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BEAM COUPLING IMPEDANCE OF HOLES IN VACUUM-CHAMBER WALLS

Protvino 1992

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Abstract

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Both the longitudinal and transverse coupling impedances produced by a small hole in vacuum-chamber walls are evaluated for wavelengths large compared to a typical size of the hole. The method developed is based on the Bethe theory of diffraction by small holes. The problem is solved for an arbitrary cross section of the chamber and arbitrary hole shape. The results are obtained in analytical form for the cases of a circular and rectangular cross section. Contributions from pumping holes to the coupling impedance of the UNK, LEP and LHC vacuum chambers are calculated.

Аннотация

С.С. Куренной. Импеданс связи пучка с отверстиями в стенках вакуумной камеры ускорителя: Препринт ИФВЭ 92-84. — Протвино, 1992. — 23 с., 6 рис., 3 табл., библиогр.: 18.

Продольный и поперечный импеданс, создаваемые малым отверстием в стенке вакуумной камеры ускорителя, вычислены на частотах, соответствующих длинам волн, значительно бóльшим характерного размера отверстия. Метод основан на теории дифракции на малых отверстиях, развитой Бете. Задача решена для камеры с произвольным поперечным сечением и для произвольной формы отверстия. В случае круглой и прямоугольной камеры ответы получены в аналитическом виде. Рассчитан вклад отверстий для откачки в импеданс камеры У , LEP и LHC.

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С.С.Куренной

Импеданс связи пучка с отверстиями в стенках вакуумной камеры ускорителя.

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INTRODUCTION

The problem of the beam-chamber interaction is very important in modern accelerators and storage rings which have very high intensity of beams. This interaction is usually studied in terms of beam-chamber coupling impedances or connected with these, of wake potentials, e.g. review [1].

There is a general tendency to minimize the coupling impedances to avoid beam instabilities and reduce wall heating. In doing so one tends to shield, with respect to electromagnetic fields, such enlargements of the vacuum chamber as vacuum boxes, bellows, etc., by metallic shields or liners. On the other hand, the requirement of the beam life-time to be long enough and beam-loss reduction puts constraints on the admissible value of the vacuum inside the beam pipe. Achieving this high vacuum requires the presence of vacuum pumping holes in these shields. The number of such elements, which are chamber discontinuities and interact with a beam, can be very large in big machines. For example, 3260 liners in the vacuum chamber of the UNK first stage have in total nearly 10^5 narrow pumping slots. In the LEP collider of the order of 10^6 holes connect the beam pipe with the parallel chamber containing getter pumps. The LHC design includes a thermal screen with $10^7 \div 2 \cdot 10^8$ small holes for pumping (the number depends on the hole size). Common wisdom tells us that such a small discontinuity contributes very little to coupling impedances since it does not interrupt essentially the lines of beam-induced currents in the chamber walls. Nevertheless, their total contribution to the beam-chamber coupling impedances can be very essential because of a large number of such elements.

Therefore, quantitative results should be obtained and the impedance evaluation for these chamber elements is of great importance. Because of lack of the axial symmetry, it is obvious that a numerical solution to the problem has to be essentially three-dimensional. This implies time-consuming computations even

in the case of simplified models. This paper presents the analytical calculation of both the longitudinal and transverse coupling impedance for small holes in the walls of the vacuum chamber which has an arbitrary cross section.

1. EVALUATION METHOD

To evaluate the coupling impedance we have to calculate the fields induced in the chamber by a given current perturbation. The fields produced by a relativistic point charge in the chamber having a constant arbitrary-shaped cross section, without hole, are evaluated easily. Then we can consider them as incident electromagnetic waves on the hole. According to the Bethe theory of diffraction by small holes [2], the diffracted fields can be obtained as those radiated by effective surface "magnetic" currents and charges or, in the case of a small hole, just by effective electric and magnetic dipoles. Thus, when this approach is applicable, one can replace the excited hole by induced dipoles, evaluate the fields radiated by them inside the chamber and obtain the coupling impedance.

1.1. Fields in Chamber without Holes

We consider an infinite cylindrical pipe with an arbitrary cross section S and perfectly conducting walls. Let z axis be directed along the pipe axis, a typical hole size h satisfy $h \ll b$, and the hole center be located at point $(\vec{b}, z = 0)$. The point charge q moves with velocity $v = c$ along the chamber axis with a transverse offset $\vec{s}$. Then the electromagnetic fields harmonics $\vec{E}^{(0)}, \vec{H}^{(0)}$, which would be produced by this charge in point $(\vec{b}, z)$ on the chamber wall without hole, can be expressed as series [1]

$$E_\nu^{(0)}(\vec{b}, z; \omega) = Z_0 H_\tau^{(0)}(\vec{b}, z; \omega) = -Z_0 q e^{ikz} \sum_{n,m} \lambda_{nm}^{-1} \psi_{nm}(\vec{s}) \nabla_\nu \psi_{nm}(\vec{b}) \ , \qquad (1)$$

where $Z_0 = 120\pi$ Ohms is the impedance of free space, $k = \omega/c$, and indices ν and τ define the normal and tangential (in a transverse cross section of the chamber) field component, respectively. In Eq. (1) λ_{nm} and $\psi_{nm}(\vec{r})$ are eigenvalues (EV) and orthonormalized eigenfunctions (EF) of the following two-dimensional Dirichlet boundary problem in region S:

$$\left(\nabla^2 + \lambda_{nm}\right) \psi_{nm} = 0 \ ; \qquad \psi_{nm}|_{\partial S} = 0 \ . \qquad (2)$$

[1] Time dependence is $\exp(-i\omega t)$.

Let us consider the ratio of the real part of the impedance to the imaginary one

$$\frac{Re\, Z(\omega)}{|Im\, Z(\omega)|} = \frac{(\alpha_e^2 + \alpha_m^2)\omega^3}{6\pi(\alpha_e + \alpha_m)c^3} \ .$$

It follows, in the framework of the applicability of our approach, i.e. when $\omega h/c \ll 1$ or $\omega l/c \ll 1$, that the inequality $Re\, Z \ll |Im\, Z|$ holds. Actually, the polarizabilities are proportional to the third power of the typical size h of the hole, and as a result

$$\frac{Re\, Z(\omega)}{|Im\, Z(\omega)|} \propto \left(\frac{\omega h}{c}\right)^3 \ll 1 \ .$$

APPENDIX

On real part of longitudinal impedance of hole

The purely inductive longitudinal impedance of a hole in perfectly conducting walls of the vacuum chamber was obtained, see Eq. (13). However, some part of the beam energy is lost to excite the hole, and hence, the impedance has to have a nonzero real part. One can calculate it in an indirect way, by evaluating the energy radiated by the hole. This evaluation is, in essence, the same one as in paper [5], where beam energy losses produced by the beam-hole interaction are calculated.

Let a bunch with normalized longitudinal charge distribution $\lambda(s)$ move along the chamber axis with velocity $v \simeq c$. In the case of a point charge $\lambda(s) = \delta(s)$. The total energy radiated by the effective dipoles $\vec{M}(t)$ and $\vec{P}(t)$ is

$$\Delta U = \int_{-\infty}^{\infty} dt W(t) = \frac{Z_0}{6\pi c^2} \int_{-\infty}^{\infty} dt \left[\frac{1}{c^2} \left(\frac{d^2}{dt^2} \vec{M}(t) \right)^2 + \left(\frac{d^2}{dt^2} \vec{P}(t) \right)^2 \right] .$$

Since $E_\nu(z-ct) = Z_0 H_\tau(z-ct) = q\lambda(z-ct)e_\nu/\varepsilon_0$, for the dipole values we get $P(t) = \alpha_e q e_\nu \lambda(z-ct)$ and $M(t) = \alpha_m q e_\nu c \lambda(z-ct)$. Then we rewrite ΔU in ω-domain as

$$\Delta U = \frac{Z_0 q^2 (\alpha_e^2 + \alpha_m^2)}{12\pi^2 c^4} e_\nu^2 \int_{-\infty}^{\infty} d\omega \omega^4 |\lambda(\omega)|^2 ,$$

where $\lambda(\omega) = \int ds \exp(i\omega s/c)\lambda(s)$ is the bunch spectrum ($\lambda(\omega) = 1$ for a point charge).

On the other hand $\Delta U = kq^2$, where the loss factor k is defined as

$$k = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\omega Re\, Z(\omega) |\lambda(\omega)|^2 .$$

So

$$\Delta U = \frac{1}{2\pi} q^2 \int_{-\infty}^{\infty} d\omega Re\, Z(\omega) |\lambda(\omega)|^2 .$$

Comparing two expressions of ΔU and taking into account that the integrands are positively defined we obtain the real part of the longitudinal impedance of the hole

$$Re\, Z(\omega) = Z_0 \frac{(\alpha_e^2 + \alpha_m^2)\omega^4}{6\pi c^4} e_\nu^2 .$$

The energy loss of a Gaussian bunch calculated using this expression in the case of a circular chamber where $e_\nu = 1/(2\pi b)$, coincides with the result by Sands [5].

Other field components of the relativistic charge vanish on the chamber walls. For example, in the case a circular cross section with radius b the nonvanishing field harmonics on the hole, i.e. in point $(r = b, \varphi = \varphi_h, z = 0)$, can be written as

$$\begin{aligned} E_r^{(0)}(b, \varphi_h, 0; \omega) &= Z_0 H_\varphi^{(0)}(b, \varphi_h, 0; \omega) \\ &= \frac{Z_0 q}{2\pi b} \left[1 + 2 \sum_{n=1}^{\infty} \left(\frac{s}{b} \right)^n \cos n(\varphi_h - \varphi_s) \right] , \end{aligned} \tag{3}$$

where φ_s is the azimuthal angle of the beam in the chamber cross section.

1.2. Hole excitation by beam fields

Let us discuss conditions which have to be satisfied to consider the hole excitation by fields (1) in the spirit of the Bethe theory. Bethe's paper [2] investigates the diffraction of a plane electromagnetic wave by the hole in an infinite perfectly-conducting plane, its thickness is assumed to be negligible. The boundary conditions on the hole are satisfied by introduction of effective surface "magnetic" charge and current. Their densities ρ_{mag} and $\vec{J}_{mag}$ depend both on the hole shape and size and on the fields of the incident wave. To evaluate fields produced by these charge and current at distances R from the hole which are large compared to a typical hole size h, one can replace them by effective dipole moments. It is obvious that the applicability conditions for Bethe's approach in our problem are as follows:

- $h \ll b$ — one can neglect the curvature of the chamber wall;
- $\omega h/c \ll 1$ — field values can be considered as equal in different hole points and the approximation of incident plane wave can be used;
- $\Delta \ll h$, where Δ is the wall thickness.

It is clear that for small holes in chamber walls, say, for pumping holes, these conditions are usually satisfied. The restriction on frequency from the second condition is weak and the approach, as a rule, is applicable at frequencies above cutoff as well.

The effective magnetic dipole moment of the hole is defined [3] as

$$\vec{M} \equiv \frac{1}{\mu_0} \iint_{hole} dS \rho_{mag} \vec{r} ,$$

the electric one as

$$\vec{P} \equiv \frac{1}{2} \iint_{hole} dS \vec{J}_{mag} \times \vec{r} .$$

An important point is that these effective moments can be easily expressed in terms of the beam fields on the hole. Let us consider an elliptic hole with semiaxes l_1 and l_2 ($l_1 \geq l_2$), and introduce a local hole coordinate (u, v) with the axis u along the major axis of the ellipse. Then the effective moments can be written [3] as

$$\begin{aligned} \vec{M} &= \vec{a}_u \alpha_{m\|} H_{\tau u}^{(0)} + \vec{a}_v \alpha_{m\perp} H_{\tau v}^{(0)} \; ; \\ \vec{P} &= \varepsilon_0 \alpha_e \vec{\nu} E_\nu^{(0)} \; , \end{aligned} \tag{4}$$

where $\vec{a}_u$ and $\vec{a}_v$ are the local coordinate unit vectors, $\vec{\nu}$ is the normal to the hole plane (u, v). Fields $E_\nu^{(0)}$ and $\vec{H}_\tau^{(0)}$ are defined by Eq. (1). Electric α_m and magnetic α_e polarizabilities of the hole are purely geometric factors and can be calculated or measured. For the elliptic hole [3] they are

$$\begin{aligned} \alpha_{m\|} &= \frac{\pi l_1^3 \varepsilon^2}{3[K(\varepsilon) - E(\varepsilon)]} \; , \\ \alpha_{m\perp} &= \frac{\pi l_1^3 \varepsilon^2 (1-\varepsilon^2)}{3[E(\varepsilon) - (1-\varepsilon^2) K(\varepsilon)]} \; , \\ \alpha_e &= -\frac{\pi l_1^3 (1-\varepsilon^2)}{3E(\varepsilon)} \; , \end{aligned} \tag{5}$$

where $\varepsilon = \sqrt{1 - l_2^2/l_1^2}$ is the eccentricity, $K(\varepsilon)$ and $E(\varepsilon)$ are complete elliptic integrals of the first and second kind.

Let α be the angle between the chamber axis and ellipse major axis. Then

$$\begin{aligned} \vec{M} = [\; & \vec{a}_\tau (\alpha_{m\|} \sin^2\alpha + \alpha_{m\perp} \cos^2\alpha) \; + \\ + \; & \vec{a}_z (\alpha_{m\perp} - \alpha_{m\|}) \sin\alpha \cos\alpha \;] \; H_\tau^{(0)} \; , \end{aligned} \tag{6}$$

where $\vec{a}_\tau$ is a unit tangential vector to the boundary of the chamber cross section.

For the specific case of a circular hole with radius h we get $\varepsilon = 0$ and

$$\begin{aligned} \alpha_{m\|} &= \alpha_{m\perp} = 4h^3/3 \; , \\ \alpha_e &= -2h^3/3 \; . \end{aligned} \tag{7}$$

Another important particular case is a narrow slot. In the case of a longitudinal slot ($\alpha = 0$) with width w and length l, $w \ll l$, the limit of $\varepsilon \to 1$ in Eqs. (5) yields [2]

$$\begin{aligned} \alpha_m &= \alpha_{m\perp} = \pi l w^2/24 \; , \\ \alpha_e &= -\pi l w^2/24 \; . \end{aligned} \tag{8}$$

[2]This value, $\pi l w^2/24$, for a narrow slot is cited also in [4]. Paper [5] and book [6] give another value, $\pi l w^2/16$, which is obtained from the solution for an infinite narrow slot in a plane shield.

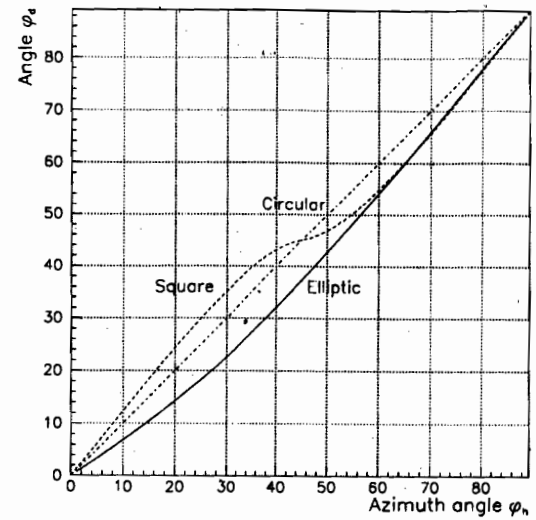

Figure 5. Dependence of angle ϕ_d on hole position

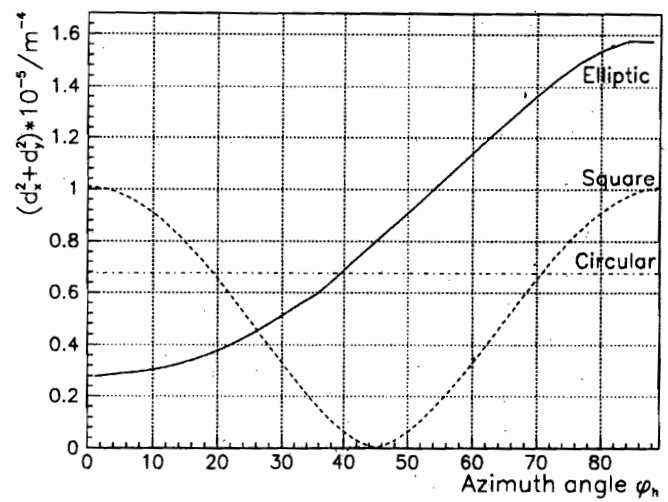

Figure 6. Dependence of $Z_\perp$ on hole position

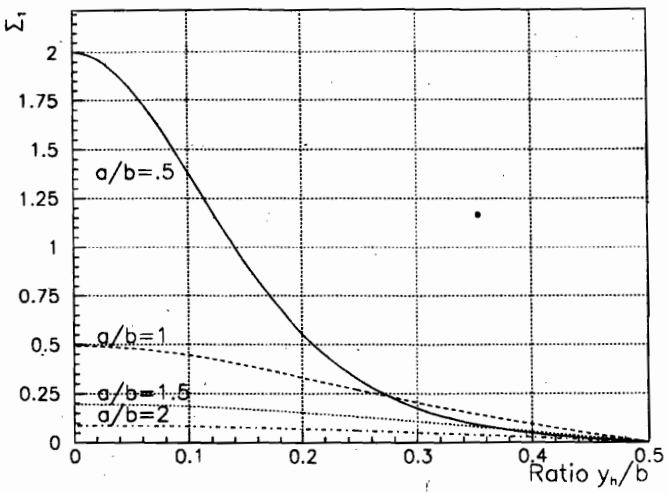

Figure 3. Function Σ_1 versus hole position

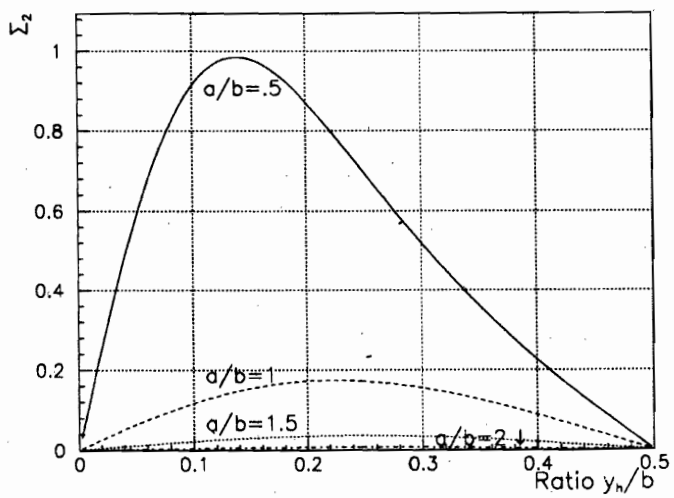

Figure 4. Function Σ_2 versus hole position

The applicability condition for the Bethe theory here is $\omega l/c \ll 1$. For a transverse narrow slit ($\alpha = \pi/2$) with width w and length l, $w \ll l$, it follows from (5) and (6) that

$$\begin{aligned} \alpha_m &= \alpha_{m\|} = \frac{\pi l^3}{24\,(\ln(4l/w) - 1)}\,, \\ \alpha_e &= -\pi l w^2/24\,. \end{aligned} \tag{9}$$

In the latter case, in contrast to a longitudinal slot, the condition $l \ll b$ is very essential. When it fails, the hole dimensions are comparable to the radius of curvature of the chamber walls, and the approximation of a small hole in Bethe's approach does not work.

1.3. Fields radiated by hole in chamber

We look for fields radiated into the chamber by induced effective currents and charges in the form of a series over the waveguide eigenmodes [3]

$$\vec{F} = \sum_{nm} \left(a_{nm} \vec{F}^{+}_{nm} \theta(z) + b_{nm} \vec{F}^{-}_{nm} \theta(-z) \right)\,, \tag{10}$$

where $\vec{F}$ means either $\vec{E}$ or $\vec{H}$ and $\theta(z)$ is the Heaviside step function. The superscripts '$\pm$' denote fields with propagation factors $\exp(\mp\Gamma_{nm} z)$, $\Gamma_{nm} = (\lambda^2_{nm} - k^2)^{1/2}$, radiated respectively in the straightforward (index '+', $z > 0$) or reverse ('−', $z < 0$) direction. The unknown coefficients a_{nm} and b_{nm} can be found by applying the Lorentz reciprocity theorem and using the eigenfunction orthogonality, e.g. [3]:

$$\begin{aligned} 2i\frac{k}{Z_0}\lambda_{nm}\Gamma_{nm}\begin{Bmatrix} a_{nm} \\ b_{nm} \end{Bmatrix} &= \iint_{hole} dS\, \vec{J}_{mag} \vec{H}^{\mp}_{nm} \\ &= -i\omega(\mu_0 \vec{H}^{\mp}_{nm} \vec{M} - \vec{E}^{\mp}_{nm} \vec{P} + \ldots)\,, \end{aligned} \tag{11}$$

where $\vec{M}$ and $\vec{P}$ are defined by Eqs. (4). The expansion of the integral in the RHS over dipole moments is justified for a small hole. However, sometimes the higher multipoles (quadrupole, etc.) have to be taken into account as well.

2. LONGITUDINAL IMPEDANCE

The longitudinal coupling impedance of the hole can be defined as

$$Z(\omega; \vec{s}, \vec{t}) = -\frac{1}{q}\int_{-\infty}^{\infty} dz e^{-ikz} E_z(\vec{t}, z; \omega)\,, \tag{12}$$

where $\vec{t}$ is the transverse offset of a test charge. The component E_z of the field radiated by the hole into the chamber, depends on $\vec{s}$ since beam fields on the hole and, as a result, the effective currents depend on $\vec{s}$. The usual impedance definition corresponds to the particular case of $\vec{s} = \vec{t} = 0$, i.e. to the lowest term of expansion over $\vec{s}, \vec{t}$, for details see review [7].

Let us substitute E_z from Eq. (10) into definition (12). When $\omega < \omega_{cut}$, all $\Gamma_{nm} > 0$ and one can easily integrate over z. It is easy to recognize that at frequencies above cutoff the difficulties of integration over z on infinite intervals are only formal ones and by introducing a small attenuation we get the same result as for $\omega < \omega_{cut}$. Then taking into account Eqs. (11) and (4) we obtain the longitudinal impedance of a small hole in the following form

$$Z(\omega; \vec{s}, \vec{t}) = -iZ_0 \frac{\omega}{c}(\alpha_e + \alpha_m) e_\nu(\vec{s}) e_\nu(\vec{t}) \ . \tag{13}$$

Here $\alpha_m = \alpha_{m\|} \sin^2\alpha + \alpha_{m\perp} \cos^2\alpha$ for holes with a symmetry axis at angle α to the chamber axis, and the following notation is introduced

$$e_\nu(\vec{r}) \equiv \frac{E_\nu^{(0)}(\vec{r})}{Z_0 q} = -\sum_{n,m} \lambda_{nm}^{-1} \psi_{nm}(\vec{r}) \nabla_\nu \psi_{nm}(\vec{b}) \ . \tag{14}$$

It is seen that $e_\nu(\vec{r})$ is just a normalized electric field of the beam on the hole. In other words, it is a solution to the standard two-dimensional electrostatic problem in S: to find the electric field on a conducting boundary produced by a charge which is placed in point $\vec{r}$. For simple cases e_ν can be easily found by means of the Gauss theorem.

It should be noted that impedance (13) is proportional to frequency ω and to sum of hole polarizabilities $\alpha_e + \alpha_m$. In the accepted notations [3] α_e is negative, and this sum has to be considered as a difference of contributions from the magnetic and electric effective dipoles. It is a typical low-frequency impedance in the case of $\alpha_e + \alpha_m > 0$. Such an impedance can cause the longitudinal instability of intense bunches when the impedance value is above some admissible threshold [8]. It is well known that low-frequency ($\omega \ll \omega_{cut}$) impedances from various inhomogeneities are additive. Naturally, the following idea arises: the total inductive impedance of the accelerator chamber can be lowered either by choosing the shape of pumping holes in such a manner that condition $\alpha_e + \alpha_m < 0$ holds, or by introducing special compensating holes of such a form. Unfortunately, for small holes the inequality $\alpha_e + \alpha_m > 0$ seems always to hold. Specifically, in paper [9] for the holes possessing a symmetry with respect to two perpendicular axes the inequality $|\alpha_e|^{-1} \geq \alpha_{m\|}^{-1} + \alpha_{m\perp}^{-1}$ was proved. The equality holds here for an elliptic hole, cf. Eq. (5) and particular

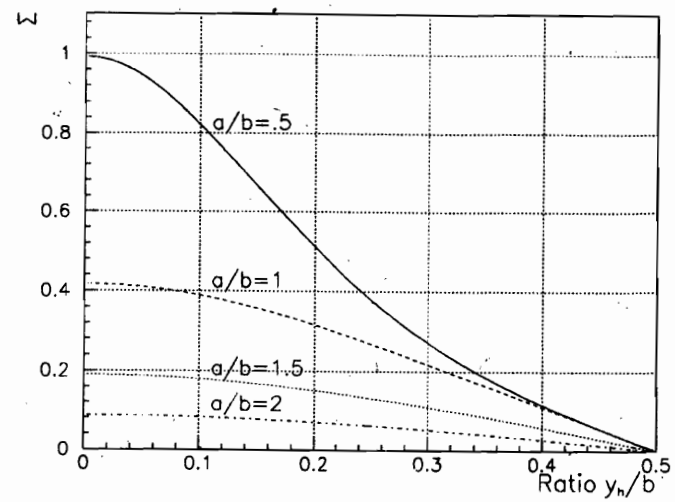

Figure 1. Function Σ versus hole position

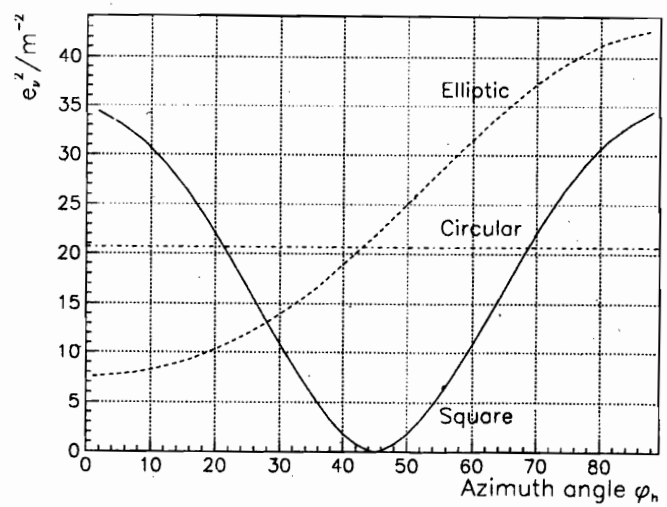

Figure 2. Dependence of Z on hole position

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Received June 10, 1992.

cases (7)-(9). It follows

$$\alpha_m + \alpha_e \geq \frac{\alpha_{m\|}^2 \sin^2\alpha + \alpha_{m\perp}^2 \cos^2\alpha}{\alpha_{m\|} + \alpha_{m\perp}} > 0 \,. \tag{15}$$

Circular chamber

Let us consider the chamber with a circular cross section of radius b. Taking into account the axial symmetry we can easily obtain from the Gauss theorem that $e_\nu(0) = 1/(2\pi b)$, cf. Eq. (3). Substitution into Eq. (13) gives the longitudinal impedance of a small hole having an arbitrary shape, in the form

$$Z(\omega) = -iZ_0 \frac{\omega}{c} \frac{(\alpha_e + \alpha_m)}{4\pi^2 b^2} \,. \tag{16}$$

This result was obtained first by a direct summation over EFs in paper [10] and later by another method in [11].

In the particular case of a circular hole with radius h taking into account Eqs. (7) for polarizabilities we get a simple expression

$$Z(\omega) = -i \frac{Z_0}{6\pi^2} \frac{\omega h^3}{c b^2} \,. \tag{17}$$

It shows an inductive contribution of the hole. The usually quoted in cyclic accelerators quantity, the so called reduced impedance Z/n, is

$$\frac{Z(n\omega_0)}{n} = -i \frac{Z_0}{6\pi^2} \frac{h^3}{R b^2} \,, \tag{18}$$

where $n = \omega/\omega_0 = \omega R/c$ is the harmonic number of revolution frequency ω_0 and R is the machine radius.

The longitudinal impedance of a circular hole in the circular-chamber walls was studied by a direct numerical method as well. In paper [12] by using the 3-D code T3 in MAFIA [13] the longitudinal wake potential produced by such a discontinuity was computed. The low-frequency impedance was estimated by fitting the numerical results. The obtained formula, $Z/n \approx -i0.017 Z_0 h^3/(Rb^2)$, coincides well with the exact analytical expression (18) ($1/(6\pi^2) \simeq 1.69 \cdot 10^{-2}$).

It should be noted that this dependence of the longitudinal impedance on the hole radius h and that of chamber b can be simply derived from qualitative considerations[3] see [12].

[3] K. Bane, private communication.

For the case of a circular hole the effect of a finite chamber-wall thickness on the hole impedance was investigated. Numerical results [14] show that the hole polarizabilities decrease slowly when wall thickness Δ increases from zero, reaching a constant value when $\Delta > 2h$. For such or thicker wall the impedance is equal to 0.56 multiplied by its value (17) for a thin wall, see [11]. So taking into account the finite thickness of the chamber walls slightly reduce the value of the hole impedance.

For a very narrow longitudinal slot taking into account polarizabilities (8) leads to $Z(\omega) = 0$, i.e. the impedance vanishes to the first-order approximation of our approach. It seems natural since such a slot does not interrupt essentially beam-induced wall currents. However, it should be noted that such a cancellation of contributions from the electric and magnetic dipoles takes place only as a limiting case, cf. (15) with $\alpha = 0$ and $\alpha_{m\|} \gg \alpha_{m\perp}$. The account of next-to-leading terms in expansion (5) at $\varepsilon \to 1$ leads, when $w \ll l < b$, to the following expression

$$Z(\omega) = -i\frac{Z_0}{96\pi}\frac{\omega}{c}\frac{w^4}{b^2 l}\left(\ln\frac{4l}{w} - 1\right) . \tag{19}$$

The contribution of the quadrupole induced moment to impedance in this case vanishes as well [10].

Narrow transverse slot, in contrast to the longitudinal one, disturbs drastically the currents which are induced in chamber walls by the beam. It is clear that the impedance produced by such a discontinuity is large. Substituting (9) into (16) and neglecting the contribution from the electric dipole which, in this case, is small as compared to the magnetic one, cf. (9), we obtain

$$Z(\omega) \simeq -i\frac{Z_0}{96\pi}\frac{\omega}{c}\frac{l^3}{b^2\left(\ln(4l/w) - 1\right)} . \tag{20}$$

This formula can be applied when $w \ll l \ll b$.

Rectangular chamber

Consider now the chamber having a rectangular cross section with width a and height b. Without restriction of generality, let us consider a hole in the side wall ($x_h = \pm a/2$) which is displaced from the horizontal plane of the chamber symmetry, $y = 0$, at distance y_h, $|y_h| \le b/2$. The two-dimensional problem to find $e_\nu(\vec{r})$ can be analytically solved in a rectangular region and substituting $e_\nu(0)$ into Eq. (13) yields

$$Z(\omega) = -iZ_0\frac{\omega}{c}\frac{(\alpha_e + \alpha_m)}{b^2}\Sigma^2 , \tag{21}$$

The author would like to thank V.I. Balbekov, R. Gluckstern, S.V. Ivanov, F. Fuggiero and B. Zotter for useful discussions and remarks, as well as I.I. Petrenko and S.V. Purtov for the help in using their code MGD2.

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of the chamber the results are obtained in a simple analytical form, Eqs. (16) and (29), a semianalytic answer is given for a rectangular chamber, and in the general case some simple computations are to be done.

The results obtained allow one to give both the numerical estimates of hole contributions to the chamber impedances, and some practical recommendations for the vacuum chamber design:

- Comparing Eqs. (17) and (19), (30) and (31) we conclude that both the longitudinal and transverse impedance of a narrow longitudinal slot are lower than those of a circular hole with the same area. The result is independent of the chamber cross section.
- Both the longitudinal and transverse impedances of a hole in the wall of a rectangular chamber vanish when the hole is near one of the chamber corners. Naturally, since the hole has a finite transverse size its impedance reduces essentially when the hole is near a corner rather than goes to zero.
- In the chamber having the cross section of a general shape it is better, from the viewpoint of impedance minimization, to arrange holes in places where beam fields are minimal, i.e. in corners, dips, etc. In doing so the hole is weakly excited by the beam and produces lower coupling impedance.

The behavior of hole impedances at very high frequencies when the perturbation wavelength is smaller than or of the order of the hole size, has remained beyond the framework of the present paper. In this frequency range the beam fields penetrate through the holes and the interaction with elements which are placed beyond the shields or liners, i.e. cavities, bellows, etc., becomes essential. This can lead to resonances whose parameters seem to be determined by specific cavity layout. The study of such resonances is especially important for electron accelerators where the bunch length is very short. In principle, it is possible to calculate impedances at high frequencies numerically, but the problem is very hard and time-consuming since the geometry is essentially the three-dimensional one.

The special case is the LHC vacuum chamber where the inner wall of the chamber together with the outer surface of the thermal screen form a coaxial structure. Through the pumping holes the beam can excite a TEM-wave in the coaxial structure. Since such a wave has a phase velocity which is equal to the speed of light, it is synchronous to the ultrarelativistic beam and interacts with the beam in a resonant manner. This interaction is studied in paper [18].

* * *

where function Σ is defined by the fast-converging series

$$\Sigma \equiv \sum_{m=0}^{\infty} \frac{\cos(2m+1)\pi y_h/b}{\cosh(2m+1)\pi x_h/b} \ . \tag{22}$$

The dependence of Σ on the hole coordinate y_h is shown in Fig. 1 for a few values of the ratio a/b. It should be mentioned that in a rectangular chamber the impedance of a small hole vanishes when the hole is near one of the corners: $\Sigma \to 0$ when $|y_h| \to b/2$. The explanation is that since the beam fields near the corner tend to zero the effective currents on the hole are not induced.

Chamber with an arbitrary cross section

To calculate the longitudinal impedance of a hole in the wall of the chamber with an arbitrary cross section S it is necessary to find $e_\nu(\vec{r})$ by solving the electrostatic problem. The evaluation of the electric field which is produced by a thin uniformly-charged filament on the cylindrical surface with cross section S is the standard two-dimensional boundary problem in the case of interest when the filament is parallel to the surface generatrix. There are a number of codes for numerical solution to the problem. We use the code MGD2 [15] which is based on the multigrid variant of the finite element method. The field produced on the boundary of region S by a unit "point" source is computed. Fig. 2 shows the values of e_ν^2 for square 70×70 mm chamber and elliptic one, with semiaxes 40 and 30 mm. For comparison we cite results for a circular chamber with radius b =35 mm.

Numerical results for the square chamber coincide well with the analytic solution (21) at $a/b = 1$. Note, the hole impedance strongly depends on the hole position in the chamber cross section. It was shown above in analytical way for the case of a rectangular chamber. For the elliptic chamber under consideration the impedance of a hole in the horizontal plane ($\varphi_h = 0$) is 5.6 times smaller than that of the same hole in the vertical plane ($\varphi_h = \pi/2$) while the ratio of their squared distances from the axis is equal to 16/9.

We may not care about solution normalization if the longitudinal impedance is computed as follows. One can solve numerically the two-dimensional problem for e_ν in region S and then in the circle of radius b. After that we get the result simply from a comparison with the hole impedance in the circular chamber as

$$Z = Z_{circ}\,(e_\nu/e_{circ})^2 \ , \tag{23}$$

where Z_{circ} is given by Eq. (16).

3. TRANSVERSE IMPEDANCE

The transverse impedance is defined as

$$\vec{Z}_\perp(\omega;\vec{s},\vec{t}) = -\frac{i}{qs}\int_{-\infty}^{\infty} dz e^{-ikz}\left[\vec{E} + Z_0\vec{\beta}\times\vec{H}\right]_\perp , \qquad (24)$$

where in the RHS the fields produced by the hole are taken at $(\vec{t}, z; \omega)$, $\vec{\beta} \to (0,0,1)$ and the limit of $s \to 0$, $t \to 0$ is usually understood. It is obvious that both the E- and H-modes contribute to the integral unlike the longitudinal impedance (12). After some calculations which are similar to those for the longitudinal case we get the expression

$$\begin{aligned}\vec{Z}_\perp(\omega;\vec{s},\vec{t}) &= -\frac{i}{qs}\sum_N \Big[\lambda_N^{-1}(\vec{\nabla}\psi_N)_t(\vec{\nabla}\psi_N\cdot\vec{P}/\varepsilon_0)_h \\ &+ \kappa_N^{-1}(\vec{\beta}\times\vec{\nabla}\chi_N)_t(\vec{\nabla}\chi_N\cdot Z_0\vec{M})_h\Big] , \qquad (25)\end{aligned}$$

where $N = \{nm\}$ is a generalized index, κ_{nm} and $\chi_{nm}(\vec{r})$ are respectively the eigenvalues and EFs of boundary problem (2) with the Neumann boundary condition, i.e. $\partial_\nu\chi_{nm}|_{\partial S} = 0$. Here we mean that the hole effective moments in the RHS are induced by the dipole component of the beam fields, i.e. $P, M \propto s$, cf. $n = 1$ term in Eq. (3). Substituting expressions (4) and (6) of the moments and taking into account the following relation between the beam field components, $Z_0\vec{H}_\tau^{(0)} = \vec{a}_z \times \vec{E}_\nu^{(0)}$, we obtain

$$\begin{aligned}\vec{Z}_\perp(\omega;\vec{s},\vec{t}) &= \frac{iZ_0}{s}\sum_N \Big[\alpha_e\lambda_N^{-1}(\vec{\nabla}\psi_N)_t(\vec{\nabla}\psi_N\cdot\vec{e}_\nu^{dip})_h \\ &+ \alpha_m\kappa_N^{-1}(\vec{a}_z\times\vec{\nabla}\chi_N)_t(\vec{a}_z\times\vec{\nabla}\chi_N\cdot\vec{e}_\nu^{dip})_h\Big] . \qquad (26)\end{aligned}$$

Since the longitudinal impedance $Z(\omega;\vec{s},\vec{t})$ has been calculated above as a function of $\vec{s}$ and $\vec{t}$, see Eq. (13), one can obtain the transverse impedance by means of the Panofsky-Wenzel theorem as $\vec{Z}_\perp(\omega;\vec{s},\vec{t}) = c\vec{\nabla}_t Z(\omega;\vec{s},\vec{t})/(\omega s)$, e.g. see [7]. This way leads to the expression

$$\vec{Z}_\perp(\omega;\vec{s},\vec{t}) = -iZ_0(\alpha_e + \alpha_m)\frac{e_\nu^{dip}(\vec{s})}{s}\cdot\vec{\nabla}_t e_\nu(\vec{t}) , \qquad (27)$$

as well as above here $s \to 0$ and $t \to 0$. Using the completeness of the EF sets CΦ χ_{nm} and ψ_{nm} one can prove the equality, up to factors α_e, α_m, of the contributions from the electric and magnetic dipoles in Eq. (26), and transform this equation to form (27). It should be noted that the transverse impedance

LHC

In the LHC design it is supposed to shield the chamber walls by an internal thermal screen with pumping holes. The total pumping area has to be approximately 0.05 of the screen surface. For estimates we accept the mean radius of the thermal screen $b = 1.5$ cm, the accelerator radius $R = 4243$ m. Let us consider the design version [17] with $N = 10^7$ circular holes of radius $h = 2$ mm. In this case there are $M = 4$ holes in one cross section of the chamber, the longitudinal spacing between such cross sections is $d = 1$ cm. There is another version, with $N = 1.6 \cdot 10^8$ very small circular holes, $h = 0.5$ mm, burned by laser.[4] We assume $d = 2.5$ mm and $M = 8$ for this version. The values of the impedances produced by the pumping holes are shown for the both design variants in Table 3.

Table 3. Impedances Produced by Holes in LHC Thermal Screen

	$\lvert Z/n\rvert$/ Ohm		$Z_\perp$/ (Ohm/m)	
	$h = 0.5$ mm	$h = 2$ mm	$h = 0.5$ mm	$h = 2$ mm
One hole	$8.3 \cdot 10^{-10}$	$5.3 \cdot 10^{-8}$	0.063	4.0
One cross section	$6.6 \cdot 10^{-9}$	$2.1 \cdot 10^{-7}$	0.25	8.0
Total	0.13	0.53	$5 \cdot 10^6$	$2 \cdot 10^7$

Taking into account the finite thickness of the screen wall leads, according to [11], to multiplication all the values in Table 3 by a factor of 0.56. Nevertheless, the values of both the longitudinal and transverse impedances are high, especially for the variant with larger holes. The latter variant can not be considered as acceptable. The present design should be modified, for example, by replacing the circular holes by longitudinal slots.

CONCLUSIONS

The method developed allows one to calculate the coupling impedances of small holes in the vacuum-chamber walls of an accelerator at the frequencies which are important in practice, i.e. when the wavelength is large compared to a typical hole size. The problem is solved for an arbitrary cross section of the chamber and general hole shape. In the specific case of a circular cross section

[4] F. Ruggiero, private communication

approximating the slot by a long elliptic hole with length l and effective width $w_{eff} = 4w/\pi$. For upper estimate we use the results of the polarizabilities measurements in the range $w/l = 0.2 \div 1.0$ for the rectangular aperture $w \times l$ from paper [16], cited in [9]. They are well fitted by the linear dependence $(\alpha_m+\alpha_e)/(w^2 l) \simeq (0.033+0.27w/l)$. The extrapolation to the value of interest, $w/l = 0.1$, gives $(\alpha_m + \alpha_e) \simeq 0.06w^2 l$. For $Z_\perp$ the maximal absolute values of the impedance corresponding to the factor $\cos(\varphi_b - \varphi_d) = 1$ are shown.

Table 1. Impedance Estimates for the UNK Slots

	$\lvert Z/n\rvert$ / Ohm	$Z_\perp$ / (Ohm/m)
One slot	$(0.4 \div 3.1) \cdot 10^{-7}$	$0.5 \div 3.3$
One liner	$(1.1 \div 6.6) \cdot 10^{-6}$	$6.0 \div 42.9$
Total	$(3.6 \div 26) \cdot 10^{-3}$	$(0.2 \div 1.4) \cdot 10^5$

The impedance values obtained are quite permissible from the viewpoint of the beam stability.

LEP

The vacuum chamber of the electron-positron collider LEP has the circumference nearly 27 km, its cross section is the ellipse with semiaxes 131 and 65 mm. About 21 km of the chamber have pumping holes (40 per meter) of rectangular shape 20 × 8 mm with rounded corners. For estimates we approximate the hole shape by the ellipse with semiaxes $l_1 = 11.3$ mm and $l_2 = 4.5$ mm having the same area. The pumping holes are placed in the horizontal plane, $\varphi_h = 0$, connecting the beam pipe with the parallel chamber which contains the getter pumps. For such a layout the transverse impedance has only the horizontal nonzero component. The results are shown in Table 2.

Table 2. Impedance Estimates for LEP Pumping Holes

	$\lvert Z/n\rvert$ / Ohm	Z_x / (Ohm/m)
One hole	$2.7 \cdot 10^{-8}$	0.11
Total	0.023	$9.3 \cdot 10^4$

These figures contribute less than 10% of the measured impedances of the LEP vacuum chamber.

of a hole, as well as the longitudinal one, is proportional to the sum of the hole polarizabilities.

Substituting $e_\nu^{dip}(\vec{s}) = \vec{s} \cdot \vec{\nabla}_s e_\nu(\vec{s})$ into Eq. (27) and going to the limit $s = 0$, $t = 0$ we get

$$\vec{Z}_\perp(\omega) = -iZ_0(\alpha_e + \alpha_m)(d_x^2 + d_y^2)\vec{a}_d \cos(\varphi_b - \varphi_d) \ . \tag{28}$$

Here x, y are the horizontal and vertical coordinates in the chamber cross section; $d_x \equiv \partial_x e_\nu(0)$, $d_y \equiv \partial_y e_\nu(0)$; $\varphi_b = \varphi_s = \varphi_t$ is the azimuthal angle of the beam position in the cross-section plane; $\vec{a}_d = \vec{a}_x \cos\varphi_d + \vec{a}_y \sin\varphi_d$ is a unit vector in this plane in direction φ_d, which is given by conditions $\cos\varphi_d = d_x/\sqrt{d_x^2 + d_y^2}$, $\sin\varphi_d = d_y/\sqrt{d_x^2 + d_y^2}$. It is seen from Eq. (28) that the angle φ_d shows the direction of the transverse-impedance vector $\vec{Z}_\perp$ and, therefore, of the beam-deflecting force. Moreover, the value of $Z_\perp$ is maximal when the beam is deflected along this direction and vanishes when the beam offset is perpendicular to it.

Circular chamber

In the case of a circular cross section of the chamber one can easily derive from Eq. (3) that $d_x = \cos\varphi_h/(\pi b^2)$ and $d_y = \sin\varphi_h/(\pi b^2)$. As a result, $\varphi_d = \varphi_h$, $\vec{a}_d$ coincides $\vec{a}_h$, which is the direction to the hole, and the transverse impedance is

$$\vec{Z}_\perp(\omega) = -iZ_0 \frac{\alpha_m + \alpha_e}{\pi^2 b^4} \vec{a}_h \cos(\varphi_h - \varphi_b) \ . \tag{29}$$

This formula was first obtained by a direct summation over EFs in [10]. It is clear that the deflecting force is directed toward the hole (or in the opposite direction) and the force value depends on the angle between the beam offset and the direction to the hole. It is worth noting that in axisymmetric structures the beam-deflecting force has to be directed along the beam offset. The presence of a hole breaks this symmetry.

For particular cases of the hole shape we obtain from Eq. (29) the transverse impedance of

circular hole with radius h, cf. (5),

$$\vec{Z}_\perp(\omega) = -iZ_0 \frac{2h^3}{3\pi^2 b^4} \vec{a}_h \cos(\varphi_h - \varphi_b) \ ; \tag{30}$$

narrow longitudinal slot with width w and length l, $w \ll l < b$, cf. (8),

$$\vec{Z}_\perp(\omega) = -iZ_0 \frac{w^4}{24\pi b^4 l} \left(\ln\frac{4l}{w} - 1\right) \vec{a}_h \cos(\varphi_h - \varphi_b) \ ; \tag{31}$$

narrow transverse slot with width w and length l, when $w \ll l \ll b$, cf. (9),

$$\vec{Z}_\perp(\omega) \simeq -iZ_0 \frac{l^3}{24\pi b^4 (\ln(4l/w) - 1)} \vec{a}_h \cos(\varphi_h - \varphi_b) \ . \tag{32}$$

If we consider M ($M \geq 3$) holes uniformly spaced in one cross section, as a result of vector summation of M expressions (29) the transverse impedance is

$$\vec{Z}_\perp(\omega) = -iZ_0 \frac{\alpha_m + \alpha_e}{\pi^2 b^4} \frac{M}{2} \vec{a}_b \ , \tag{33}$$

where $\vec{a}_b$ is a unit vector in the direction of the beam transverse offset. One can see that the deflecting force is now directed along the beam offset, i.e. some kind of the axial symmetry restoration occurs. The maximal value of $Z_\perp$ for M holes which are uniformly spaced in one cross-section is only $M/2$ times larger than that for $M = 1$. Moreover, the well-known empirical relation $Z_\perp = (2R/b^2)Z/n$ (cf. (16) and (33)), which is justified only for axisymmetric structures, holds in this case also.

Rectangular chamber

In the case of the rectangular ($a \times b$) chamber cross section one can derive analytical expressions $d_x = \pi\Sigma_x/b^2$ and $d_y = \pi\Sigma_y/b^2$ in the form of series

$$\begin{aligned} \Sigma_x &\equiv \sum_{m=0}^{\infty} \frac{(2m+1)\cos(2m+1)\pi y_h/b}{\sinh(2m+1)\pi x_h/b} \ ; \\ \Sigma_y &\equiv \sum_{m=0}^{\infty} \frac{2m \sin 2m\pi y_h/b}{\cosh 2m\pi x_h/b} \ . \end{aligned} \tag{34}$$

These series are fast-convergent ones. The dependence of functions defined by these series on the hole position for various values of a/b is shown in Figs. 3 and 4. The transverse impedance of the hole in the rectangular-chamber wall is

$$\vec{Z}_\perp(\omega) = -iZ_0 \frac{\pi^2(\alpha_m + \alpha_e)}{b^4} (\Sigma_x^2 + \Sigma_y^2) \vec{a}_d \cos(\varphi_b - \varphi_d) \ , \tag{35}$$

where $\tan\varphi_d = \Sigma_y/\Sigma_x$. Note, that in the rectangular chamber φ_d is not equal, in general, to φ_h. Since $\Sigma_x \to 0$ and $\Sigma_y \to 0$ when $|y_h| \to b/2$ the hole transverse impedance in the rectangular chamber goes to zero, as well as the longitudinal one, when the hole is placed near one of the chamber corners.

Chamber with an arbitrary cross section

To evaluate the transverse impedance of a hole in the walls of the chamber with an arbitrary cross section S it is necessary to find quantities d_x and d_y defined by (28). In doing so we compute, using the code MGD2 [15], the electric field produced on the region boundary ∂S by a dipole, which consists of two opposite-sign charges spaced by small distance $2s$ and is directed along the axis x or y, respectively. In the case when region S has some symmetry with respect to x or y it is convenient to solve numerically the problem for a single charge in the quarter of the region with given relevant boundary conditions of the 1st and 2nd kind on the axes. After dividing the result by the dipole length $2s$ we get d_x and d_y as functions of the hole position on the chamber wall.

Computations for circular and rectangular chambers show a good agreement with the analytical results cited above. As an example, the results for the elliptic chamber with semiaxes 40 and 30 mm are shown also in Figs. 5 and 6. It is worth noting that the dependencies of the value of the transverse impedance on the hole position (Fig. 6) are very similar to those for the longitudinal impedance (Fig. 2).

4. ESTIMATES

At frequencies below cutoff the contributions to the chamber impedance from small discontinuities are additive. One can give a formal proof of this fact for many holes if the effective moments induced on each hole are independent of the presence of neighboring holes. The latter circumstance is not very restrictive even in the case of holes which are very close to each other, see [5]. In making estimates we will assume this additivity of the contributions from different holes to the inductive impedance of vacuum chambers. Nevertheless, at higher frequencies the problem of coherent effects is still open. The real part of the longitudinal impedance produced by a hole in the chamber wall is evaluated in Appendix.

UNK, 1st stage

Let us estimate the impedances produced by slots in the liners of the UNK 1st stage. Approximately $N = 3260$ vacuum boxes with bellows will be shielded by the liners and each liner has $M = 26$ slot with width $w = 0.6$ cm and length $l = 6$ cm. For estimates we take the chamber radius $b = 3.5$ cm and the machine one $R = 3306$ m. Note, in this case $l > b$ and therefore our estimates will be rough. The figures are cited in Table 1. The lower bound is obtained by