$\frac{\delta L_g}{\delta \tilde{g}^{\mu\nu}} = -\frac{1}{16\,\pi} R_{\mu\nu} + \frac{m^2}{32\,\pi}(g_{\mu\nu} - \gamma_{\mu\nu}), \tag{55}$$

$$\frac{\delta L_M}{\delta \tilde{g}^{\mu\nu}} = \frac{1}{2\sqrt{-g}}\left(T_{\mu\nu} - \frac{1}{2} g_{\mu\nu}\, T\right), \tag{56}$$

then the system of equations (53), (54) may be given in the form

$$(R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R) + \frac{m^2}{2}[g^{\mu\nu} + (g^{\mu\alpha} g^{\nu\beta} - \frac{1}{2} g^{\mu\nu} g^{\alpha\beta})\, \gamma_{\alpha\beta}] = \frac{8\,\pi}{\sqrt{-g}} T^{\mu\nu}, \tag{57}$$

$$\nabla_\lambda T^{\lambda\nu} = 0. \tag{58}$$

In virtue of the Bianchi identity

$$\nabla_\mu(R^{\mu\nu} - \frac{1}{2} g^{\mu\nu}\, R) = 0$$

from equations (57) we have

$$m^2\sqrt{-g}(g^{\mu\alpha}g^{\nu\beta} - \frac{1}{2}\,g^{\mu\nu}g^{\alpha\beta})\nabla_\mu\,\gamma_{\alpha\beta} = 16\,\pi\,\nabla_\mu\,T^{\mu\nu}. \tag{59}$$

Taking into consideration the expression

$$\nabla_\mu\,\gamma_{\alpha\beta} = -G^\sigma_{\mu\alpha}\,\gamma_{\sigma\beta} - G^\sigma_{\mu\beta}\,\gamma_{\sigma\alpha}, \tag{60}$$

where $G^\sigma_{\mu\alpha}$ is defined by formula (26) we find

$$(g^{\mu\alpha}g^{\nu\beta} - \frac{1}{2}\,g^{\mu\nu}g^{\alpha\beta})\,\nabla_\mu\,\gamma_{\alpha\beta} = \gamma_{\mu\lambda}g^{\mu\nu}(D_\sigma\,g^{\sigma\lambda} + G^\beta_{\alpha\beta}\,g^{\alpha\lambda}). \tag{61}$$

However since

$$\sqrt{-g}(D_\sigma\,g^{\sigma\lambda} + G^\beta_{\alpha\beta}\,g^{\alpha\lambda}) = D_\sigma\,\tilde{g}^{\lambda\sigma}, \tag{62}$$

expression (61) takes the form

$$\sqrt{-g}(g^{\mu\alpha}g^{\nu\beta} - \frac{1}{2}\,g^{\mu\nu}g^{\alpha\beta})\,\nabla_\mu\,\gamma_{\alpha\beta} = \gamma_{\mu\lambda}\,g^{\mu\nu}\,D_\sigma\,\tilde{g}^{\lambda\sigma}. \tag{63}$$

Using (63) one can rewrite expression (59) in the form

$$m^2\gamma_{\mu\lambda}\,g^{\mu\nu}D_\sigma\tilde{g}^{\lambda\sigma} = 16\pi\,\nabla_\mu\,T^{\mu\nu}.$$

This expression can be rewritten in the form

$$m^2 D_\sigma\tilde{g}^{\lambda\sigma} = 16\,\pi\,\gamma^{\lambda\nu}\nabla_\mu\,T^\mu_\nu. \tag{64}$$

With the help of this relation equation (58) may be replaced by the equation

$$D_\sigma\tilde{g}^{\nu\sigma} = 0. \tag{65}$$

Therefore the system of equations (57) and (58) reduces to a system of gravitational equations in the form

$$(R^{\mu\nu} - \frac{1}{2}\,g^{\mu\nu}R) + \frac{m^2}{2}[g^{\mu\nu} + (g^{\mu\alpha}g^{\nu\beta} - \frac{1}{2}\,g^{\mu\nu}\,g^{\alpha\beta})\gamma_{\alpha\beta}] = \frac{8\,\pi}{\sqrt{-g}}\,T^{\mu\nu}, \tag{66}$$

$$D_\mu\,\tilde{g}^{\mu\nu} = 0. \tag{67}$$

These equations are form-invariant w.r.t. the Lorentz transformations, i.e. in any inertial (Galilean) coordinate system the phenomena are described with the same equations.

A concrete inertial Galilean coordinate system is singled out by the statement of the physical problem itself (initial and boundary conditions). The description of the given physical problem in different inertial (Galilean) coordinate systems, is of course different, however this does not contradict the relativity principle. If we introduce the tensor

$$N^{\mu\nu} = R^{\mu\nu} - \frac{m^2}{2}[g^{\mu\nu} - g^{\mu\alpha}g^{\nu\beta}\gamma_{\alpha\beta}],\;\; N = N^{\mu\nu}g_{\mu\nu},$$

then the system of equations (66) and (67) may be presented in the form

$$N^{\mu\nu} - \frac{1}{2} g^{\mu\nu} N = \frac{8\pi}{\sqrt{-g}} T^{\mu\nu}, \tag{66a}$$

$$D_\mu \tilde{g}^{\mu\nu} = 0. \tag{67a}$$

It can also be presented in the form

$$N^{\mu\nu} = \frac{8\pi}{\sqrt{-g}} (T^{\mu\nu} - \frac{1}{2} g^{\mu\nu} T), \tag{68}$$

$$D_\mu \tilde{g}^{\mu\nu} = 0, \tag{69}$$

or

$$N_{\mu\nu} = \frac{8\pi}{\sqrt{-g}} (T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T), \tag{68a}$$

$$D_\mu \tilde{g}^{\mu\nu} = 0. \tag{69a}$$

It should particularly be stressed that the Minkowski metric tensor enters both system (68) and system (69).

The coordinate transformations leaving the Minkowski metric form-invariant connect the physically equivalent reference frames. The simplest among them are inertial ones. Therefore possible gauge transformations satisfying the Killing conditions

$$D_\mu \varepsilon_\nu + D_\nu \varepsilon_\mu = 0,$$

do not take us out the class of the physically equivalent reference frames.

If we admit the possibility of the experimental measurement of the characteristics of the Riemannian space and of matter motion to arbitrary high accuracy, then proceeding from equations (68a) and (69a) we can define the Minkowski metric and find the Galilean (inertial) coordinate systems. Hence, basically the Minkowski space becomes observable.

The existence of the Minkowski space is reflected in the conservation laws; therefore their verification in physical phenomena is, at the same time, the verification of the structure of space-time.

The system of gravitational equations can be given another equivalent form:

$$\gamma^{\alpha\beta} D_\alpha D_\beta \tilde{\phi}^{\mu\nu} + m^2 \tilde{\phi}^{\mu\nu} = 16\pi t^{\mu\nu}, \tag{70}$$

$$D_\mu \tilde{\phi}^{\mu\nu} = 0, \tag{71}$$

where $t^{\mu\nu} = -2\delta L/\delta\gamma_{\mu\nu}$ is the conserved energy-momentum tensor density of matter and gravitational field in the Minkowski space. The form of these equations is similar to the equations of electrodynamics with a photon mass $\mu$ in the absence of gravitation:

$$\gamma^{\alpha\beta} D_\alpha D_\beta A^\nu + \mu^2 A^\nu = 4\pi j^\nu, \tag{72}$$

$$D_\nu A^\nu = 0. \tag{73}$$

In electrodynamics the source of the vector field $A^\nu$ is the conserved electromagnetic current $j^\nu$ produced by charged bodies, whereas in RTG the source of the tensor field is the conserved total energy-momentum tensor of matter and gravitational field. Therefore the gravitational equations will be nonlinear even for the proper gravitational field.

It should be particularly stressed that in addition to the known cosmological term in equations (66) there appeared another one containing the Minkowski metric $\gamma_{\mu\nu}$, with the two terms having the common constant coinciding with the graviton mass, that is therefore extremely small. The second mass term in equations (66) containing the metric $\gamma_{\mu\nu}$ is responsible for the production of repulsion forces, rather large in strong gravitational fields. This circumstance changes the nature of collapse and evolution of the Universe.

As we saw before, the presence of the graviton rest mass has a paramount importance for the construction of the field theory of gravitational field. Just due to this, it follows from the theory that the homogeneous and isotropic Universe can be but just flat.

In conclusion to this Section it is worth to note, that the theory of the tensor gravitational field in the Minkowski space, that introduces effective Riemannian space-time, is possible only under condition, that the gravitational field does possess the rest mass.

## 5. The Causality Principle in RTG

The RTG, like the theories of other physical fields, has been constructed within the frames of the Special Relativity Theory (SRT). According to the latter, any motion of a point-like test body takes place inside the light causality cone of the Minkowski space. Hence, noninertial reference frames realized by test bodies should also be inside the causality cone of the pseudo-Euclidean space-time. This defines the whole class of possible noninertial reference frames. The 3-dimensional inertia and gravitation forces acting upon a material point will be locally equal if the light cone of the effective Riemannian space does not escape outside the causality light cone in the Minkowski space. Just only in this case the 3-dimensional force of a gravitational field acting upon a test body can be compensated locally by changing over into an admissible noninertial reference frame connected with this body.

If the light cone of the effective Riemannian space escaped the light causality cone of the Minkowski space this would mean that for such "gravitational field" there is no admissible noninertial reference frame in which this "field of the force" acting upon a material point could be compensated. Putting it in other words, the local compensation of a 3-dimensional gravitation force by the inertia force is possible only if gravitational field as a physical one acting upon particles, does not take their world lines beyond the causality cone of the pseudo-Euclidean space-time. This condition should be treated as the causality principle, allowing us to select the solutions to the system of equations (66) and (67), having a physical sense and corresponding to gravitational fields.

The causality principle is not fulfilled automatically because the gravitational interaction enters the coefficients of the second-order derivatives in the field equations, i.e. varies the initial pseudo-Euclidean space-time geometry. This is an inherent feature of gravitational field only. The interaction of all other known physical fields typically does

not effect the second-order derivatives in the field equations and therefore does not vary the initial pseudo-Euclidean space-time geometry.

Now let us formulate analytically the causality principle in the RTG. Since in this theory the motion of matter under the action of gravitational field in the pseudo-Euclidean space-time is equivalent to that in the relevant effective Riemannian one then for causally related events, i.e. for the world lines of particles and light, we should, on the one hand, have the condition

$$d\,s^2 = g_{\mu\nu}\,dx^\mu\,dx^\nu \;\geq\; 0, \tag{74}$$

and, on the other hand, for such events the inequality

$$d\sigma^2 = \gamma_{\mu\nu}\,dx^\mu\,dx^\nu \;\geq\; 0 \tag{75}$$

should necessarily hold true. For the chosen reference frame realized by physical bodies the condition

$$\gamma_{oo} \;>\; 0 \tag{76}$$

must be fulfilled. Let us single out in expression (75) the time– and space–like terms:

$$d\,\sigma^2 = \left(\sqrt{\gamma_{oo}}\,dt + \frac{\gamma_{oi}\,dx^i}{\sqrt{\gamma_{oo}}}\right)^2 - s_{ik}\,dx^i\,dx^k. \tag{77}$$

Here the Latin indices $i$, $k$ range the values 1, 2, 3,

$$s_{ik} = -\,\gamma_{ik} + \frac{\gamma_{oi}\gamma_{ok}}{\gamma_{oo}}, \tag{78}$$

$s_{ik}$ is a metric three-dimensional space tensor in the four-dimensional pseudo-Euclidean space-time. The spatial distance squared is determined by the expression

$$d\,l^2 = s_{ik}\,dx^i\,dx^k. \tag{79}$$

Now present the velocity $v^i = dx^i/dt$ as $v^i = ve^i$, where $v$ is the velocity value, $e^i$ is an arbitrary unity vector in the three-dimensional space

$$s_{ik}e^i\,e^k = 1. \tag{80}$$

In the absence of gravitational field the velocity of light in the chosen coordinate system is easily found from expression (77) by putting this expression equal to zero:

$$\left(\sqrt{\gamma_{oo}}\,dt + \frac{\gamma_{oi}\,dx^i}{\sqrt{\gamma_{oo}}}\right)^2 = s_{ik}\,dx^i\,dx^k.$$

Whereof we find that

$$v = \sqrt{\gamma_{oo}}\Big/\left(1 - \frac{\gamma_{oi}\,e^i}{\sqrt{\gamma_{oo}}}\right). \tag{81}$$

Hence, the arbitrary four-dimensional isotropic vector $u^\nu$ in Minkowski space is

$$u^\nu = (1,\, v\, e^i). \tag{82}$$

For conditions (74) and (75) to be fulfilled simultaneously it is necessary and sufficient that for any isotropic vector

$$\gamma_{\mu\nu}\, u^\mu\, u^\nu = 0 \tag{83}$$

the causality condition

$$g_{\mu\nu}\, u^\mu\, u^\nu \;\leq\; 0 \tag{84}$$

should be fulfilled. This condition just means that the light cone of the effective Riemannian space does not escape the light causality cone of the pseudo-Euclidean space-time. The causality conditions can be put down in the following form:

$$g_{\mu\nu}\, v^\mu\, v^\nu = 0, \tag{83a}$$

$$\gamma_{\mu\nu} v^\mu\, v^\nu \;\geq\; 0. \tag{84a}$$

In the GRT, of physical sense are the solutions to the Hilbert-Einstein equations satisfying at each point of space-time the inequality

$$g \quad < \quad 0,$$

and also the requirement called the energy dominance condition formulated as follows. For any time–like vector $K_\nu$ the inequality

$$T^{\mu\nu}\, K_\mu\, K_\nu \;\geq\; 0$$

should hold true and for this vector the quantity

$$T^{\mu\nu}\, K_\nu$$

should form a nonspace-like vector.

In our theory, only such solutions to equations (68a) and (69a) have physical sense which, alongside with these requirements, also satisfy causality conditions (83a) and (84a). Proceeding from equations (68a), the last condition can be written as

$$R_{\mu\nu}\, K^\mu K^\nu \;\leq\; \frac{8\,\pi}{\sqrt{-g}}(T_{\mu\nu} - \frac{1}{2}\, g_{\mu\nu}\, T)\, K^\mu K^\nu + \frac{m^2}{2}\, g_{\mu\nu}\, K^\mu K^\nu. \tag{85}$$

If we take the density of energy-momentum tensor for matter in the form:

$$T_{\mu\nu} = \sqrt{-g}[(\rho + p)U_\mu U_\nu - p g_{\mu\nu}],$$

then from (68a) it is possible to derive the following relation between the interval of Minkowski space $d\sigma$ and effective Riemannian space $ds$:

$$\frac{m^2}{2} d\sigma^2 = ds^2[4\pi(\rho + 3p) + \frac{m^2}{2} - R_{\mu\nu} U^\mu U^\nu],$$

where $U^\mu = dx^\mu/ds$.

Due to causality principle there is an inequality:

$$R_{\mu\nu}U^\mu U^\nu < 4\pi(\rho + 3p) + \frac{m^2}{2},$$

which is a partial case of inequality (85) or

$$\sqrt{-g}R_{\mu\nu}\, v^\mu v^\nu \;\leq\; 8\,\pi T_{\mu\nu}v^\mu v^\nu. \tag{85a}$$

In 1918 A.Einstein formulated the equivalence principle in the following way: *"Inertia and gravity are identical; from this as well as from the results of Special Relativity Theory it inevitably follows, that symmetric <<fundamental tensor>> $g_{\mu\nu}$ determines the metric properties of space, inertial motion of bodies in it and also the action of gravitation."* Identification in the GRT the gravitational field and metric tensor $g_{\mu\nu}$ of the Riemannian space allows one through the choice of the coordinate system to make all the components of the Christoffel symbol equal to zero on all the points of an arbitrary line. However in this case the gravitational field is not excluded in GRT through the coordinate system choice, since the motion of two close material points will not be free because of the presence of the curvature tensor, that, owing to the tensor properties can never be turned into zero by choosing the coordinate system.

In RTG the gravitational field is a physical field in Faraday-Maxwell spirit, therefore the gravitational force is described by the four-vector, and consequently only by conditions (83) and (84) fulfilled one can balance with inertia forces the three-dimensional part of the gravitation force by choosing the relevant coordinate system. The continence of the equivalence principle in RTG changes drastically and reduces to conditions (83) and (84) that provide a possibility to choose such a coordinate system, where the gravitational force will be balanced by the inertia force. The motion of the material point in the gravitational field, no matter what coordinate system we have, can never be free. The last statement is evident especially if we write the geodesic line equation in the form [1]:

$$\frac{DU^\nu}{d\sigma} = -G^\rho_{\alpha\beta}U^\alpha U^\beta(\delta^\nu_\rho - U^\nu U_\rho).$$

Here $$d\sigma^2 = \gamma_{\mu\nu}dx^\mu dx^\nu, \qquad U^\nu = \frac{dx^\nu}{d\sigma}.$$

A free motion in the Minkowski space is described by the equation:

$$\frac{DU^\nu}{d\sigma} = \frac{dU^\nu}{d\sigma} + \gamma^\nu_{\mu\lambda}U^\mu U^\lambda = 0,$$

$\gamma^\nu_{\mu\lambda}$ are Christoffel symbols of the Minkowski space. We see that the motion along a geodesic line of the Riemannian space is the motion of a test body under the action of a force $F^\nu$:

$$F^\nu = -G^\rho_{\alpha\beta}U^\alpha U^\beta(\delta^\nu_\rho - U^\nu U_\rho),$$

and this force has a four-vector nature. We have here the same situation as in the case for other known physical forces.

In SRT there is a principal difference between the inertia and physical forces (electromagnetic, nuclear, etc.). The inertia forces may always be turned into zero by a simple choice of the reference frame, while physical forces cannot be turned into zero by any choice of the reference frame, because they are of vector nature in the Minkowski space. In GRT gravitational forces are locally identical to the inertia forces, therefore they greatly differ from any other physical forces. In RTG contrary to GRT, gravitational forces, as all other physical forces, have one and the same, vector nature in four-dimensional space-time.

Einstein recognized a profound reason for the equality of the inertial and gravitational masses from the local identity of inertia and gravitation. However, in our opinion, as it is seen from equations (70) the reason for this equality is in the fact that the source of the gravitational field is the conserved total density of the tensor of matter and gravitational field. That is why the equality of the inertial and gravitational masses does not demand of the local identity of the gravitation and inertia forces. However geometrization, determined by the geometrization principle, turns out to be necessary.

## 6. Some Physical Conclusions of RTG

The RTG system of equations (66) and (67) leads to completely different, qualitatively new physical conclusions than the GRT. For example, the picture of collapse changes completely. It turns out that during collapse of a spherically symmetric body of an arbitrary mass the contraction process in the region close to the Schwarzschild sphere ceases to be replaced by further extension. This means that in addition to contracting objects in the Nature there should exist extending ones. Therefore, according to the RTG, the possibility for "black holes", i.e. objects having no material boundaries and "cut" off out of the external world, to exist in the Nature is ruled out completely.

Another important physical conclusion concerns the evolution of the homogeneous and isotropic Universe. It follows from equations (66) and (67) as well as from causality conditions (83) and (84) that homogeneous and isotropic Universe has been existing for an infinite period of time and its three-dimensional geometry is Euclidean. This Universe evolves cyclewise from the maximal finite density till the minimal one, then till the maximal again (in the absence of dissipation), etc. Our theory predicts the existence in the Universe of a large "hidden" mass of matter because, according to equations (66) and (67) the total matter density is presently equal to

$$\rho = \rho_c + \frac{1}{16\,\pi\,G}\left(\frac{m\,c^2}{\hbar}\right)^2. \tag{86}$$

Whereof it is seen that the matter density even for a sufficiently small graviton mass is close to the critical density $\rho_c$ determined by the Hubble constant $H$ and equal to

$$\rho_c = \frac{3\,H^2}{8\,\pi\,G}. \tag{87}$$

The RTG explains all the known gravitational experiments in the Solar system and, as we saw before, makes it possible to introduce for gravitational field the notion of energy-momentum tensor, as it can be done for other physical fields. As due to geometrization the conserved total tensor density of matter and gravitational field is the source of the gravitational field, from equations (70) directly follows, that inertial mass of a static body is exactly equal to its active gravitational mass. Here this equality does not suggest the local identification of gravitation and inertia.

From the other side a motion of a neutral test body in the given gravitational field does not depend on the body mass, as it is along the geodesic line of the effective Riemannian space. So, it should be concluded that the passive gravitational mass of the test body is also equal to its inertial mass and, therefore, the passive gravitational mass of the test body is equal to its active gravitational mass. The energy-momentum tensor density $-2\delta L_g/\delta g_{\mu\nu}$ of gravitational field in the Riemannian space outside matter is, according to equations (66), equal to zero. However, this does not mean the absence of gravitational radiation because the gravitational wave carrying energy moves along the effective gravitational background.

As to the gravitational radiation of massive gravitons it was expounded in article [4], in which the author showed that the calculations made earlier were based on the general expression for intensity obtained incorrectly. Those deducing this expression did not take into account the important fact that gravitons actually propagated in the effective Riemannian space rather than in the Minkowski one. Consideration of this fact led the author to the statement that the intensity of the gravitational radiation of massive gravitons was a positive-definite value. Its expression is presented in paper [4]. The system of gravitational equations (66) and (67) opens up new possibilities for both specific and comprehensive studies of these or those gravitational phenomena.

In conclusion one should make some important remarks. Can one set the graviton mass equal to zero? Since in our theory the graviton mass eliminates the degeneration in the gauge group, putting it equal to zero directly in equations (66) and (67) is incorrect. In our theory it must not be equal to zero. The system of gravitational equations (66) and (67) is hyperbolic, with the causality principle ensuring the existence in the whole space of a space-like surface crossed by each nonspace-like curve in the Riemannian space only once. Putting it in other words, there exists a global Cauchy surface on which the initial physical conditions are given for any specific problem.

Penrose and Hawking [5] proved the theorems on existence of singularity in the GRT under certain general conditions. In virtue of causality conditions (85a) and proceeding from equations (68a) outside of matter the inequality

$$R_{\mu\nu}\, v^{\mu}\, v^{\nu} \leq 0 \tag{88}$$

holds true for the isotropic vectors in Riemannian space, therefore the conditions of the theorems on the existence of singularity are not fulfilled in the RTG and, hence, their statements are unacceptable for the RTG. In our theory those events which are space-like in the absence of gravitational field can never become under the action of gravitational

field time-like ones. In virtue of the causality principle the effective Riemannian space–time in the RTG will possess the isotropic and time-like geodesic completeness.

The aforementioned reasonings allow us to make the following general conclusion. If we accept in virtue of the universal nature of gravitation that the source of gravitational field in the Minkowski space is the conserving energy-momentum tensor of matter and massive gravitational field, then the field will manifest itself as a second-rank tensor one. Similarly to electrodynamics the field equations can naturally be written as

$$\Box\phi^{\mu\nu} + m^2\,\phi^{\mu\nu} = \lambda\, t^{\mu\nu}, \qquad \partial_\mu\,\phi^{\mu\nu} = 0.$$

But this system of equations follows from the Lagrangian formalism only in the case if the matter–gravitational field interaction takes place according to the geometrization principle, that just reduces the action of this field to the effective space-time geometry.

Hence, with the conserving matter energy-momentum tensor accepted as the universal source of gravitational field, this necessarily leads to the effective Riemannian geometry.

As the field gravitation theory requires that the graviton mass be introduced, and, according to the theory structure it is close to electrodynamics, then it may well be that the photon rest mass is not equal to zero either.

## 7. Mach Principle

When formulating the laws of mechanics Newton introduced the notion of absolute space, that always remains the same and motionless. And right with respect to this space he defined the acceleration of a body. This acceleration was of an absolute character. The introduction of such an abstract notion, as absolute space, turned out to be rather fruitful. In particular, the notion of inertial reference frames in the whole space, relativity principle for mechanical processes sprung out from this idea. Besides it favoured the formation of the idea of physically distinguished states of motion. In connection with this in 1923 Einstein wrote: *"The coordinate systems found in such states of motion differ in the fact that the laws of Nature formulated in these coordinates acquire the most simple form"* and then he went on: *"...in accordance with the classical mechanics there exists the relativity of the velocity, but not the relativity of the acceleration."* Since then the idea of inertial reference frames was established in the theory. In these inertial reference frames the material points not subjected to the action of the forces do not undergo the acceleration and are in their state of rest or of uniform and straightforward motion. However the absolute Newton space or the inertial reference frames were in fact introduced a priori, without taking into consideration the distribution of matter in the Universe.

Mach was brave enough to seriously criticize the basic postulates of Newton mechanics. As he himself recognized later, it was very difficult for him to publish his ideas. Though Mach has not constructed a physical theory, free from the drawbacks pointed out himself, nevertheless he has greatly influenced the development of the physical theory. He attracted the attention of scientists to the analysis of the basic physical notions.

Let us quote some statements by Mach [6], which later on were called Mach principle. *"No one can say anything about absolute space and absolute motion, it is only something*

*imagined, not observable in experiments."* Then: *"Instead of relating a moving body to space (i.e. to some coordinate system) we should rather consider its relation to the bodies of the world, which are the only ones allowing us to determine the coordinate system... even in the simplest case, when we supposedly consider the interaction of only two masses we cannot neglect the rest part of the world.... If a body rotates w.r.t. the sky with motionless stars, then there arise centrifugal forces, however if it rotates w.r.t. some other body, rather than w.r.t. the stars, no centrifugal forces arise. I have nothing against calling the first rotation an absolute one, if we always bear in mind that it does not imply anything else but the rotation w.r.t. the motionless stars sky."*

This is why Mach wrote: *" ...there is not any necessity to relate the inertia law with any special absolute space. The most natural approach of a true scientist is as follows: first we are to consider the inertia law as a rather approximate one, to relate it spatially with the motionless starry sky,... and then we are to expect some corrections or development of our knowledge on the basis of our further experience. Recently Lange has published an article saying how one might introduce a new coordinate system using his principles for the case when usual rough reference to the motionless starry sky turned out to be inapplicable due to more precise astronomic observations. There is no disagreement between me and Lange in the theoretical formal value of Lange's conclusions, i.e., the motionless starry sky is in fact the only applicable reference system at present, as well as there is no disagreement concerning the method of defining a new reference system through step-by-step corrections."* Then Mach quotes S.Neumann: *"Since all the motions are to be referred to the alpha system (inertial system), it obviously represents some indirect relation among all the processes going on in the Universe, and consequently, one may say, it contains so a mysterious as a complicate universal law."* Mach makes a remark in this relation: *"I think, anyone will agree with this."*

From Mach's remarks it is clear that since we are speaking about the inertia law, according to which, if we follow Newton *"...any separately taken body, since it is left to itself, keeps its state of rest or of uniform straightforward motion..."* then quite naturally there arises the question about inertial reference frames and their connection with the distribution of matter. Mach and his contemporaries understood quite clearly, that such connection should exist in Nature. This is the meaning that will be embedded into the notion of "Mach principle".

Mach wrote: *"Though I think that astronomic observations will entail first only rather insignificant corrections, I still admit, that the inertia law, in its simple form, given by Newton, has a limited and temporary significance for us."* As we will see in what follows, Mach was mistaken here. Mach did not formulate his idea mathematically, that is why numerous authors put their own ideas into Mach's principle. Here we will try to preserve the meaning, that Mach himself implied.

Poincaré, and later Einstein generalized the relativity principle for all physical phenomena. In Poincaré's formulation [2] it reads: *"...the relativity principle, in agreement with which all the laws of physical phenomena should be identical for motionless observer and for the one, moving uniformly and straightforwardly, so that we have no means and we cannot have those means to determine, whether we perform this motion or not."* The

application of this principle to the electromagnetic phenomena brought Poincaré, and then Minkowski to the discovery of the pseudo–Euclidean geometry of space-time and thus gave an even more support to the hypothesis on the existence of inertial reference frames in the whole space. Such reference frames are physically preferred, and therefore the acceleration w.r.t. them has an absolute meaning.

In the General Relativity Theory there are no inertial systems in the whole space. On this point Einstein wrote in 1929: *"The starting point for the theory is the statement, that there is no physically distinguished state of motion, i.e. neither the velocity, nor acceleration has absolute meaning."*

Mach principle in his formulation was not used in the GRT. However, it should be noted, that the ideas of inertial reference frames in the whole space are quite enough grounded experimentally, since, for example, passing from the coordinate system, connected with the Earth, to the coordinate system, connected with the Sun, and then to the Metagalaxy, we approach the inertial reference frame with higher and higher precision. Therefore there are no serious grounds to give up such an important notion, as the inertial reference frame. On the other hand the existence of such fundamental laws as energy-momentum and angular momentum conservation laws also leads us by necessity to the fact of the existence of the inertial reference frames in the whole space. Pseudo–Euclidean geometry of space-time reflects the general dynamic properties of matter, at the same time it introduces the inertial reference frames. Though the pseudo–Euclidean geometry of space-time arose when studying the matter, and hence it is indispensable from it, nevertheless one may formally speak about the Minkowski space in the absence of matter. However as before in Newtonian mechanics, the Special Relativity Theory does not answer the question in what way the inertial reference frames are connected with the distribution of matter in the Universe.

The discovery of the pseudo–Euclidean geometry of space-time allowed one to consider in an unified way not only the inertial, but accelerated reference frames as well. A great difference sprung between inertia forces and forces caused by physical fields. Its essence lies in the fact, that the inertia forces may always be made equal to zero by choosing proper reference frame, while the forces, caused by the physical fields, cannot in principle be turned into zero through a choice of the reference frame, for they have a vector nature in the four-dimensional space-time. Since in the RTG the gravitational field is a physical field in Faraday–Maxwell spirit, the forces caused by this field, cannot be turned into zero by choosing the reference frame.

Another situation takes place in the General Theory of Relativity. Here gravitational forces have no vector nature in the four-dimensional space–time, and hence they can locally be turned into zero by choosing proper reference frame. Owing to the presence of the rest mass of the gravitational field the basic equations of the RTG (66) and (67) contain alongside with the Riemannian metric a metric tensor of the Minkowski space, but it means that the metric of this space can, in principle, be expressed via geometrical characteristics of the effective Riemannian space, as well as via the quantities characterizing the distribution of matter in the Universe. It can easily be realized, if we pass from

contravariant quantities to covariant ones in (66). Thus we obtain

$$\frac{m^2}{2}\gamma_{\mu\nu}(x) = \frac{8\pi}{\sqrt{-g}}(T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T) - R_{\mu\nu} + \frac{m^2}{2}g_{\mu\nu}. \tag{89}$$

As we see, the r.h.s. of the equation contains only geometric characteristics of the effective Riemannian space and the quantities that define the distribution of matter in this space.

Experimental studies of the motion of particles and light in the Riemannian space may, in principle, allow us to find a metric tensor of the Minkowski space, and consequently, to construct an inertial reference frame. Hence, the RTG constructed in the framework of the Special Relativity Theory, makes it possible to formulate mathematically the Mach principle. As is seen, the special relativity principle has a global meaning independent of the form of matter.

As to the gravitational field, its requirements are expressed through the form–invariance condition of equations (66) and (67) w.r.t. the Lorentz group. Lorentz form-invariance of the physical equations remains to be the basic physical principle when constructing the theory, for namely this principle gives us a possibility to introduce universal characteristics for all forms of matter.

In 1950 A. Einstein wrote: *"... shouldn't we finally try to conserve the notion of the inertial system and give up all attempts to explain the fundamental property of gravitational phenomena, that manifests itself in Newton's system as equality of the inertial and gravitational masses?"* In Section 6 we have established that the equality of the inertial and gravitational masses is a consequence of equations (70), where the conserved total density of the energy-momentum tensor of matter and gravitational field is, owing to geometrization, the source of the gravitational field, and this equality does not exclude, to the least extent, the notion of the inertial system. This notion remains in full in the RTG and it reflects general dynamic properties of matter–energy-momentum and angular momentum conservation laws. Hence the equivalence of the inertial and gravitational masses does not make one reject the notion of the inertial system. Contrary to our conclusion A. Einstein answered the question in the following way: *"The one, who believes in the cognoscibility of Nature, should say — no."*

Mach's ideas greatly influenced Einstein's views of gravitation when constructing the General Relativity Theory. In one of his articles Einstein wrote: *"Mach principle: G-field is completely determined by the masses of the bodies."* But it turns out that neither this postulate is fulfilled in the GRT, since there exist solutions in the absence of matter. An attempt to overcome this difficulty by introducing a $\lambda$ term was not a success. The desirable result was not achieved. It happened so that even the equations with the $\lambda$ term will have solutions, different from zero in the absence of matter. As it becomes clear Einstein embedded quite a different sense into the notion "Mach principle". However even this interpretation did not allow Mach's principle to find its place in the GRT.

Does the Mach principle take place in Einstein's formulation in RTG? Contrary to the GRT in this theory one has space-like surfaces over the whole space — global Cauchy surfaces (which is due to causality principle). And if one of these surfaces lacks matter, then on the basis of the energy-dominance requirement, imposed on the matter tensor,

the matter will always be absent [5]. Since matter does exist in Nature, then it follows, that the system of gravitational equations homogeneous in the whole space, does not have solutions, realizable in Nature. On other words all the solutions of this system have no physical sense under the present development of the Universe. Such rejection of the solutions to the system of homogeneous gravitational equations became possible not only due to the equations, but also to the character of the real Universe.

In principle, the equations of the theory do not reject the Universes, built of the gravitational field without matter. They are rejected by the development of matter itself. Why our Universe turned out to be with matter — the theory does not give an answer to this question. Only solutions of the system of inhomogeneous gravitational equations have physical sense, when in some part of the space or in the whole space there is matter. It means, that the gravitational field and the effective Riemannian space in the real Universe could not have been produced without matter generating them. We see that in Einstein's formulation also Mach principle is realized in the Relativistic Theory of Gravitation.

However there is quite a noticeable difference in understanding G-field in our theory and in the GRT. By the G-field Einstein understood Riemannian metric, while in our understanding the gravitational field is a physical field. Such field enters the Riemannian metric alongside with the flat metric, hence in the absence of matter and gravitational field the metric does not vanish, and remains a metric of the Minkowski space. In literature one can find other formulations of Mach's principle, that differ in their meaning from the ideas of Mach and Einstein. But since in our opinion they have not been formulated sufficiently definitely, we do not consider them. Since the gravitational forces in the RTG are due to the physical field of Faraday-Maxwell type, then we cannot speak about any unified essence of the inertia and gravitation forces.

Sometimes one sees the essence of Mach principle in the fact, that the inertia forces are as if determined by the interaction with the Universe matter. From the field point of view, such principle cannot take place in Nature. The thing is that though the inertial reference frames, as was seen above, are connected with the distribution of matter in the Universe, inertia forces are not the results of the interaction with the Universe matter, for any interaction of matter can proceed only via physical fields, but this means that the forces caused by these fields, cannot be made zero just by choosing the reference frame, which is due to their vector nature. Hence the inertia forces are directly determined not by the physical fields, but by a strictly determined structure of geometry and the choice of the reference frame.

The pseudo–Euclidean geometry of space-time, which reflects the dynamic properties common for all forms of matter, from one side has confirmed the hypothesis on the existence of the inertial reference frames, and from the other side it has shown, that the inertia forces, arising under a corresponding choice of the reference frame, are expressed via Christoffel symbols of the Minkowski space. Therefore they do not depend on the nature of the body. All this became evident when it was shown that the SRT was applicable not only in the inertial reference frames, but in non-inertial (accelerated) frames as well. This made possible to give in article [7] a more general formulation of the relativity principle: *"No matter what the physical reference frame we choose (inertial or non-inertial) one*

*can always point out an infinite set of other reference frames, such that all the physical phenomena would proceed identically to the initial reference frame, so we do not have and we cannot have any experimental possibilities to distinguish, in what particular reference frame out of all this infinite set we find ourselves."*

In the RTG there is a great difference between inertial and gravitational forces: the farther we are from the bodies the weaker the gravitational field is, while the inertia forces may be arbitrarily large depending on the choice of the reference frame. And only in the inertial reference frame they are equal to zero. Therefore it would be wrong to say, that we cannot separate the inertia forces from the gravitation ones. In our everyday life their difference is almost evident.

The construction of the RTG allowed one to establish the connection between the inertial reference frame and distribution of matter in the Universe, thus making deeper our understanding of the nature of the inertia forces and their difference from material forces. In our theory the inertia forces are to play the same role, as they play in any other field theories.

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