Abstract  The impact of the precision tests on the theory of the electroweak interactions is discussed in view of the preliminary 93 LEP data and of the polarization asymmetry measurement at SLAC. A particular correlation between the top and the Higgs mass that could occur in supersymmetry is illustrated.

Talk given at the Rencontres de la Vallee d'Aoste, La Thuile, Italy, March 1994.
The preliminary analysis of the 1993 LEP data and the recent measurement at SLAC by the SLD collaboration of the left-right $e^+ - e^-$ asymmetry allow a test of the electroweak theory of increasing significance. The collection of these data, including also an improved value of the ratio $M_w/M_z$ from hadron colliders, is summarized in Table 1. As a special effect of the 93 run at LEP, one notices a reduction by almost a factor of 2 of the error in the Z-width with respect to the previous data. Worth of note is also the fact that the precise value of the ratio of the Z-electron couplings $g_y/g_A$ inferred from the still preliminary SLD result ($g_y/g_A = 0.0838 \pm 0.0040$) differs by almost $3 \sigma$ from the average value of the same ratio obtained from the various asymmetry measurements at LEP ($g_y/g_A = 0.0711 \pm 0.0020$), maybe indicating an experimental problem.

Table 1. List of experimental values used in the text$^{1,2,3}$

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_z$ (GeV)</td>
<td>91.1899 ± 0.0044</td>
</tr>
<tr>
<td>$M_w/M_z$ (UA2+CDF)</td>
<td>0.8814 ± 0.0021</td>
</tr>
<tr>
<td>$\Gamma_\gamma$ (MeV)</td>
<td>2497.1 ± 3.8</td>
</tr>
<tr>
<td>$R = \Gamma_h/\Gamma_t$</td>
<td>20.79 ± 0.04</td>
</tr>
<tr>
<td>$\sigma_h = 12\pi\Gamma e\Gamma_h/m_z^2\Gamma_t^2$ (nb)</td>
<td>41.51 ± 0.12</td>
</tr>
<tr>
<td>$\Gamma_t$ (MeV)</td>
<td>83.98 ± 0.18</td>
</tr>
<tr>
<td>$\Gamma_h$ (MeV)</td>
<td>1746 ± 4</td>
</tr>
<tr>
<td>$\Gamma_b$ (MeV)</td>
<td>385.9 ± 3.4</td>
</tr>
<tr>
<td>$R_{bh} = \Gamma_b/\Gamma_h$</td>
<td>0.2210 ± 0.0019</td>
</tr>
<tr>
<td>$A_{FB}^t$</td>
<td>0.0170 ± 0.0016</td>
</tr>
<tr>
<td>$A_{pol}$</td>
<td>0.150 ± 0.010</td>
</tr>
<tr>
<td>$A$</td>
<td>0.120 ± 0.012</td>
</tr>
<tr>
<td>$A_{FB}^t$</td>
<td>0.0970 ± 0.0045</td>
</tr>
<tr>
<td>$\mathcal{R}$</td>
<td>0.312 ± 0.003</td>
</tr>
<tr>
<td>$Q_w$</td>
<td>-71.04 ± 1.81</td>
</tr>
<tr>
<td>$A_{LR} (SLD)$</td>
<td>0.1668 ± 0.0079</td>
</tr>
</tbody>
</table>

To have a better appreciation of the impact of these data on the theory of the electroweak interactions, I choose to contrast two possible and, to a large extent, equally legitimate viewpoints. For reference, I shall attribute them to an "optimist" and to a "sceptic".

As expected, the "optimist" has the first word, which he spends to illustrate the success of the Standard Model (SM) in reproducing the data. Table 2 summarises the result of the overall fit in the SM of all LEP data and of all LEP+SLD data for two different values of the Higgs mass. The variables of the fit are the top mass and $\alpha_s(M_Z)$. 
Table 2. Standard Model fits of all LEP and of all LEP+SLD data for fixed Higgs mass.

<table>
<thead>
<tr>
<th></th>
<th>$m_H = 65\text{GeV}$</th>
<th>$m_H = 1\text{TeV}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>LEP</td>
<td>$m_t(\text{GeV}) = 146 \pm 13.5$</td>
<td>$m_t(\text{GeV}) = 183 \pm 12$</td>
</tr>
<tr>
<td>LEP + SLD</td>
<td>$m_t(\text{GeV}) = 158 \pm 11$</td>
<td>$m_t(\text{GeV}) = 194 \pm 10$</td>
</tr>
<tr>
<td></td>
<td>$\alpha_s(M_Z) = 0.123 \pm 0.0044$</td>
<td>$\alpha_s(M_Z) = 0.127 \pm 0.0044$</td>
</tr>
</tbody>
</table>

The light Higgs fit has a slightly better $\chi^2$ ($\Delta \chi^2 = 3.5$). More significant is the separation, in the two fits, between the central values of $m_t$, especially if this separation is compared with the theoretical error, $\Delta m_t^{\text{th}} = 6 \pm 8\text{GeV}$, that is estimated to affect the determination of $m_t$ from a pure $t-\bar{t}$ cross section measurement at the Tevatron. This comparison may become a relevant source of information on the Higgs mass in a not too distant future. To this end, a more complete information is contained in the isoplot of the $\chi^2$ in the $m_t$, $m_H$ plane, given in Fig. 1, for $\alpha_s(M_Z) = 0.118 \pm 0.007$, and using all $e^+ - e^-$ data (LEP + SLD).

![Figure 1. Isoplot of the $\chi^2$ of the SM fit of all $e^+ - e^-$ data (LEP + SLD), with $\alpha_s(M_Z) = 0.118 \pm 0.007$. The lines give constant values of the $\chi^2$ ($\chi^2 = \chi^2_{\text{min}} + 1^2, 2^2, 3^2$ respectively)
All this does not quite show, however, how precisely sensitive are the data to the structure of the radiative corrections in the SM. It is in fact well known - the "sceptic" notes - that the bulk of the radiative corrections is anyhow of purely electromagnetic origin and, as such, cannot be doubted at the per mille level. For a better appreciation of this point, it is useful to analyze the data in a model independent way, following Ref. 5. To this end, I will write any of the precision observables as

\[ O_i = O_i^0 \left( 1 + \sum_{j=1}^{4} a_j \epsilon_j \right) \]  

where:

i) \( O_i^0 \) is the corresponding prediction of the theory in the Born approximation and **including** the QED radiative-corrections effects;

ii) \( \epsilon_j \) are four dimensionless parameters containing all the "genuine" electroweak radiative-correction effects, dependent, as such, on \( m_t \) or \( m_H \), or on any other parameter of similar nature;

iii) \( a_j \) are fixed numerical constants.

Up to now, the only restriction on the possible theories is that they should all give the same \( O_i^0 \) as the SM. By now, in view of the present data, this is a very interesting set of theories. To define the \( \epsilon_j \), \( j = 1, 2, 3, b \), one picks up four observables of particular interest, \( M_w / M_Z \), \( \Gamma_l \) (the Z leptonic width into charged leptons), \( A^l_{FB} \) (the forward-backward asymmetry at the Z peak for charged leptons) and \( \Gamma_b \) (the Z width into a \( b\bar{b} \) pair) and puts them in suitable correspondence with the four \( \epsilon_j \). Making reference to 5 for the actual definition of the \( \epsilon \)-parameters, such correspondence is set according to the following principles:

i) Two sectors in the physics of the electroweak interactions are considered as more likely to deviate from the SM and, as such, more interesting: the gauge-boson vacuum polarization amplitudes and the \( Z \rightarrow b\bar{b} \) vertex.

ii) In defining the \( \epsilon_j \), one should avoid having to specify the top-quark mass, which is at present a main source of uncertainty in the comparison of theory and experiment.

With this in mind, in an effective Lagrangian description, 4 coefficients are of particular interest, three of them related to the masses and the kinetic terms of the gauge bosons

\[ L^\tau_{VB} = -\frac{1}{2} W^\tau_{\mu \nu} W^{\tau}_{\mu \nu} - \frac{1}{4} (1 - e_2) W^3_{\mu \nu} W^{3}_{\mu \nu} - \frac{1}{4} B_{\mu \nu} B^{\mu \nu} - \frac{1}{2} c e_3 B_{\mu \nu} W^3_{\mu \nu} \]

\[ -\frac{1}{2} M_Z^2 (c W^2_{\mu \nu} - s B_{\mu \nu})^2 - M_Z^2 c^2 (1 + e_1) W^3_{\mu \nu} W^{\tau}_{\mu \nu} \]  

and a fourth one to the GIM-violating \( Z \rightarrow b\bar{b} \) vertex
\[ V_{\mu}^{\text{GIM}}(Z \to b \bar{b}) = \frac{g}{2c} e_b \gamma_\mu \frac{1-\gamma_5}{2} \]  \hfill (3)

The \( \epsilon_j \) are in a one-to-one correspondence with these \( \epsilon_j, j = 1,2,3,b \), in the sense that

\[
\begin{align*}
\epsilon_1 &= \epsilon_1 + \ldots, \\
\epsilon_2 &= \epsilon_2 + \ldots, \\
\epsilon_3 &= \epsilon_3 + \ldots, \\
\epsilon_b &= \epsilon_b + \ldots,
\end{align*}
\]  \hfill (4)

where the dots stand for other effects, related to higher dimensional vacuum polarization operators, vertex corrections, etc.\(^6\).

Up to now we have focused on \( M_W/M_Z, \Gamma, A_{FB}^l \), and \( \Gamma_b \). Without any new assumption other than the universality of the \( Z \) coupling to the charged leptons, also all the asymmetries measured at LEP can be parametrized in terms of the \( \epsilon_j \). On the other hand, the inclusion of the hadronic observables at the \( Z \) peak requires that all deviations from the Standard Model be only contained in vacuum polarization diagrams (without demanding a truncation of the \( q^2 \) dependence of the corresponding functions) and/or in the \( Z \to b \bar{b} \) vertex [principle (i) becomes an assumption]. Finally, a mild \( q^2 \) dependence of the vacuum polarization amplitudes is also required, if one wants to include in the analysis the lower energy data as well (only \( \epsilon_1, \epsilon_2, \epsilon_3 \) and \( \epsilon_b \) can deviate from their SM values). Under this hierarchy of assumptions, the data can be accordingly fitted\(^5\).

The results of the various fits are collected in Table 3. We also display in Fig.s 2-4 the projected ellipses in the planes \( \epsilon_3 - \epsilon_1, \epsilon_b - \epsilon_1, \epsilon_3 - \epsilon_b \) corresponding to 1\( \sigma \) errors, together with the SM predictions. The parameter \( \epsilon_2 \), affected by a larger uncertainty, is shown in Fig. 5.

<table>
<thead>
<tr>
<th>( \epsilon ), ( 10^3 )</th>
<th>Defining Variables</th>
<th>Defining Variables + Asymms.s</th>
<th>All ( e^+ - e^- ) data (LEP+SLD)</th>
<th>&quot;All data&quot;</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \epsilon_1 )</td>
<td>4.8 ± 2.2</td>
<td>5.1 ± 2.2</td>
<td>3.5 ± 1.8</td>
<td>3.0 ± 1.7</td>
</tr>
<tr>
<td>( \epsilon_2 )</td>
<td>-7.0 ± 5.3</td>
<td>-7.7 ± 5.0</td>
<td>-9.2 ± 5.0</td>
<td>-9.5 ± 5.0</td>
</tr>
<tr>
<td>( \epsilon_3 )</td>
<td>3.5 ± 3</td>
<td>4.9 ± 2.0</td>
<td>3.8 ± 1.9</td>
<td>3.4 ± 1.8</td>
</tr>
<tr>
<td>( \epsilon_b )</td>
<td>5.0 ± 4.8</td>
<td>5.2 ± 4.8</td>
<td>0.9 ± 4.2</td>
<td>1.3 ± 4.1</td>
</tr>
</tbody>
</table>
Figure 2 Plot of $\varepsilon_3$ versus $\varepsilon_1$ from the fit of the data, with (LEP+SLD) or without the inclusion of the SLD result (LEP). The projections of the ellipsis on the axes correspond to the 1 $\sigma$ errors displayed in Table 3.

Figure 3 Plot of $\varepsilon_b$ versus $\varepsilon_1$ as in figure 2.
All LEP (and SLD) Data

Figure 4. Plot of $\varepsilon_3$ versus $\varepsilon_b$ as in figure 2.

Figure 5. Determination of $\varepsilon_2$ from all data, compared with the SM prediction.
One main point emerges from these numbers: three of the $\varepsilon$-parameters show at about $2\sigma$ level a deviation from zero. The data are not described anymore by the pure "Born" approximation, as it was still the case for the data of the last year. This constitutes the first evidence for the "genuine" electroweak corrections, which agree within one standard deviation with the expectation of the SM.

Two of the three parameters that deviate from zero, $\varepsilon_1$ and $\varepsilon_3$, are determined, essentially from $e^+e^-$ measurements, at 1-2 per mille level. The third one has a larger uncertainty, reflecting the less precise knowledge of $M_W/M_Z$. The only parameter which does not yet show a deviation from zero is the correction to the $Z \to b\bar{b}$ vertex, $\varepsilon_4$. At the same time its central value is higher than expected in the SM, for whatever value of the top and the Higgs masses. This reflects a high value of the ratio $R_{bb} = \Gamma_{bb}/\Gamma_h$, which is more than $2\sigma$ away from the expectation in the SM. Note the significant change of $\varepsilon_4$, when going from the second to the third column in Table 3. It should be remembered that the value of $\varepsilon_4$ is strongly correlated with that of $\alpha_s(M_Z)$.

The evidence for the "genuine" electroweak corrections is clearly a remarkable achievement - the "sceptic" admits - but - he immediately adds - this is still a long way from settling the really open issue in the physics of the electroweak interactions: the origin of mass or of the symmetry breaking. To support this view, he points out the weak dependence of the radiative corrections on the Higgs mass, which, in the SM, is the physical parameter related to the symmetry breaking sector.

According to the "optimist", this viewpoint can be contrasted with two different orders of arguments. The first one is pretty simple and indeed rather clear from Fig.1. Much of the issue of the sensitivity on the Higgs mass in the SM is related to the independent determination of the top mass at the hadron colliders. On the other hand, as shown in Fig.s 2-5, the main dependence of the radiative corrections on the Higgs mass is contained in $\varepsilon_3$, mostly determined from the asymmetry measurements, which can still be improved, both at LEP and at SLAC. Theoretically, the knowledge of $\varepsilon_3$ is only limited by the uncertainty on the electromagnetic fine structure constant at the $Z$-pole, which contributes to an error on $\varepsilon_3$ of about $0.7 \cdot 10^{-3}$.

The second argument, although certainly more indirect, is, in the opinion of the "optimist", at least as important as the first one. To judge of the significance of the precision tests on the symmetry breaking issue by only looking at the Higgs mass dependence in the SM, is a limited viewpoint, if not a logical mistake. From this point of view, it seems in fact much more appropriate to compare alternative theories of the symmetry breaking. In the case of the Minimal Supersymmetric Standard Model (MSSM) and using the same parameters $\varepsilon_i$ defined above, this has been done in Ref. 8. The main feature of the radiative corrections in the MSSM is that they can differ from those of the SM with a light Higgs only for very special values of the parameters, corresponding to some of the supersymmetric particles...
being just around the corner. The point is however that the MSSM is not at all alternative to the SM as far as the symmetry breaking mechanism is concerned. Actually, the MSSM is largely motivated by the very need to put on a sounder basis the same Higgs mechanism. On the contrary, it is generally found in theories without a Higgs mechanism that the radiative corrections to the precision observables do differ in a significant way from those of the SM. In the language of the $\epsilon$, in the cases when they can be not too unreliably estimated, one often finds effects of the order of the per cent, well above the sensitivity that is being reached by the experiments and, in fact, far larger than the same values actually measured. In other words, to infer from the weak dependence of the SM radiative effects on the Higgs mass a general blindness of the precision tests on the symmetry breaking mechanism does not appear justified by the theoretical studies that have been made on this subject.

* * *

At this point, the "optimist" feels that he has won his fight with the "sceptic" over all the line. He thinks that the precision measurements are giving indirect but significant support to the view that the origin of mass in the theory of the electroweak interactions is of perturbative nature. To him, the SM appears likely to be literally true even in its most uncertain sector, the one of the Higgs system. Of course he realizes at the same time that the Higgs case would be objectively strengthened if it were possible to predict the mass of the Higgs boson. In fact, the supersymmetric extension of the SM has something to say on this issue.

The general upper bound holding on the Higgs mass in the MSSM, as function of the top mass, is well known and is of course a very significant property of the MSSM at all \cite{10}. It is less well known, but nevertheless true, that it is possible to find a more special correlation between the top and the Higgs masses, if the MSSM is supplemented with a further hypothesis. In turn this hypothesis originates from considerations that have been first developed in the past in a general, non supersymmetric context. The idea is that the top Yukawa coupling, $y_t$, be at its infrared fixed point \cite{11}.

In the SM, this possibility does not seem to be entertainable anymore, precisely in view of the precision tests that we have illustrated, since it requires a value of the top mass above 220 GeV or so \cite{12}. On the contrary, in the MSSM, due to the different particle content, the top mass value is lowered \cite{13-14}. More precisely, if one requires that the ultraviolet explosion in $y_t$ should not occur before the unification scale, $\approx 10^{16} \text{GeV}$, this leads to the correlation between the Higgs and the top quark masses shown in Fig 6. Such correlation is only determined by the value of the usual parameter $\tan \beta$ and, less directly, by the value of $\alpha_s(M_Z)$. To be precise, the Higgs mass shown in Fig 6 could actually be lowered, if also the usual pseudoscalar state would be equally light.
Figure 6. Correlation between the top and the lightest Higgs mass for $\alpha_s(M_Z) = 0.110 \pm 0.125$ (region between the dotted lines). Also shown is the general upper bound on the Higgs mass in the MSSM.

It is also interesting to note that the same conclusion holds as a consequence of another independent suggestion, again first put forward in a non supersymmetric context $^{15}$. Unified Theories often suggest that the bottom and $\tau$ Yukawa couplings are equal at the Grand scale. In a non supersymmetric theory of normal type such an hypothesis is no longer tenable, since, this time, consistency with the observed $b/\tau$ mass ratio would need a top lighter than about 90 GeV. On the contrary, at least for non extreme values of $tg\beta$, in the supersymmetric case a top Yukawa coupling at, or very close to, the fixed point is required $^{16-14}$.

The "optimist" might conclude that finding a Higgs with a mass correlated with that of the top in the way shown in Fig 6 would prove that the Higgs hypothesis and supersymmetry are both true and they go indeed together, as they should. Fig. 7 illustrates what the precision tests have to say at present with respect to this conjecture, under the hypothesis that radiative corrections due to supersymmetric particles are irrelevant. Time will tell if it is true.
Figure 7. The same as in figure 6, compared with the isoplot of the $\chi^2$ from the precision tests, as in figure 1.

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