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TWO-DIMENSIONAL INTEGRABLE MAPPINGS AND EXPLICIT FORM OF EQUATIONS OF (1+2)-DIMENSIONAL HIERARCHIES OF INTEGRABLE SYSTEMS

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Abstract

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The equations of $(1+2)$ integrable systems belonging to Darboux–Toda, Heisenberg and Lotky–Volterra hierarchies which are invariant with respect to discrete transformations of the corresponding integrable mappings are represented in the explicit form.

Аннотация

Дерягин В.Б. и др. Двумерные интегрируемые подстановки и явная форма уравнений (2+1)-мерных иерархий интегрируемых систем: Препринт ИФВЭ 95-26. – Протвино, 1995. – 9 с., библиогр.: 7.

В статье представлены в явной форме уравнения (2+1)-мерных интегрируемых систем, принадлежащих к иерархиям Дарбу-Тоды, Гайзенберга и Лотки-Вольтерра и являющиеся инвариантными относительно дискретного преобразования соответствующей интегрируемой подстановки.

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1. Introduction

In this paper we will investigate $(1+2)$-dimensional integrable systems [1,2] in terms of their groups properties of integrable mappings [3].

This program has been proposed in [4,5] and consists in the following construction. With each local invertible substitution of the form

$$\overleftarrow{u} \equiv \phi(u) = \phi(u, u', u'', ...) \tag{1.1}$$

(where u is the s-dimensional vector function , $u'^{\cdots}$ its derivatives of an arbitrary order with respect to independent arguments) it is possible to connect Frechet derivative [6]

$$\phi'(u) = \frac{\partial \phi}{\partial u} + \frac{\partial \phi}{\partial u'} D + \frac{\partial \phi}{\partial u''} D^2 + ... \tag{1.2}$$

As it follows from definition of (1.2) $\phi'(u)$ is $s \times s$ matrix operator.

Then it is necessary to consider the following functional equation with deviated arguments:

$$\overleftarrow{F} \equiv F(\phi(u)) = \phi'(u) F(u), \tag{1.3}$$

where F is unknown s–dimensional vector function, components of which depend on vector function u and its derivatives up to some limited order.

Equation (1.3) always possesses one (trivial) solution $F(u) = u'$ in which it is possible to get convinced by differentiation (1.1) with respect to one of independent arguments of the problem.

If (1.3) possesses some other solution different from the trivial one such substitution was called in [3] as integrable substitution or integrable mapping.

With each of solutions of (1.3) it is possible to connect the evolution type equation:

$$u_t = F(u), \tag{1.4}$$

which is obviously invariant relatively to transformation (1.1). In [4,5] hope was expressed that the background of future theory of integrable systems is the theory of representations of the groups of integrable mappings.

The goal of the present paper is to investigate of two-dimensional integrable mappings and construct on this base the explicit form of integrable systems belonging to the corresponding hierarchies.

2. Two-dimensional integrable mappings

Below we will discuss three concrete examples of two-dimensional integrable mappings which can be considered by the similar methods.

2.1. Darboux–Toda substitution

The explicit form of the direct and inverse D–T integrable substitution is the following:

$$\overleftarrow{u} = \frac{1}{v}, \quad \overleftarrow{v} = v(uv - (\ln v)_{xy}),$$

$$\overrightarrow{v} = \frac{1}{u}, \quad \overrightarrow{u} = u(vu - (\ln u)_{xy}). \tag{2.1}$$

The function $f(u,v)$ after application of the s-times direct transformation we will denote $\overset{\leftarrow s}{f}$, after s-times inverse transformation as $\overset{s\rightarrow}{f}$, with the following agreement $\overset{\leftarrow(-m)}{f} \equiv \overset{m\rightarrow}{f}, m \geq 0$.

As a direct corollary of (2.1) there follows an important for our further considerations Toda-like recurrent relation for function $T_0 = uv$

$$(\ln T_0)_{xy} = -\overleftarrow{T_0} + 2T_0 - \overrightarrow{T_0}. \tag{2.2}$$

The corresponding to (2.1) Frechet derivative [6] has the form

$$\phi'(u) = \begin{pmatrix} 0 & -\frac{1}{v^2} \\ v^2 & 2(uv) - \frac{v_x v_y}{v^2} + \frac{v_x}{v} D_y + \frac{v_y}{v} D_x - D_{xy} \end{pmatrix}, \tag{2.3}$$

where $D_y \equiv \frac{\partial}{\partial y}, D_x \equiv \frac{\partial}{\partial x}$.

System (1.3) in the concrete case of D–T substitution may be rewritten as

$$\overleftarrow{F_1} = -\frac{1}{v^2}, F_2 \quad \overleftarrow{F_2} = v^2 F_1 + (2(uv) - \frac{v_x v_y}{v^2} + \frac{v_x}{v} D_y + \frac{v_y}{v} D_x - D_{xy}) F_2. \tag{2.4}$$

It is not difficult to check by the direct computations that $F_0 = (u, -v)$ is the solution of the last equation and so substitution (2.1) is integrable in the sense of [3].

After introducing the new functions $F_1 = uf_1$, $F_2 = vf_2$, the system (2.4) takes the form of a single equation for only one unknown function f_2

$$(\overleftarrow{uv})(\overleftarrow{f_2} - f_2) - (uv)(f_2 - \overrightarrow{f_2}) = -D_{xy} f_2, \quad f_1 = -\overrightarrow{f_2}. \tag{2.5}$$

The meaning of notations in the last equation is explained after formula (2.1).

In further transformations of (2.5) we will use the fact that condition of invariance of some function with respect to the discrete transformation $\overleftarrow{F} = F$ is equivalent to the $F \equiv const$. This is in some sense the analogues of Liouville theorem in the theory of analicity functions. Using this fact for function T ($f_2 = \int dy(\overleftarrow{T} - T)$) we obtain the Toda chain-like equation:

$$-T_x = T_0 \int dy(\overleftarrow{T} - 2T + \overrightarrow{T}), \quad T_0 = uv. \tag{2.6}$$

In terms of solution of (2.6) the evolution type equation (1.4) (invariant with respect to D–T substitution (2.1)) takes the form:

$$v_t = v \int dy(\overleftarrow{T} - T) \qquad u_t = u \int dy(\overrightarrow{T} - T). \tag{2.7}$$

2.2. Two-dimensional Heisenberg substitution

Under this term we will understand the direct and inverse transformations of two functions (u, v) of the form:

$$\begin{aligned} \overleftarrow{u} &= v^{-1}, \qquad \frac{1}{1+\overleftarrow{u}v} = \frac{1}{1+uv} + \frac{\phi_{xy}}{\phi_x\phi_y}, \quad \phi = \ln v, \\ \overrightarrow{v} &= u^{-1}, \qquad \frac{1}{1+\overrightarrow{u}v} = \frac{1}{1+uv} + \frac{\psi_{xy}}{\psi_x\psi_y}, \quad \psi = \ln u. \end{aligned} \tag{2.8}$$

One can get convinced that functions t_m $\left(t_1 = \frac{u_y v_x}{(1+uv)^2} = -\frac{(\overrightarrow{v})_y v_x}{(\overrightarrow{v}+v)^2}, t_2 = \frac{v_y u_x}{(1+uv)^2} = -\frac{(\overrightarrow{v})_x v_y}{(\overrightarrow{v}+v)^2}\right)$ satisfy the Toda-like recurrent relations

$$(t_m)_x = t_m \int dy \Delta_m, (m = 1, 2), \tag{2.9}$$

where $\Delta_m = \overleftarrow{t_m} - 2t_m + \overrightarrow{t_m}$.

The explicit form of the Frechet derivative operator is the following:

$$\phi'(u) = \begin{pmatrix} 0 & -v^{-2} \\ (\frac{v\overleftarrow{R}^{-1}}{R})^2 & -(1 + (\frac{vu\overleftarrow{R}^{-1}}{R})^2 + (\overleftarrow{R}^{-1})^2\delta(\phi_x^{-1}D_x + \phi_y^{-1}D_y - \frac{v}{v_{xy}}D_{xy}) \end{pmatrix}, \tag{2.10}$$

$$\delta = \frac{vv_{xy}}{v_x v_y}, \quad R = 1 + uv, \quad \overleftarrow{R} = 1 + \overleftarrow{u}v\,.$$

By a short calculation it is possible to show that equation (1.3) possesses nontrivial solution $F_1 = u, F_2 = -v$ and so Heisenberg substitution by definition is integrable.

Now we can rewrite equation (1.3) in more observable form. Let us denote $F_1 = uB,\ F_2 = vA$. From the first equation (1.3) we obtain immediately $B = -\overrightarrow{A}$. The second equation after some transformations may be rewritten in the form of a single equation for function A:

$$(\frac{\overleftarrow{u}v}{(1+uv)^2})(\overleftarrow{A} - A) - \frac{uv}{(1+uv)^2}(A - \overrightarrow{A}) = (\phi_x\phi_y)^{-1}(\frac{\phi_{xy}}{\phi_x}A_x + \frac{\phi_{xy}}{\phi_y}A_y - A_{xy}). \quad (2.11)$$

As we know from introduction the main equation (1.3) always possesses the trivial solution $F_1 = u_x, (u_y); F_2 = v_x, (v_y)$ or $A = \phi_x, (\phi_y)$. Let us look for the solution of (2.10) in the form $A = \phi_x\alpha$. Instead of (2.10) we obtain the equation for α:

$$(\frac{u_x\overleftarrow{v}_x}{(1+uv)^2})(\overleftarrow{\alpha} - \alpha) - \frac{u_xv_x}{(1+uv)^2}(\alpha - \overrightarrow{\alpha}^1) = (\frac{\alpha_y}{\theta})_x, \qquad \theta = \frac{\phi_y}{\phi_x}. \quad (2.12)$$

Resolving (2.12) by the substitution:

$$(\frac{\alpha_y}{\theta})_x = \overleftarrow{T} - T$$

we obtain the equation for determining T:

$$T_x = T_0 \int dy[\theta(\overleftarrow{T} - T) - \overrightarrow{\theta}(T - \overrightarrow{T})], \quad (2.13)$$

where

$$T_0 = \frac{u_xv_x}{(1+uv)^2}.$$

2.3. Lotky-Volterra substitution

In this case direct and inverse transformations, have the form

$$\overleftarrow{u} = u + (\ln v)_x, \quad \overleftarrow{v} = v + (\ln \overleftarrow{u})_y,$$
$$\overrightarrow{u} = u - (\ln \overrightarrow{v})_x, \quad \overrightarrow{v} = v - (\ln u)_y. \quad (2.14)$$

As in the previous case the functions $t_1 = uv,\ t_2 = \overleftarrow{u}v$ Toda-like recurrent relations (2.9).

The Frechet operator in this case has the form:

$$\phi'(u) = \begin{pmatrix} 1 & D_xv^{-1} \\ D_y(\overleftarrow{u})^{-1} & 1 + D_y(\overleftarrow{u})^{-1}D_xv^{-1} \end{pmatrix}. \quad (2.15)$$

By the same technique as in the previous subsections we obtain the single equation for the unknown function T and expressions of the equations of hierarchy via this solution

$$T_y = v \int dx[\overleftarrow{u}^1(\overleftarrow{T} - T) + u(T - \overrightarrow{T})] \quad (2.16)$$

and at last

$$u_t = u(T - \overrightarrow{T}), \quad v_t = D_yT.$$

3. Solution of the main equation

In spite of an essential difference of Frechet operators in the three above cases the main equations of problems (2.6),(2.14) and (2.16) have the same structure and may be solved by the similar methods. We shall demonstrate these methods on the more complicated example of Heisenberg substitution and represent the results of calculations for other cases.

First of all let us notice that equation (2.14) has the partial solution

$$T = T_0$$

in what one can get convinced with the help of the below equality which is the direct corollary of (2.8) and (2.9)

$$\overleftarrow{T_0} - T_0 = 2\phi_x(\frac{1}{1+uv})_x + 2\phi_{xy}\frac{\phi_x}{\phi_y}\frac{1}{1+uv} + \phi_x(\frac{\phi_{xy}}{\phi_x\phi_y})_x - \phi_{xy}\frac{\phi_x}{\phi_y} + \frac{\phi_{xy}^2}{\phi_y^2}.$$

Let us seek now the solution of (2.14) as $T = T_0 \int dy \alpha_0$. Instead of equation (2.14) we obtain equation for determining the α_0 function

$$(\alpha_0)_x + \alpha_0 \int dy[\overleftarrow{t_1} - t_1 + \overrightarrow{t_2} - t_2] = \overleftarrow{t_1} \int dy(\overleftarrow{\alpha_0} - \alpha_0) + \overrightarrow{t_2} \int dy(\overrightarrow{\alpha_0} - \alpha_0). \tag{3.1}$$

As it will be shown below this equation will arise many times and thus for us two possible ways of its further evolution will be important. Let us use the following ansatz

$$\alpha_0 = \overleftarrow{t_1}\alpha_1 + \overrightarrow{t_2}\beta_1.$$

After substituting these expressions into (3.1) and equating in front of the term $\overleftarrow{t_1}, \overrightarrow{t_2}$ to zero coefficients (this is some additional assumption) we come to equations for unknown functions α_1, β_1:

$$\begin{aligned} (\alpha_1)_x + \alpha_1 \int dy[\overleftarrow{t_1}^2 - \overleftarrow{t_1} + \overrightarrow{t_2} - t_2] &= \int dy(\overleftarrow{\alpha_0}\alpha_0), \\ (\beta_1)_x + \beta_1 \int dy[\overleftarrow{t_1} - t_1 + \overrightarrow{t_2}^2 - \overrightarrow{t_2}] &= \int dy(\overrightarrow{\alpha_0} - \alpha_0). \end{aligned} \tag{3.2}$$

Summarizing the second equation (3.2) shifted by the direct transformation with the first one we obtain

$$(\alpha_1 + \overleftarrow{\beta_1})_x + (\alpha_1 + \overleftarrow{\beta_1}) \int dy[\overleftarrow{t_1}^2 - \overleftarrow{t_1} + \overrightarrow{t_2} - t_2] = 0$$

and we see that the system (3.2) has the partial solution $\overset{+}{\alpha_1}\beta_1 = 0$, which we will use in what follows.

For this solution system (3.2) is equivalent to a single equation for the unknown function α_1:

$$(\alpha_1)_x + \alpha_1 \int dy[\overset{\leftarrow 2}{t_1} - \overset{\leftarrow}{t_1} + \overset{\rightarrow}{t_2} - t_2] = \int dy[(\overset{\leftarrow 2}{t_1}\overset{\leftarrow}{\alpha_1} - t_2\alpha_1) - (\overset{\leftarrow 1}{t_1}\alpha_1 - \overset{\rightarrow}{t_2}\overset{\rightarrow}{\alpha_1})],$$

The last equation has the obvious solution $\alpha_1 = 1$. As a corollary we obtain the second partial solution of our main equation:

$$T_1 = T_0 \int dy(\overset{\leftarrow}{t_1} - \overset{\rightarrow}{t_2}),$$

A further evolution of equation for α_1 is connected with the representation of unknown function in integral form $\alpha_1 \to \int dy\alpha_1$ (we conserve the same symbol for unknown function because it can't lead to misunderstanding in below considerations

$$(\alpha_1)_x + \alpha_1 \int dy[\overset{\leftarrow 2}{t_1} - \overset{\leftarrow}{t_1} + \overset{\rightarrow}{t_2} - t_2] = \overset{\leftarrow 2}{t_1} \int dy(\overset{\leftarrow}{\alpha_1} - \alpha_1) + \overset{\rightarrow}{t_2} \int dy(\overset{\rightarrow}{\alpha_1} - \alpha_1), \tag{3.3}$$

which up to obvious replacement $\overset{\leftarrow}{t_1} \to \overset{\leftarrow 2}{t_1}$ coincide with the equation for α_0 (3.1).

We can repeat the same trick with this equation as with the equation for α_0 and after k steps will come to substitution

$$\alpha_k = \overset{\leftarrow (k+1)}{t_1}\alpha_{k+1} - \overset{\rightarrow}{t_2}\overset{\rightarrow}{\alpha_{k+1}}$$

and equation for α_{k+1}

$$(\alpha_{k+1})_x + \alpha_{k+1} \int dy[\overset{\leftarrow k+2}{t_1} - \overset{\leftarrow k+1}{t_1} + \overset{\rightarrow}{t_2} - t_2] = \int dy[(\overset{\leftarrow k+2}{t_1}\overset{\leftarrow}{\alpha_{k+1}} - t_2\alpha_1) - (\overset{\leftarrow k+1}{t_1}\alpha_1 - \overset{\rightarrow}{t_2}\overset{\rightarrow}{\alpha_1})]$$

with the obvious solution $\alpha_{k+1} = 1$.

Collecting all the results together we obtain the partial solution of the main equation in the following formal formulae

$$T_n = T_0 \prod_{i=1}^{n}(1 - L_i \exp[-(i+1)d_i - \sum_{k=i+1}^{n} d_k]) \int dy \overset{\leftarrow 1}{t_1} \int dy \overset{\leftarrow 2}{t_1} ..... \int dy \overset{\leftarrow n}{t_1}, \tag{3.4}$$

where symbol $\exp d_s$ means the shift by unity of the argument of s-repeated integral $(.... \int dy \overset{h\rightarrow}{t_1} ... \to ... \int dy \overset{h+1\rightarrow}{t_1} ...)$ in (2.13) and symbol L_p means the exchange $\overset{r\rightarrow}{t_1}$ on the $\overset{r\rightarrow}{t_2}$ in the p-repeated integral $... \int dy \overset{r\rightarrow}{t_1} ... \to ... \int dy t_2 r....$

Expression (3.4) is directly applicable to Heisenberg and Lotky-Voltera integrable hierarchies. In the case of D–T hierarchy it is necessary to put all operators $L_i = 1$ and keep in mind the equality $t_1 = t_2 = T_0$.

4. Examples

In this section we represent the simplest integrable systems in usual unknown functions u, v corresponding to the lowest solutions T_n of the main equation for D–T, Heisenberg and L–V substitutions.

4.1. Darboux–Toda substitution

4.1.1. n=0

$$T_0 = uv, \quad u_t = au_x + bu_y, \quad v_t = av_x + bv_y.$$

In the examples below we shall choose $a = 1, b = 0$ keeping in mind that it is alvays possible to add the term (with arbitrary numerical coefficient) in which x is changed by y and vise versa.

4.1.2. n=1

$$T_1 = vu_x - v_x u,$$

$$u_t = u_{xx} - u\int dy(uv)_x, \quad -v_t = v_{xx} - v\int dy(uv)_x.$$

This is the Davey-Stewartson equation in its original form [5].

4.1.3. n=2

$$T_2 = (uv)_{xx} - 3u_x v_x - 3uv\int dy(uv)_x,$$

$$u_t = u_{xxx} - 3u_x\int dy(uv)_x - 3u\int dy(u_x v)_x,$$

$$v_t = v_{xxx} - 3v_x\int dy(uv)_x - 3v\int dy(v_x u)_x.$$

This is the equation of Veselov-Novikov [6].

4.1.4. n=3

$$T_3 = -(T_1)_{xx} - 2(u_x v_{xx} - v_x u_{xx}) + 2uv\int dy(T_1)_x + 4T_1\int dy(uv)_x,$$

$$v_t = -v_{xxxx} + 4v_{xx}\int dy(uv)_x - 2v_x(\int dy(T_1)_x - 2\int dy(uv)_{xx}) + 2v(\int dy(uv)_{xxx} -$$

$$\int dy(u_x v_x)_x + \int (uv_{xx})_x - ([\int dy(uv)]^2)_{xx} - [\int dy(uv)_x]^2).$$

Equation for u may be obtained from the equation for v under the changing of $u \to v, v \to u, t \to -t$.

4.2. Heisenberg substitution

4.2.1. n=0

$$v_t = -v_{xx} + 2v_x \int dy(\frac{uv_y}{1+uv})_x, \quad -u_t = -u_{xx} + 2u_x \int dy(\frac{vu_y}{1+uv})_x.$$

4.2.2. n=1

$$\begin{aligned} v_t + v_{xxx} - 3v_{xx} \int dy(\frac{uv_y}{1+uv})_x + 3v_x[\int dy(\frac{uv_y}{1+uv})_x]^2 + \\ +3v_x \int dy(\frac{u_x v_y}{(1+uv)^2})_x - 3v_x \int dy(\frac{uv_y}{1+uv})_{xx} \\ u_t + u_{xxx} - 3u_{xx} \int dy(\frac{vu_y}{1+uv})_x + 3u_x[\int dy(\frac{vu_y}{1+uv})_x]^2 + \\ +3u_x \int dy(\frac{v_x u_y}{(1+uv)^2})_x - 3u_x \int dy(\frac{vu_y}{1+uv})_{xx} \cdot \end{aligned}$$

4.3. Lotky–Volterra substitution

4.3.1. n=0

In the case of $T_0 = v$ we obtain the trivial system with the help of (2.2)

$$u_t = u_y, \quad v_t = v_y \cdot$$

4.3.2. n=1

In this case

$$S_1 = v \int dx(\overleftarrow{t_1} - \overrightarrow{t_2}) = v_y + v^2 + 2v \int dx(u_y) \cdot$$

The corresponding integrable system has the form

$$u_t = -u_{yy} + 2(uv)_y + 2u_y \int dx(u_y) \quad v_t = (v^2 + v_y + 2v \int dx(u_y))_y \cdot$$

In one dimensional case $D_x = D_y$ this system is a partial case of a wider integrable system described in [7].

4.3.3. n=2

In this case

$$S_2 = v^3 + 3vv_y + v_{yy} + 3vD_x^{-1}(uv)_y + 3(v_y + v^2)D_x^{-1}(u_y) + 3v(D_x^{-1}(u_y))^2.$$

The corresponding integrable system is the following

$$\begin{aligned} u_t = D_y(u_{yy} - 3(vu_y) + 3v^2u - 3(u_y - uv)D_x^{-1}(u)_y) + \\ + D_x(3D_x^{-1}(u_y)D_x^{-1}(uv)_y + (D_x^{-1}(u_y))^3), \\ v_t = D_y(v^3 + 3vv_y + v_{yy} + 3vD_x^{-1}(uv)_y + 3(v_y + v^2)D_x^{-1}(u_y) + 3v(D_x^{-1}(u_y))^2). \end{aligned}$$

5. Conclusion

In order to appreciate the results of the present paper let us come back to the main equation (1.3). This equation contains two unknown s–dimensional vector functions $\phi(u)$ and $F(u)$. The principle problem connected with this equation is to find substitution $\phi(u)$ in such a way that equation (1.3) would have some other solution except the trivial one. This problem has not been considered in this paper. We have taken at hoc integrable substitutions (Darboux–Toda, Heisenberg and Lotky–Volterra) and found for them the solution) for equation (1.3). This is only the second part of the problem as it has been formulated in [4,5].

From the explicit form of integrable equations we see that for their construction of them it is only necessary to know maximally two functions $t_{1,2}$. The only thing we need besides this is to have explicit formulas for multi–times discrete transformations and the technique of repeated integrals. We have also seen also that in the usual variables u, v all formulas became much more complicated and unobservable. So we may conclude that the discrete transformations are the principal point of the theory of integrable systems. We can suppose that in order to understand finally the theory of integrable systems it is necessary to have (or create) the complete theory of representations of the group of integrable mappings.

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