IFUP-TH-43/92
Fermilab Library
0 1160 0027832 9

IFUP-TH-43/93

10

2+1 GRAVITY AND CLOSED TIME-LIKE CURVES*

P. Menotti
Dipartimento di Fisica dell' Università, Pisa 56100, Italy and
INFN, Sezione di Pisa

and

D. Seminara
Scuola Normale Superiore, Pisa 56100, Italy and
INFN, Sezione di Pisa

Talk given at the Workshop on "Constraint Theory and Quantization Methods"
Montepulciano, Italy, June 1993

*Work partially supported by M.U.R.S.T

IFUP-TH-43/93

2+1 GRAVITY AND CLOSED TIME-LIKE CURVES*

P. Menotti
Dipartimento di Fisica dell' Università, Pisa 56100, Italy and
INFN, Sezione di Pisa

and

D. Seminara
Scuola Normale Superiore, Pisa 56100, Italy and
INFN, Sezione di Pisa

Talk given at the Workshop on "Constraint Theory and Quantization Methods"
Montepulciano, Italy, June 1993

*Work partially supported by M.U.R.S.T

ABSTRACT

In this paper we report some results obtained by applying the radial gauge to 2+1 dimensional gravity. The general features of this gauge are reviewed and it is shown how they allow the general solution of the problem in terms of simple quadratures. Then we concentrate on the general stationary problem providing the explicit solving formulas for the metric and the explicit support conditions for the energy momentum tensor. The chosen gauge allows, due to its physical nature, to exploit the weak energy condition and in this connection it is proved that for an open universe conical at space infinity the weak energy condition and the absence of closed time like curves (CTC) at space infinity imply the total absence of CTC. It is pointed out how the approach can be used to examine cosmological solution in 2+1 dimensions.

1. Introduction

Gravity in 2+1 dimensions[1] turned out to be a good theoretical laboratory both at the classical and at the quantum level. In addition to be interesting in itself, the theory is important in connections to the cosmic strings[2], as all solutions in 2+1 dimensions are special solution of 3+1 dimensional gravity.

Most attention has been devoted in the past to point like or string like sources and to stationary problems, even though some inroads[3,4] have been made in the realm of the time dependent problem.

It has been shown[3,5,6,7,8] that a special choice of gauge allows to give general resolvent formulas for the metric in terms of simple quadratures both in the case of time dependent and extended sources. The main reason is the practical identification in 2+1 dimensions of the Riemann and Ricci tensors which allows to reformulate the problem as the solution of the covariant conservation and symmetry constraints on the energy momentum tensor. The procedure of solution is such that one has a complete control on the support properties of the energy momentum tensor; still more important is the fact that due to the physical nature of the gauge, one is able to exploit the weak energy condition (WEC) without the imposition of which, Einstein's equations loose most of their content. The possibility of exploiting the WEC will be instrumental in the problem of the occurrence of closed time-like curves (CTC).

In this paper we shall give a brief survey of the techniques and the results obtained by exploiting the radial gauge, referring for details to ref. (3),(5),(6),(7) and (8).

As in connection with the problem of CTC we shall be mainly interested in the stationary case, we shall report and discuss in sec.2 in more detail the resolvent formulae and the support conditions for the stationary case, which will be dealt with by developing a variant of the general radial gauge, i.e. the reduced radial gauge which is more apt to the time independent situation.

Turning to the problem of CTC [9], in sect.3 we shall prove the following result[6,7]: for a stationary solution with rotational symmetry the imposition of i) the weak energy condition (WEC) and ii) the absence of CTC at space infinity prevents the occurrence of CTC everywhere in an open (conical) universe.

An extension is given of the same result to any stationary solution, also in absence of rotational symmetry, provided that in our coordinate system the determinant of the dreibein never vanishes[8].

2. General solution in the radial gauge

The radial gauge, which can be defined in any space-time dimensions[10], presents particular features in 2+1 dimensions due to the practical identification of the Riemann and Ricci tensors. The defining equations are

$$\xi^\mu \Gamma^a_{b\mu} = 0, \tag{1}$$

$$\xi^\mu e^a_\mu = \delta^a_\mu \xi^\mu. \tag{2}$$

These conditions define the usual Riemann-normal coordinates on the manifold and in this gauge one can express the connection and the vierbein in terms of the Riemann two-form as follows

$$\Gamma^a_{b\mu}(\xi) = \xi^\rho \int_0^1 R^a_{b\rho\mu}(\lambda\xi)\lambda d\lambda \tag{3}$$

$$e^a_\mu(\xi) = \delta^a_\mu + \xi^\rho \xi^b \int_0^1 R^a_{b\rho\mu}(\lambda\xi)\lambda(1-\lambda)d\lambda. \tag{4}$$

As in 2+1 dimensions the Riemann two-form through Einstein's equations is directly given in term of the energy momentum form Eqs. 3 and 4 express the geometry of the space in term of the sources through a simple quadrature. On the other hand the energy momentum form is not arbitrary but it is subject to the symmetry and covariant conservation conditions which are nothing else than Bianchi identities. Thus in the present approach the problem is reduced to constructing the most general energy momentum form which is symmetric and covariantly conserved. The solution of such constraints can be given through a simple quadrature[3]. For the stationary problem, with which we shall be mainly concerned here, the radial gauge as formulated above is in general not apt due to the fact that it singles out a special event in space time. One can however, for stationary problems, define a similar gauge which we shall call reduced radial gauge, through the conditions (in the following i, j, l run over space indices)

$$\xi^i \Gamma^a_{bi}(\boldsymbol{\xi}) = 0 \tag{5}$$

$$\xi^i e^a_i(\boldsymbol{\xi}) = \xi^i \delta^a_i. \tag{6}$$

This gauge has a natural interpretation as the reference frame of an observer which follows an integral curve of the (time-like) Killing field. It corresponds to the Fermi-Walker coordinates[11,5,7]. The resolving formulae analogous to Eqs. 3 and 4 are

$$\Gamma^a_{bi}(\boldsymbol{\xi}) = \xi^j \int_0^1 R^a_{bji}(\lambda\boldsymbol{\xi})\lambda d\lambda, \tag{7}$$

$$\Gamma^a_{b0}(\boldsymbol{\xi}) = \Gamma^a_{b0}(\mathbf{0}) + \xi^i \int_0^1 R^a_{bi0}(\lambda\boldsymbol{\xi})d\lambda, \tag{8}$$

$$e_i^a = \delta_i^a + \xi^j \xi^l \int_0^1 R^a_{jli}(\lambda\boldsymbol{\xi})\lambda(1-\lambda)d\lambda, \tag{9}$$

$$e_0^a = \delta_0^a + \xi^i \Gamma^a_{i0}(\mathbf{0}) + \xi^i \xi^j \int_0^1 R^a_{ij0}(\lambda\boldsymbol{\xi})(1-\lambda)d\lambda. \tag{10}$$

In 2+1 dimensions the Riemann two-form appearing in the previous equations is given in terms of the energy momentum form T_c by

$$\varepsilon_{abc} R^{ab} = -2\kappa T_c, \tag{11}$$

where $\kappa = 8\pi G$, and thus

$$R^{ab} = -\kappa \varepsilon^{abc} T_c = -\frac{\kappa}{2} \varepsilon^{abc}\ \varepsilon_{\rho\mu\nu} \tau_c{}^{\rho} dx^\mu \wedge dx^\nu. \tag{12}$$

Using such a relation one can express through a simple quadrature, the connections and the vierbeins in terms of the energy momentum tensor, which is the source of the gravitational field and thus one solves Einstein's equation. We come now to the covariant conservation and symmetry constraints on the energy momentum tensor. The problem is to construct the general conserved symmetric energy momentum tensor in the reduced radial gauge, which in addition should satisfy other physical requirements given by the support of the sources and the restrictions due to the energy condition[12].

The conservation and symmetry equations for the energy momentum tensor are

$$\mathcal{D}T^a = 0, \tag{13}$$

$$\varepsilon_{abc} T^b \wedge e^c = 0. \tag{14}$$

The most general solution of Eq. 13 is[7]

$$\begin{aligned} \tau_c^\rho(\boldsymbol{\xi}) \quad &= \frac{1}{\kappa}\Big[P^\mu \partial_\mu A_c^\rho(\boldsymbol{\xi}) - \frac{1}{\rho} A_c^\rho(\boldsymbol{\xi}) - \frac{1}{\rho}\Theta^\rho \Theta_\mu A_c^\mu(\boldsymbol{\xi}) - P^\rho\Big(\partial_\mu A_c^\mu(\boldsymbol{\xi}) - \\ &- \frac{1}{2}\varepsilon_{clm}\varepsilon_{\alpha\beta\sigma} P^\alpha A^{l\beta}(\boldsymbol{\xi}) A^{m\sigma}(\boldsymbol{\xi})\Big)\Big], \end{aligned} \tag{15}$$

where A_c^μ is an arbitrary field. The field A_c^μ is related to the connection Γ_μ^{ab} in the reduced radial gauge by

$$\Gamma_\mu^{ab}(\boldsymbol{\xi}) = \varepsilon^{abc} \varepsilon_{\mu\rho\nu} P^\rho A_c^\nu(\boldsymbol{\xi}). \tag{16}$$

More demanding is the imposition of the symmetry property Eq. 14 which however can be solved as follows. One express $A_c^\mu(\boldsymbol{\xi})$ in component form

$$A_c^\rho(\boldsymbol{\xi}) = T_c\left[\Theta^\rho \beta_1 + T^\rho \frac{(\beta_2 - 1)}{\rho}\right] + \Theta_c\left[\Theta^\rho \alpha_1 + T^\rho \frac{\alpha_2}{\rho}\right] + P_c\left[\Theta^\rho \gamma_1 + T^\rho \frac{\gamma_2}{\rho}\right]. \tag{17}$$

where $T_\mu = \frac{\partial \xi^0}{\partial \xi^\mu}$, $P_\mu = \frac{\partial \rho}{\partial \xi^\mu}$ and $\Theta_\mu = \rho \frac{\partial \theta}{\partial \xi^\mu}$ are the cotangent vectors defined by the polar variables in the (ξ^1, ξ^2) plane. This gives the following expression for $\tau^{a\rho}$

$$\tau_c^\rho = -\frac{1}{\kappa}\Big\{T_c\Big(T^\rho \frac{\beta_2'}{\rho} + \Theta^\rho \beta_1'\Big) + \Theta_c\Big(T^\rho \frac{\alpha_2'}{\rho} + \Theta^\rho \alpha_1'\Big) + P_c\Big(T^\rho \frac{\gamma_2'}{\rho} + \Theta^\rho \gamma_1'\Big) + \frac{1}{\rho}P^\rho\Big[T_c\Big(\alpha_1\gamma_2 - \alpha_2\gamma_1 - \frac{\partial \beta_1}{\partial \theta}\Big) + \Theta_c\Big(\beta_1\gamma_2 - \beta_2\gamma_1 - \frac{\partial \alpha_1}{\partial \theta}\Big) + P_c\Big(\alpha_1\beta_2 - \alpha_2\beta_1 - \frac{\partial \gamma_1}{\partial \theta}\Big)\Big]\Big\}. \tag{18}$$

Introducing the primitives of the functions $\alpha_1,\ \beta_1,\ \alpha_2,\ \beta_2$

$$A_1(\boldsymbol{\xi}) = \rho \int_0^1 \alpha_1(\lambda\boldsymbol{\xi})d\lambda - 1 \quad , \quad B_1(\boldsymbol{\xi}) = \rho \int_0^1 \beta_1(\lambda\boldsymbol{\xi})d\lambda,$$
$$A_2(\boldsymbol{\xi}) = \rho \int_0^1 \alpha_2(\lambda\boldsymbol{\xi})d\lambda \quad \text{and} \quad B_2(\boldsymbol{\xi}) = \rho \int_0^1 \beta_2(\lambda\boldsymbol{\xi})d\lambda, \tag{19}$$

the symmetry condition is reduced to the following system of differential equations

$$A_1\alpha_2 - A_2\alpha_1 + B_2\beta_1 - B_1\beta_2 = 0 \tag{20}$$
$$A_2\gamma_1 - A_1\gamma_2 + \frac{\partial B_1}{\partial \theta} = 0 \tag{21}$$
$$B_2\gamma_1 - B_1\gamma_2 + \frac{\partial A_1}{\partial \theta} = 0. \tag{22}$$

In general, in absence of rotational symmetry, caustics may develop in the sense that geodesics emerging from the origin with different θ can intersect at some point for large enough ρ. This renders the map of ρ, θ into the physical points of space not one to one, but the geometry can be still regular in the sense that a proper change of coordinates removes the singularity. For an example of how this non single valuedness can show up and how it can be removed by changing coordinates, we refer to the appendix of ref. (5). Such a problem does not arise in the case of rotational symmetry.

We recall furthermore that to give a regular geometry, the functions $\alpha_i,\ \beta_i,\ \gamma_i$ must satisfy simple regularity conditions at the origin[3,7].

Eqs. 20, 21, 22 give the whole geometry of the problem once three of the functions, e.g. $\alpha_1,\ \beta_1,\ \gamma_1$, are given as data; in fact the other three can be obtained by a single quadrature[7]. We have

$$\alpha_2 = \frac{B_1^2}{B_1^2 - A_1^2}\frac{\partial}{\partial \rho}\left(\frac{N}{B_1}\right) + 2\alpha_1 I. \tag{23}$$

$$\beta_2 = \frac{A_1^2}{B_1^2 - A_1^2}\frac{\partial}{\partial \rho}\left(\frac{N}{A_1}\right) + 2\beta_1 I, \tag{24}$$

$$\gamma_2 = \frac{B_1^2}{B_1^2 - A_1^2}\frac{\partial}{\partial\theta}\left(\frac{A_1}{B_1}\right) + 2\gamma_1 I, \tag{25}$$

where

$$N(\rho,\theta) \equiv A_2B_1 - A_1B_2 = \frac{1}{2\gamma_1}\frac{\partial}{\partial\theta}(A_1^2 - B_1^2) \tag{26}$$

and coincides with the determinant of the dreibein in polar coordinates, while I is given by

$$I = \int_0^\rho d\rho' \frac{N(A_1\beta_1 - B_1\alpha_1)}{(B_1^2 - A_1^2)^2}. \tag{27}$$

This parametrization of the source allows a simple characterization of the support properties of the energy momentum tensor. In fact one can prove[7] that if the energy momentum tensor vanishes for $\rho > \rho_0(\theta)$ one has

$$\alpha_1 B_1 - A_1\beta_1 = \text{constant} \quad \text{for} \quad \rho > \rho_0(\theta) \tag{28}$$

and

$$\alpha_1^2 - \beta_1^2 + \gamma_1^2 = \text{constant} \quad \text{for} \quad \rho > \rho_0(\theta), \tag{29}$$

where the two constants do not depend on ρ and θ. Viceversa Eqs. 28 and 29 impose that the support of $\tau^{a\rho}$ lies in $\rho < \rho_0(\theta)$.

In our formalism the metric assumes the form

$$ds^2 = (A_1^2 - B_1^2)dt^2 + 2(A_1A_2 - B_1B_2)dtd\theta + (A_2^2 - B_2^2)d\theta^2 - d\rho^2. \tag{30}$$

while the determinant of the dreibein in polar coordinates is given by

$$\det(e) = A_2B_1 - A_1B_2. \tag{31}$$

Even though we shall in the following be mainly interested in general case we want to report what happens in case of rotational symmetry. As derived in a previous work[3], in the case of rotational symmetry, all functions, as expected, do not depend on θ. Furthermore from the two last symmetry equations one obtains $\gamma_1 = \gamma_2 = 0$, under the assumption that determinant never vanishes. The regularity conditions at the origin for the functions α_i, β_i become

$$\alpha_1 = O(\rho), \qquad \alpha_2 = o(\rho^2), \qquad \beta_1 = c + o(\rho), \qquad \beta_2 = 1 + O(\rho^2). \tag{32}$$

and the only surviving symmetry equation is

$$A_1\alpha_2 - A_2\alpha_1 + B_2\beta_1 - B_1\beta_2 = 0. \tag{33}$$

The support equations simplify to

$$\alpha_i' = \beta_i' = 0 \quad \text{and} \quad \alpha_1\beta_2 - \alpha_2\beta_1 = 0 \tag{34}$$

outside the source. From these equations one can easily derive all solutions with rotational symmetry. (For more details see ref. (7)).

3. Closed time-like curves and the weak energy condition

For an arbitrary choice of the functions α_2 and β_2 the $g_{\theta\theta} = A_2^2 - B_2^2$ term in the metric is not necessarily negative even though, due to the regularity assumption, $g_{\theta\theta}$ is negative in a neighbourhood of the origin. A not negative $g_{\theta\theta}$ is a symptom of possible occurrence of CTC. In fact the existence of CTC implies that $g_{\theta\theta}(\rho,\theta)$ is positive at least for some ρ and θ. In fact given the CTC $t(\sigma)$, $\rho(\sigma)$, $\theta(\sigma)$ at the point $\bar{\sigma}$, where $t'(\bar{\sigma}) = 0$, one would have $ds^2 = g_{\theta\theta}d\theta^2 - d\rho^2 > 0$. For clearness sake we shall consider first the case of rotational invariance[6,7]. To begin with, if the determinant of the dreibein in the reduced radial gauge vanishes at certain $\bar{\rho}$ it follows that the manifold at $\rho = \bar{\rho}$ either closes or become singular. Such a conclusion is obtained through the following steps which are analyzed in detail in ref. (7). The regularity of the trace of the energy momentum tensor is an invariant

$$T^{\mu}_{\mu} = -\frac{1}{\kappa}[\frac{(\det(e))''}{\det(e)} + \frac{\alpha_1\beta_2 - \alpha_2\beta_1}{\det(e)}] = -\frac{1}{2\kappa}\, R. \tag{35}$$

On the other hand the term $\dfrac{(\alpha_1\beta_2 - \alpha_2\beta_1)}{\det(e)}$ is also an invariant being the third eigenvalue of $T_{\mu\nu}$. Thus the regularity of the remainder imposes

$$\det(e) = c\ (\bar{\rho} - \rho)(1 + O((\bar{\rho} - \rho)^2)). \tag{36}$$

Now if in $\bar{\rho}$ A_2 and/or $B_2 \neq 0$ one can easily show that the manifold is singular, while if $A_2 = B_2 = 0$ in $\bar{\rho}$ the universe closes without a singularity only if in $\bar{\rho}$ $A_1^2 - B_1^2 > 0$ and $\alpha_2^2 - \beta_2^2 = -1$. The topology of the resulting universe is that of a sphere and inside the universe $\det(e) \geq 0$.

If we now consider the WEC on the two light-like vectors $T^a + \Theta^a$ and $T^a - \Theta^a$ we obtain an inequality which is exactly integrable i.e.

$$\frac{dE^{(\pm)}}{d\rho} \leq 0, \tag{37}$$

where $E^{(\pm)}(\rho) \equiv (B_2 \pm A_2)(\alpha_1 \pm \beta_1) - (\alpha_2 \pm \beta_2)(B_1 \pm A_1)$. It is not difficult to show for a conical universe, in absence of CTC at infinity (which implies $\alpha_2^2 - \beta_2^2 \leq 0$), using $\det(e) > 0$ and the support equation $\alpha_1\beta_2 - \alpha_2\beta_1 = 0$, that $0 \leq E^{(\pm)}(\infty) \leq E^{(\pm)}(\rho)$. Then by straightforward algebra one obtains

$$\frac{d}{d\rho}\left(\frac{A_2^2(\rho) - B_2^2(\rho)}{\det(e)}\right) = -\frac{1}{2\det(e)^2}[(A_2 - B_2)^2 E^{(+)}(\rho) + (A_2 + B_2)^2 E^{(-)}(\rho)] \leq 0, \tag{38}$$

and as $g_{\theta\theta}$ is negative at the origin it is always negative and thus CTC cannot occur. Such analysis can be extended to all universes with the single exception of the cylindrical universe, generated by a string with tension and zero angular momentum.

All these reasoning can be also extended with no substantial change to the case of an open universe not invariant under rotation, provided that in our coordinates $\det(e)$ never vanishes[8]. Let us consider in fact a general external metric of the form

$$ds^2 = g_{00}(\theta)dt^2 + 2g_{0\theta}(\rho,\theta)dtd\theta + g_{\theta\theta}(\rho,\theta)d\theta^2 - d\rho^2, \tag{39}$$

i.e. the second order polynomial in ρ, $g_{00}(\rho,\theta)$ reduces outside the source (or equivalently at infinity) to a function of θ. We prove that $g_{00}(\theta) > 0$ imposes that $\alpha_1 = \beta_1 = 0$. In fact such a behaviour implies $\alpha_1^2 - \beta_1^2 = 0$ and $\alpha_1 A_1 = \beta_1 B_1$. Thus if $\alpha_1 \neq 0$ one has $A_1^2 = B_1^2$ and $g_{00} \equiv 0$ (for $\rho \geq \rho_0(\theta)$). Thus $\alpha_1 = \beta_1 = 0$. Symmetry equations (20) now gives

$$(A_1 + B_1)(\alpha_2 - \beta_2) = -(A_1 - B_1)(\alpha_2 + \beta_2) \tag{40}$$

and thus

$$(A_1^2 - B_1^2)(\alpha_2^2 - \beta_2^2) = g_{00}(\alpha_2^2 - \beta_2^2) \leq 0 \tag{41}$$

i.e. $\alpha_2^2 - \beta_2^2 \leq 0$. From $\alpha_1 = \beta_1 = 0$ we have the validity of the same support equation $\alpha_2\beta_1 - \alpha_1\beta_2 = 0$ as in the rotationally symmetric case. We are thus in the same situation as in the rotationally invariant case and thus we prove that $g_{\theta\theta}(\rho,\theta) \leq 0$. However CTC would imply that at least for a value of ρ and θ $g_{\theta\theta}(\rho,\theta) > 0$ and thus there cannot be any CTC.

With regard to the metric it is easy to prove[8] that the assumption $g_{00}(\theta) > 0$ implies that $g_{0\theta} = g_{0\theta}(\theta)$ and thus the external metric assumes the form

$$ds^2 = g_{00}(\theta)(dt + J(\theta)d\theta)^2 - (a(\theta)\rho - b(\theta))^2 d\theta^2 - d\rho^2 \tag{42}$$

because the coefficient $\gamma_{\theta\theta}$ of $d\theta^2$ is the square of the dreibein determinant divided by g_{00}. Performing the following change of variables

$$\theta' = 2\pi \frac{\int_0^\theta a(\phi)d\phi}{\int_0^{2\pi} a(\phi)d\phi} \tag{43}$$

and

$$t' = t + \left[\int_0^{\theta(\theta')} J(\phi)d\phi - \frac{\theta'}{2\pi}\int_0^{2\pi} J(\phi)d\phi\right] \tag{44}$$

we reach the metric

$$ds^2 = g_{00}(\theta)(dt' + J_0 d\theta')^2 - (a_0\rho - b(\theta'))^2 d\theta'^2 - d\rho^2 \tag{45}$$

where

$$a_0 = \frac{1}{2\pi}\int_0^{2\pi} a(\theta)d\theta \tag{46}$$

and

$$J_0 = \frac{1}{2\pi}\int_0^{2\pi} J(\theta)d\theta. \tag{47}$$

If $a_0 \neq 0$ one easily proves that g_{00} becomes a constant and we have the usual conical metric[1]. If $a_0 = 0$ we have a cylinder.

4. Conclusions

The application of the radial gauge to 2+1 dimensional gravity has been successful both in dealing with extended sources and time dependent problems. In ref. (3,5) we gave the general resolvent formulas for the time dependent problem in terms of a simple quadrature and derived the support properties of the energy momentum tensor in the case of time dependent sources with rotational symmetry. In ref. (5) we gave also explicit time dependent solutions, not necessarily invariant under rotations which satisfy all energy conditions. In the present paper we concentrated mainly on the general, non rotationally invariant, stationary problem. We wrote down the metric in terms of quadratures and gave explicit formulas for the support of the energy momentum tensor. In addition we have shown that the reduced radial gauge allows to derive important consequences of the weak energy condition. In particular we proved that for the general stationary open universe the WEC and the absence of CTC at infinity prevents the occurrence of CTC everywhere, both in presence and in absence of rotational symmetry. The radial gauge approach appears also apt to examining the time dependent situation in connection to 2+1 dimensional cosmology.

References

1. See "Quantum Gravity in Flatland" presented by R. Jackiw at 17th International Colloquium on Group Theoretical Methods in Physics, St. Adele, Canada 1988 and references therein.
   A. Staruszkiewicz, *Acta. Phys. Polon.* **24** (1963) 734; S. Deser, R. Jackiw, G. 't Hooft, *Ann. Phys. (N.Y.)* **152** (1984) 220; J. R. Gott, III and M. Alpert, *Gen. Rel. Grav.* **16** (1984) 243; S. Giddings, J. Abbot and K. Kuchar, *Gen. Rel. Grav.* **16** (1984) 751. G. Clement *Int. J. Theor. Phys.* **24** (1985) 267; S. Deser and R. Jackiw, *Ann. Phys. (N.Y.)* **192** (1989) 352; G. Grignani and C. Lee, *Ann. Phys. (N.Y.)* **196** (1989) 386; G. Clement *Ann. Phys. (N.Y.)* **201** (1990) 241.
2. A. Vilenkin, *Phys. Rev. D* **23** (1981) 852; J. Gott, *Astrophys. J.* **288** (1985) 422; J. Isper and P. Sikivie, *Phys. Rev. D* **30** (1984) 712; V. P. Frolov, W. Israel and W.G. Unruh, *Phys. Rev. D* **39** (1989) 1084. For a review on the subject see A. Vilenkin, *Phys. Rep. C* **121** (1985) 263.
3. P. Menotti and D. Seminara, *Nucl. Phys.* **B376** (1992) 411.
4. G. 't Hooft, *" The evolution of gravitating point particles in 2+1 dimensions"* preprint THU-93/02.
5. P. Menotti and D. Seminara, *Ann. Phys. (N.Y.)* **208** (1991) 449.
6. P. Menotti and D. Seminara, *Phys. Lett. B* **301** (1993) 25.
7. P. Menotti and D. Seminara, *"Stationary solutions and closed time-like curves in 2+1 dimensional gravity"* preprint IFUP-TH-18/93, to appear on Nucl. Phys. B.

8. P. Menotti and D. Seminara, to be published

9. J.R. Gott, *Phys. Rev. Lett.* **66** (1991) 1126; S. Deser, R. Jackiw and G. 't Hooft, *Phys. Rev. Lett.* **68** (1992) 2647; S.M. Carrol, E. Farhi and A.H. Guth, *Phys. Rev. Lett.* **68** (1992) 263.; Erratum: **68** (1992) 3368; S.M. Carrol, E. Farhi and A.H. Guth, *" Gott time machines cannot exist in an open (2+1) dimensional universe with timelike total momentum"* preprint MIT-CTP-2117 (1992); C. Cutler, *Phys. Rev. D* **45** (1992) 487; D. Kabat, *Phys. Rev. D* **46** (1992) 2720; A. Ori, *Phys. Rev. D* **44** (1992) R2214; G. 't Hooft, *Class. Quantum Grav.* **9** (1992) 1335; F. J. Tipler, *Ann. Phys. (N.Y.)* **108** (1977) 1; S. W. Hawking, *Phys. Rev. D* **46** (1992) 603; S. Deser and R. Jackiw, *Time Travel* preprint BRX TH 334; CTP#2101 (1992); J.D.E. Grant, *Phys. Rev. D* **47** (1993) 2388.

10. G. Modanese and M. Toller, *J. Math. Phys.* **31** (1990) 452.

11. E. Fermi, *Atti Accad. Naz. Lincei Cl. Sci. Fis. Mat. & Nat.* **31** (1922) 184, 306; A. G. Walker, *Proc. London Math. Soc.* **42** (1932) 90.

12. S.W. Hawking and G.F.R. Ellis, *The large scale structure of space-time*, (Cambridge University Press, Cambridge, 1976),p. 33.