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IFUP-TH 49/94
August 1994
C94/07/24.1

Black Holes as Quantum Membranes

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Abstract. We describe a proposal for a quantum description of black holes. Defining a path integral in the presence of a black hole only in terms of variables outside the horizon, and performing a coarse graining over short distances, we obtain an effective lagrangian describing the quantum dynamics of black holes. The leading terms are given by the boundary term in the gravitational action and by a membrane action. We show that the approach correctly reproduces the temperature and entropy, and we write down the Schroedinger equation for the black hole horizon.

Talk given at the Seventh Marcel Grossmann Meeting, Stanford, July 25-29, 1994. To appear in the Proceedings.

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ABSTRACT

We describe a proposal for a quantum description of black holes. Defining a path integral in the presence of a black hole only in terms of variables outside the horizon, and performing a coarse graining over short distances, we obtain an effective lagrangian describing the quantum dynamics of black holes. The leading terms are given by the boundary term in the gravitational action and by a membrane action. We show that the approach correctly reproduces the temperature and entropy, and we write down the Schroedinger equation for the black hole horizon.

1. Introduction

In this talk we describe a proposal for a quantitative description of quantum black holes. The general framework is provided by ideas of 't Hooft[1] and Susskind and coworkers[2]. An important consequence of this approach, named by Susskind "black hole complementarity", is the following. We are used to think that the horizon of a black hole is *not* a special place; it is only a coordinate singularity, while invariant quantities remain regular there. This is certainly true classically; however a fiducial observer (i.e., an observer static with respect to the black hole) cannot consistently use classical concepts if he is sufficiently close to the horizon; this is indicated by the fact that the local Hawking temperature measured by a fiducial observer, $T = 1/(8\pi M \sqrt{g_{00}})$, diverges on the classical horizon. A fiducial observer located at one Planck length from the horizon is in a region of Planckian temperatures and energies where quantum gravity effects are expected to be dominant. Within the principle of black hole complementarity one therefore holds the point of view that, for a fiducial observer, the horizon *is* a special place, and it is in fact the place where the quantum degrees of freedom of the black hole are to be found. Our aim is to develop a computational scheme which is inspired by this general framework.

2. Defining the path integral in the presence of a horizon

Let us first state in intuitive terms the physical idea that we would like to implement. From the point of view of a fiducial observer, only field variables outside the horizon should influence the dynamics. Thus, in a path integral, we would like to define an integration over the field variables $g_{\mu\nu}(x)$ restricted by the condition that x is outside the horizon. This is a very complicated and non-linear condition, since, as the field $g_{\mu\nu}$ fluctuates, the horizon fluctuates and therefore even the number of variables which we would like to use as integration variables fluctuates. (In a sense, this reminds of a grancanonical, rather than a canonical ensemble.) A formal implementation of this intuitive idea can be obtained as follows. Let us consider

the quantum, fluctuating, horizon, and let us draw an imaginary spherical surface around it. We place this surface at a distance from the nominal horizon such that the typical quantum fluctuations of the horizon are contained in it. A few Planck lengths (or possibly string lengths) will suffice. Let us call Ω the region of spacetime outside this surface, and $\partial\Omega$ the surface itself. If x belongs to Ω we insert $g_{\mu\nu}(x)$ as an integration variable in the path integral. The question is what to do in the small shell between the fluctuating horizon and $\partial\Omega$.

In general, when evaluating a path integral in field theory, a crucial point is the identification of the physically relevant variables, as opposed to fast varying variables which can be integrated out. The most well known examples of this come from *QCD*, for instance considering the dynamics of flux tubes of chromoelectric field, which is governed by an effective string theory, or considering chiral lagrangians governing the dynamics of hadrons. In our case it is natural to identify the "collective variables" which play the main role with the coordinates $\zeta^\mu(\theta,\phi,t)$ which define the position of the horizon. Of course, the variables ζ^μ are uniquely fixed by the metric $g_{\mu\nu}$. So, the integration variables $g_{\mu\nu}(x)$, restricted by the condition that x lies between the horizon, as determined by $g_{\mu\nu}$ itself, and the surface $\partial\Omega$, are decomposed into the physically relevant variables ζ^μ plus "fast variables". The effective action for the ζ^μ is obtained integrating over the fast variables. As a result of this integration, one gets a partition function of the form[3]

$$Z = \int \mathcal{D}\zeta^\mu \int_\Omega \mathcal{D}g_{\mu\nu}\, e^{iI}\,, \tag{1}$$

where the action I is the sum of a membrane action $I_{\rm memb}$ and of the gravitational action $I_{\rm grav}$, which also includes the boundary term on $\partial\Omega$. Furthermore, in a first approximation one can neglect the fluctuations of the metric within Ω, since the most important quantum fluctuations are the ones near the horizon, which have been taken into account by the renormalization group procedure. Thus (after rotating the membrane theory to Euclidean space), we get

$$Z \simeq \left(e^{-I_{\rm grav}} \int \mathcal{D}\zeta^\mu\, e^{-I_{\rm memb}}\right)\Big|_{g_{\mu\nu}=g^{\rm cl}_{\mu\nu}}\,. \tag{2}$$

In $I_{\rm grav}$ only the surface term survives. Having an expression for the partition function, we can now compute a number of quantities.

3. A summary of results

3.1. Temperature

To study field theory at finite temperature T, we restrict the integration in the partition function to Euclidean field configurations periodic in time with period $\beta = 1/T$. In Rindler space, with acceleration g, we find[3] a periodic solution of the membrane equations of motion with period $\beta = 2\pi/g$, so that the correct temperature $T = g/2\pi$ is reproduced. This result is highly non-trivial, and is an independent derivation of the Unruh temperature –as well as a strong check of the

method that we propose. For Schwarzschild metric we find that, if the black hole mass M is larger than a critical value, there is a periodic solution of the membrane equations of motion just with the right period, so that the Hawking temperature $T = 1/(8\pi M)$ is correctly reproduced –again, a rather non-trivial result. For masses below a critical value a qualitatively new phenomenon takes place; there is a phase transition, the specific heat becomes positive and the temperature goes to zero at some value of the mass around the Planck mass. Whether this intriguing result is physically meaningful or it is just a signal of the breakdown of our approximations is a matter for further investigations.

3.2. Entropy

Having fixed the temperature to the correct value, we can evaluate the partition function Z, the free energy and the entropy. The leading term comes from the boundary term in the gravitational action, and reproduces the correct result for the black hole entropy, $S = \text{area}/4$. The membrane action gives one-loop corrections, which are divergent and proportional to the area.

3.3. Wave function and Schroedinger equation for the horizon

We can quantize the membrane action, restricting ourselves to spherically simmetric membranes. Since we are retaining only the radial mode the problem is reduced to a quantum mechanical one and we can write down the Schroedinger equation for the membrane wavefunction[4]. We find a quasi-continuum of energy levels, and a wave-function which oscillates near the horizon and decreases exponentially at large distances. In Rindler space the dynamics of the horizon turns out to be equivalent to Liouville quantum mechanics.

3.4. 2+1 black hole

The same analysis can be repeated for the 2+1 dimensional black hole. In this case instead of a membrane we have a string; again the temperature and entropy are correctly reproduced by our approach[5].

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