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Constraints on minimal supersymmetric SU(5) grand unification

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The bounds coming from the requirement of coupling constant unification and from proton decay are analised in minimal SU(5) supersymmetric grand unification, supplemented with the requirement that the Higgs doublets be identically massless in the limit of unbroken supersymmetry. We consider the cases when this is enforced by a fine tuning or is achieved by a pseudo-Goldstone mechanism.

We find that:

i) the model is never compatible with supersymmetry breaking mass parameters lower than 1 TeV unless some of them are very unnaturally tuned;

ii) accepting supersymmetry breaking mass parameters higher than 1 TeV, the strong coupling constant α_3 is significantly constrained from above.

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1 Introduction

In supersymmetric grand unified theories the particle spectrum at the unification scale affects significantly the unification of the three gauge coupling constants [1]: in most models the unification sector contains too many unknown parameters so that it is not possible to extract precise constraints on the low energy physics from this condition.

We encounter a similar situation when studying the bounds coming from proton decay: in this case the mass of the heavy Higgs triplets that mediate the decay may be easily increased weakening the resulting bounds on the low energy physics.

In this paper, we analyse the bounds coming from the requirement of coupling constant unification and from proton decay in minimal SU(5) supersymmetric grand unification, supplemented with the requirement that the Higgs doublets be identically massless in the limit of unbroken supersymmetry. We consider the cases when this is enforced by a fine tuning [2] or is achieved by a pseudo-Goldstone mechanism [3].

We find that:

i) the model is never compatible with supersymmetry breaking mass parameters lower than 1 TeV unless some of them are very unnaturally tuned;

ii) accepting supersymmetry breaking mass parameters higher than 1 TeV, the strong coupling constant α_3 is significantly constrained from above.

Minimal SU(5) grand unification [2] is problematic in its predictions for the fermion masses of the first two generations. Furthermore, to explain the observed baryon asymmetry, in view of the B-violating sphaleron induced processes [4], a rather unnatural initial condition at temperatures above the unification scale may be required. We assume that the necessary modifications of the theory do not alter in a significant way the analysis of this work. This will have to be checked in any given extension of the theory itself.

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In the minimal SU(5) model [2] the Higgs sector of the superpotential contains 4 unknown parameters:

$$\frac{\lambda}{3}\,\mathrm{Tr}\,\Sigma^3 + \frac{M}{2}\,\mathrm{Tr}\,\Sigma^2 + \lambda'\,\bar{H}\Sigma H + M'\bar{H}H$$

where $\Sigma, H, \bar{H}$ are chiral supermultiplets in $24, 5, \bar{5}$ representation of SU(5). In order to get light Higgs doublets we have to fine tune the parameters as follows

$$M' = 3M\frac{\lambda'}{\lambda} + m \tag{1}$$

where M and M' are of order of the unification scale and m is the Higgs doublet supersymmetric mass, which must be of order of the weak scale. The superheavy particle spectrum is made of:

a) the massive vector supermultiplet, containing the usual SU(5) heavy vector bosons, X and Y, of mass $M_V = 5\sqrt{2}g_5 M/\lambda$, where g_5 denotes the unification gauge coupling constant;

b) the two Higgs colour triplet supermultiplets in H and $\bar{H}$, of mass $M_H = 5M\lambda'/\lambda$;

c) the colour octet and the SU(2) triplet in Σ with mass $M_\Sigma = \frac{5}{2}M$.

d) the SU(3) $\otimes$ SU(2) $\otimes$ U(1) singlet in Σ, with mass of order M, that does not play any role in the unification of the gauge couplings at one loop order.

The condition that the dimensionless couplings λ and λ' do not blow up below the Planck scale results in the constraints [5]

$$M_H < 2.0 M_V \qquad M_\Sigma < 1.8 M_V \tag{2}$$

In the following we will use the weaker bounds $M_H, M_\Sigma < 3M_V$.

Only the two combinations M_H and $M_V^2 M_\Sigma$ of the three masses enter the unification condition of the gauge coupling constants. We choose to write the two equations as [5,6]

$$\begin{aligned}\ln\frac{M_H}{M_Z} &= -\frac{\pi}{6}\left(-\frac{5}{\alpha_1}+\frac{15}{\alpha_2}-\frac{10}{\alpha_3}\right) - H_2 - \frac{5}{3}\ln\frac{M_3}{M_2} + \\ &\quad +\frac{1}{6}\ln\frac{m_H}{M_Z} + \frac{2}{3}\ln\frac{m_{\tilde{H}}}{M_Z} + \frac{1}{12}\ln\frac{m_{\tilde{Q}}^4 m_{\tilde{L}}^2}{m_{\tilde{u}}^3 m_{\tilde{d}}^2 m_{\tilde{e}}}\end{aligned} \tag{3}$$

$$\begin{aligned}\ln\frac{M_V}{M_H} &= \frac{\pi}{3}\left(\frac{5}{\alpha_1}-\frac{12}{\alpha_2}+\frac{7}{\alpha_3}\right) + (V_2 - H_2) + \frac{1}{2}\ln\frac{M_H}{M_\Sigma} + \\ &\quad -\frac{8}{3}\ln\frac{M_2}{M_Z} + \frac{7}{3}\ln\frac{M_3}{M_Z} - \frac{1}{4}\ln\frac{m_H}{M_Z} - \ln\frac{m_{\tilde{H}}}{M_Z} - \frac{1}{4}\ln\frac{m_{\tilde{Q}}^7 m_{\tilde{L}}^3}{m_{\tilde{u}}^5 m_{\tilde{d}}^3 m_{\tilde{e}}^2}\end{aligned} \tag{4}$$

where the gauge coupling constants are measured at the M_Z pole and it is understood that all masses lighter than M_Z must be substituted with M_Z itself. The symbols M_3 and M_2 represent the masses of the SU(3) and SU(2) gauginos, respectively; we neglect their mixing with the Higgsinos, supposed to make two degenered doublets of mass $m_{\tilde{H}}$. The four real Higgs scalar fields heavier than M_Z are assumed to make an almost degenerate doublet with mass m_H. We will justify all these assumptions in the following. The squarks and sleptons masses are denoted by $m_{\tilde{Q}}, m_{\tilde{u}}, m_{\tilde{d}}, m_{\tilde{L}}, m_{\tilde{e}}$. Finally $H_2 \approx -3.9$ and $V_2 \approx 2.2$ represent the two loop contributions.

The proton mean life can be written as [5,7]

$$\tau(p \to K^+\bar{\nu}_\mu) = 6.2\ 10^{30}\ \mathrm{yr} \times \left| x\frac{M_H}{10^{17}\ \mathrm{GeV}}\frac{\mathrm{TeV}^{-1}}{f(u,d)+f(u,e)}\right|^2 \qquad x \equiv \frac{0.01\,\mathrm{GeV}^3}{\beta_p}\frac{0.67}{A_S}\frac{\sin 2\beta}{|1+y^{tK}|} \tag{5}$$

where β_p is a suitably normalized matrix element between the proton and the vacuum of the relevant three quark operator, which is calculated to be in the range $\beta_p = (0.003 \div 0.03)\,\mathrm{GeV}^3$; $\tan\beta$ is the ratio of

M_H/M_Σ	M_H	M_V	m_0	μ_0
1	$0.5\,10^{16}$ GeV	$2.5\,10^{16}$ GeV	11 TeV	16 TeV
5	$1.0\,10^{16}$ GeV	$3.4\,10^{16}$ GeV	7 TeV	50 TeV
50	$3.4\,10^{16}$ GeV	$5.6\,10^{16}$ GeV	3.4 TeV	260 TeV
500	$11\,10^{16}$ GeV	$10\,10^{16}$ GeV	1.6 TeV	1200 TeV
5000	$40\,10^{16}$ GeV	$20\,10^{16}$ GeV	0.8 TeV	6000 TeV

Table 1: Possible spectra for $\alpha_3 = 0.11$, $s^2 = 0.2333$, $m_{1/2} = 100\,\mathrm{GeV}$, $A - B = 1$ and $x = 1$. m_0 is the lightest value compatible with p-decay.

the two Higgs vacuum expectation values; A_S is the top quark dependent renormalization effect between the unification scale and the supersymmetry breaking scales, f is a rather lengthy function of the squarks and slepton masses and y^{tK} is the suitably normalized third generation contribution to the decay, ranging in the interval $|y^{tK}| = 0.04 \div 1.7$. The 90% confidence limit is $\tau(p \to K^+\bar{\nu}) > 1.0\,10^{32}$ yr [8]. The kind of information we get from this bound is a lower limit on $m_S M_H$ where m_S is a certain combination of the supersymmetric particles masses described by the function f.

How does this limit combine with the unification conditions (3) and (4)? We are especially interested in an upper bound on M_H, since that would turn into a lower bound on the supersymmetric particles. An information on M_H can be inferred from (3), which is rather sensitive to the values of gauge coupling constants. If we take the largest 1σ values of α_3 and s^2 [9,10,11] the unification is achieved with the largest possible mass of the Higgs triplet $M_H \approx 5 \cdot 10^{17}\,\mathrm{GeV}$. The bound $M_H < 3M_V$ can be automatically satisfied, choosing a low M_Σ, since only $M_\Sigma M_V^2$ is fixed by equation (4). Thus, clearly, the present nucleon decay limit, even if near to the actual rate, only weakly constrains the low energy physics.

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We shall now try to improve on the situation discussed in the previous section, by focusing on the naturalness problem of the light Higgs masses. More specifically we require that these masses vanish in the supersymmetric limit, in such a way that the Fermi scale (or the W mass) be related to the supersymmetry breaking scale. In the context of standard minimal SU(5) [2] this amounts to set $m = 0$ in (1). In this way, of course, the masslessness of the Higgs bosons in the supersymmetric limit is only achivied by a (stable) fine tuning.

A more attractive possibility is that the Higgs doublets are massless because they are pseudo-Goldstone particles of a spontaneously broken SU(6) global symmetry of the Higgs sector [3]. In this case the SU(6) symmetry gives the additional constraint $M_H = M_\Sigma$ [12]. We again quantify the condition that the dimensionless coupling does not blow up below the Planck scale as $M_H < 3M_V$. In this case, due to $M_H = M_V$, the unification of the gauge coupling constants (3) and (4) is sufficient to fix both M_H and M_V in terms of the low energy physics, so that this bound can be used to constrain the theory.

The main consequences of this new constraint are:

- high values of α_3 and s^2 that bring to a too high M_H are forbidden;
- for the remaining lower values of α_3 and s^2, M_H cannot reach its maximum, so that the lower bound on the supersymmetric particles masses is increased.

Before discussing in detail these implications of the new constraint, we now show that they hold even in the minimal SU(5) model with $m = 0$.

If we assume that supersymmetry breaking arise from supergravity couplings [13], the tree level potential at the Planck scale, in terms of the superpotential W, is

$$V = \sum_i \left[\left| \frac{\partial W}{\partial z_i} \right|^2 + m_0^2 |z_i|^2 \right] + m_0 [A W^{(3)} + B W^{(2)} + \mathrm{h.c.}]$$

where z_i is any matter field, and the gauginos receive a common mass $m_{1/2}$. By minimizing the potential V, the SU(5)-breaking vacuum expectation value of the Σ field gets shifted from the value it has in the supersymmetric limit. As a consequence a mass term of effective supersymmetric form, $W^{(2)} = -\mu_0 \bar{H} H$, is induced for the Higgs doublets. In the minimal SU(5) model, with $m = 0$, we find

$$\mu_0 = \frac{3}{2}\frac{M_H}{M_\Sigma} m_0 (A - B), \tag{6}$$

whereas in the SU(5) model with Higgs as pseudo-Goldstones $\mu_0 = m_0(A - B)$.

We now discuss the impact of this relation. For each value of the unknown ratio M_H/M_Σ we obtain, in the minimal SU(5) model, bounds similar to those we find in pseudo-Goldstone model, where $M_H = M_\Sigma$. We now show that as a consequence of the relation (6), both the proton decay and this new condition give weaker bounds on the low energy physics when M_H/M_Σ is minimum.

About the proton decay bound, we observe that to reduce the bound on the 'mean' supersymmetric mass m_S, requires increasing M_H/M_Σ. In this way, however, we may reduce the masses of the squarks and the sleptons, but only at the price of increasing μ_0, which, in turn, increases the mass of Higgs scalars and Higgsinos giving a more unnatural spectrum. Infact the Higgs doublets potential contains masses of order μ_0 that is increased by this operation. In any case, $\mu_0 \gg m_0$ must be avoided because the resulting Higgs doublet potential is unsuitable for the SU(2) radiative breaking and because the third generation $\tilde{Q}$ and $\tilde{u}$ squared masses are easily drived to be negative even by low values of the top quark Yukawa coupling [14].

Even the $M_H < 3M_V$ bound becomes stronger when increasing M_H/M_Σ, because, due to the $m_{\tilde{H}}$ and m_H terms in (4), of order μ_0, M_H is increased more rapidly than M_V.

We give an example of these assertions in table 1, in which even the most natural spectrum has $m_0 \approx \mu_0 \approx 10\,\mathrm{TeV}$.

4

We now discuss the bounds on the coupling constants. In both models under discussion, α_3 and s^2 can take their maximum allowed values when $M_H = 3M_V$. In the minimal SU(5) model we have the additional freedom of varying M_Σ: the weakest bound is obtained when M_Σ is maximum. Since M_Σ has the same upper limit than M_H, in both the models, the bounds are weaker when $M_H = M_\Sigma = 3M_V$. At this point, we have only to find the values of the remaining free parameters $m_0, m_{1/2}, A, B, m_t$ that allow the gauge coupling constants to take their maximum possible values.

We restrict the analysis to the region $m_0 \gg m_{1/2}$ in which the p-decay bound allows the lighter spectrum. In this case the desired bound takes a very simple form: the unification conditions approximately depend on A, B, m_t only through the combination

$$\ell = \ln \frac{m_{\tilde{H}}}{m_0} + \frac{1}{4} \ln \frac{m_H}{m_0} \tag{7}$$

since all the supersymmetric particles other than the Higgs and the Higgsinos have masses largely independent of A, B, m_t [14], with the exception of the third generation squarks that, however, occur with small coefficients in the evolution equations. An investigation of the evolution equations shows that ℓ is only weakly m_0 and $m_{1/2}$ dependent.

Imposing the $M_H = M_\Sigma = 3M_V$ bound we get from (4) an upper bound on m_0:

$$\ln \frac{m_0}{M_Z} = \frac{4\pi}{15}(5\alpha_1^{-1} - 12\alpha_2^{-1} + 7\alpha_3^{-1}) + \frac{4}{5}(V_2 - H_2 + \ln 3 - \ell) + \frac{28}{15} \ln^* \frac{M_3}{M_Z} - \frac{32}{15} \ln^* \frac{M_2}{M_Z} \tag{8}$$

where $\ln^* x = \ln x$ for $x > 1$ and $\ln^* x = 0$ for $x \leq 1$. In this region the proton decay limit may be written as

$$\ln \frac{x M_H}{M_Z} > 40.0 - 2 \ln \frac{m_0}{M_Z} + \ln \left[\frac{M_2}{M_Z} \ln \frac{m_0}{M_2} \right] \tag{9}$$

with M_H given by (3) and x defined in (5).

In figure 1 we show these two constraints for different α_3 values in the $(m_{1/2}, m_0)$ plane: we see that increasing α_3 the $M_H = M_\Sigma = 3M_V$ bound becomes quickly stronger until it requires that the supersymmetric particles are so light that the proton decay becomes too fast. The maximum α_3 and the minimum m_0 are obtained when the two bounds cross each other and M_2 is as light as possible.

Inserting the m_0 value (8) in the proton decay bound (9) we get the constraint on the gauge coupling constants

$$\pi\left(\frac{56}{3\alpha_1} - \frac{197}{10\alpha_2} + \frac{163}{15\alpha_3}\right) + \frac{163}{15}\ln^* \frac{M_3}{M_Z} - \frac{197}{15}\ln^* \frac{M_2}{M_Z} - 3\ln\left[\frac{M_2}{M_Z}\ln\frac{m_0}{M_2}\right] > \frac{24}{5}\ell + 83. - 3\ln x$$

This constraint is shown in figure 2, and compared with the present values of α_3 and s^2 for $\alpha_{\rm em}^{-1} = 127.9$ [9,10,11], $x = 1$, M_2 taking its lower experimental value $M_2 = 45\,\mathrm{GeV}$ [15], and different values of ℓ, the only remaining free parameter. The bound is weaker for negative ℓ values that can be obtained by fine tuning $A - B \ll 1$, corresponding to an Higgsino lighter than m_0. At these limiting values of the gauge coupling constants, m_0 takes its lowest possible values in the model, always higher than 3 TeV as shown in figure 3. Since m_0 and μ_0 are always much greater than the Z mass, the mixings following from the SU(2) breaking are negligible, as we have assumed in section 2.

In figure 4 we show how the bound on α_3 gets enforced choosing M_2 values higher than the lower experimental limit.

It is possible that a second region in which $m_{1/2} \gg m_0$ survives for even higher values of α_3 and s^2. In this region also the squarks, the sleptons and the Higgses acquire a mass of order $m_{1/2}$ due to radiative corrections. We do not consider this possibility since $m_{1/2}$ is always required to be heavier than 50 TeV.

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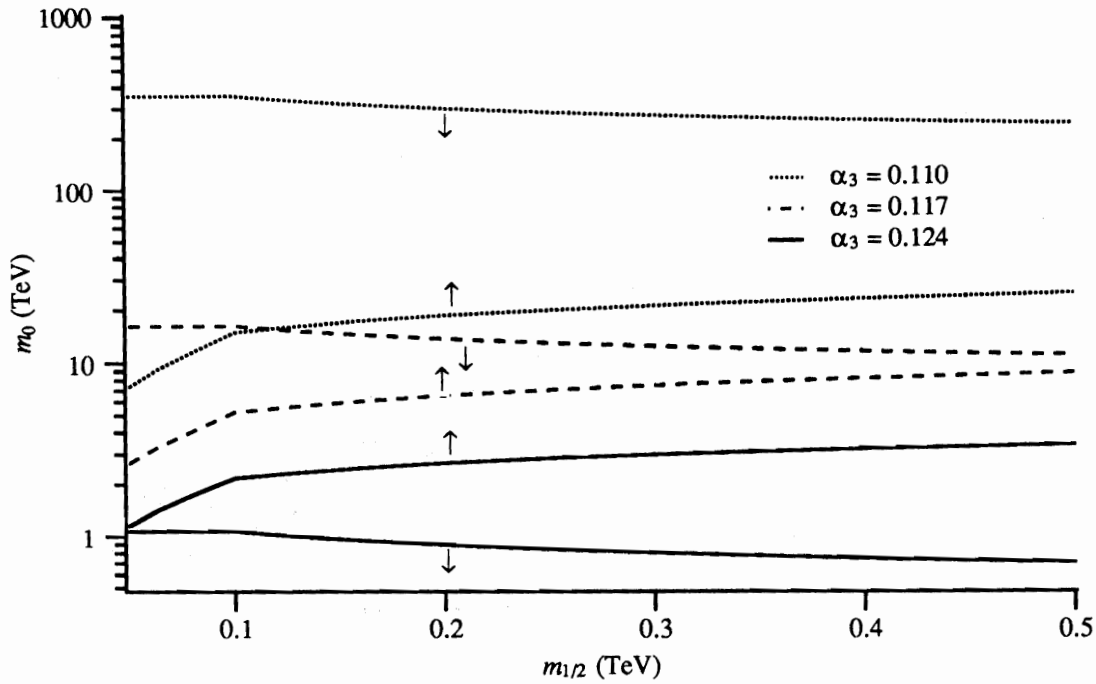

Figure 1: The limit $M_H < 3M_V$ (down arrows) and the limit from p-decay (up arrows) for different α_3 values and $s^2 = 0.233$, $\ell = 0$, $M_H = M_\Sigma$, $x = 1$.

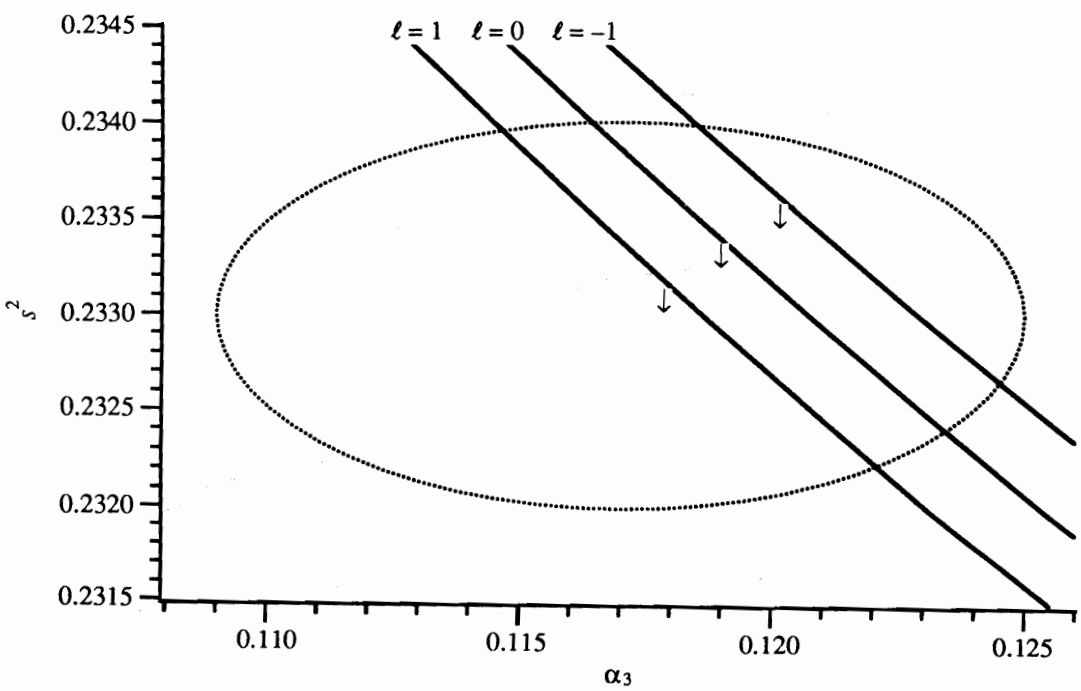

Figure 2: Bounds on α_3 and s^2.

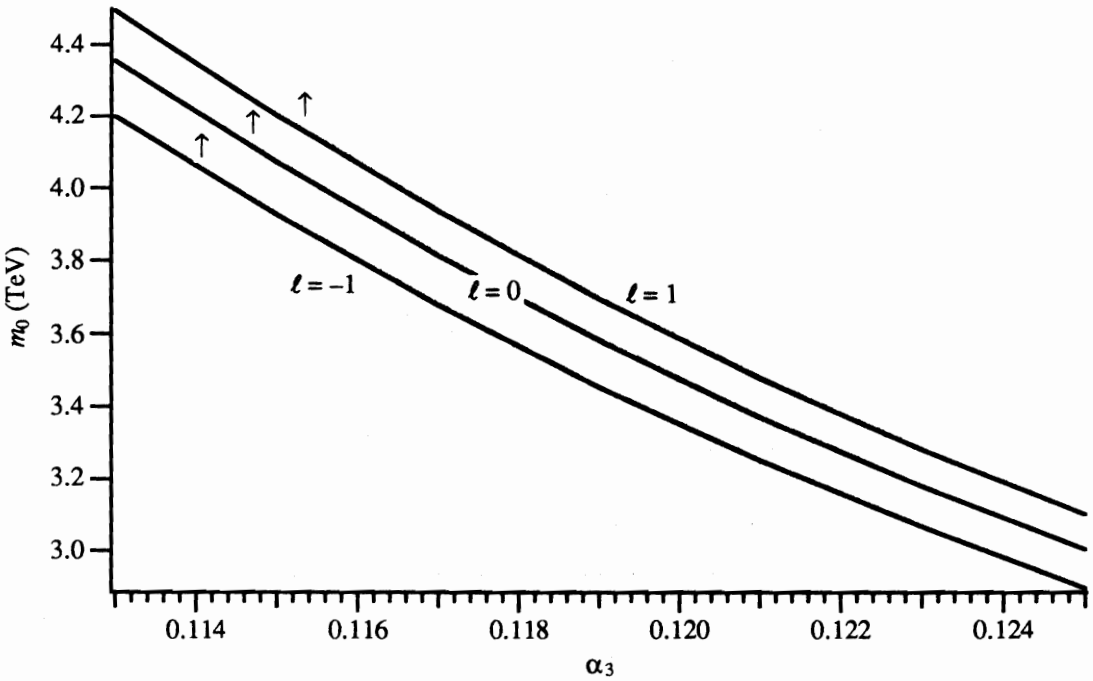

Figure 3: The lower values of m_0 for different ℓ-values as functions of α_3.

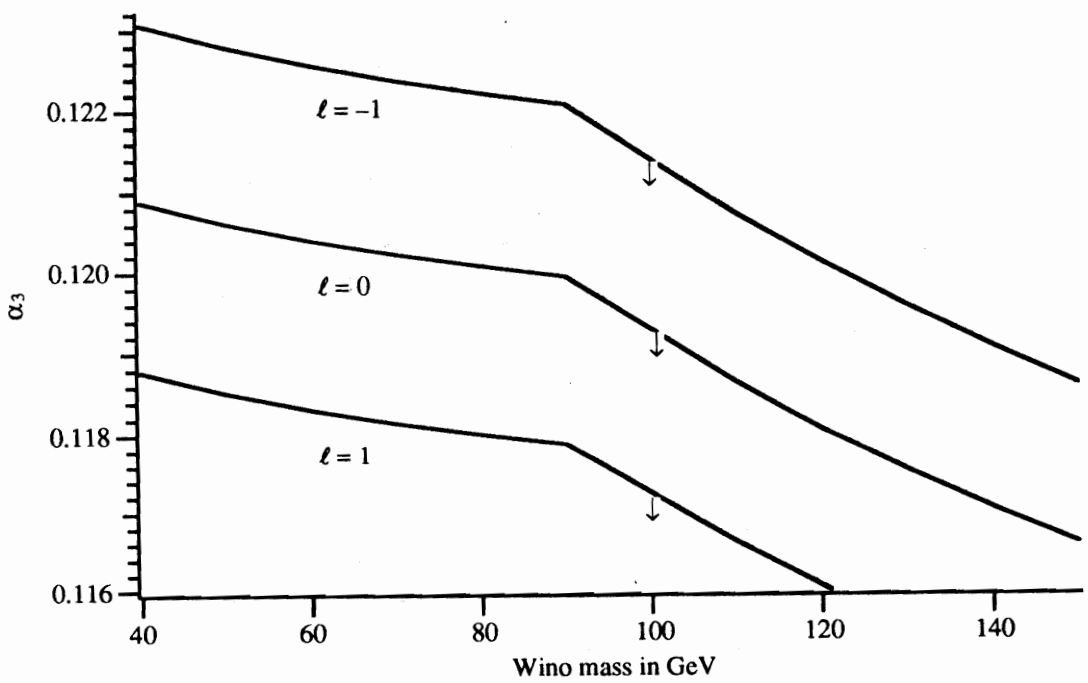

Figure 4: Upper bounds on α_3 for different ℓ-values as functions of the chargino mass M_2 ($s^2 = 0.233$).