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Lectures given at the Symposium on Particle Physics at the Fermi scale, Beijing, May 27 -June 4, 1993

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1. INTRODUCTION

The experimentation at the Z resonance at LEP makes it possible, for the first time, to study with great precision some properties of the electroweak interactions at distances of the order of the inverse Fermi scale, $G_F^{1/2}$. An immediate question arises: What is it that we learn from these studies? The answer is not an obvious one. It may in fact require some time and some complementary studies (both experimental and theoretical) to be able to fully appreciate the significance of these precision tests of the electroweak interactions. What is sure is that a judgment of the impact of these tests requires an appreciation of several different aspects of the theory of the electroweak interactions themselves. In particular, this appreciation is not achieved, I believe, by only making a comparison of the Standard Model (SM)[1] predictions with the experimental results.

This is the physical background that motivates these lectures. It is actually my opinion that the experimental program being carried out at LEP1, especially in association with the direct search for the top quark at Fermilab, although indirectly, may shed light on the symmetry breaking issue in electroweak physics. My purpose, rather than trying to convince the reader of this statement, is to make him able to judge by himself.

To discuss this subject, one has to master the problem of calculating the radiative corrections in the SM, as well as in alternative more or less defined theories, a technically heavy and often boring task. For this reason, when discussing these issues, one frequently ends up by making reference to other people's work for detailed calculations and similar technical things. This is an attitude that I would like to avoid here, aiming

rather at being as self-contained as possible. The reader is however expected to be familiar with the basic properties and techniques of gauge field theories as well as with the Lagrangian of the SM.

2. DEFINITION OF THE THEORY - BASIC OBSERVABLES

As I have made it clear in the Introduction, we do not want to restrict our attention to the SM only. Rather I consider a generic theory that fulfils the following requirements:
i) The gauge group is SU(2)XU(1);
ii) The spectrum includes the standard three generations of fermions with the usual SU(2)XU(1) assignments;
iii) At the tree level, the vector boson masses are related to the gauge couplings and to a scale parameter v by the relations

$$M_W^2 = \frac{g^2 v^2}{2}, \quad M_Z^2 = \frac{\left(g^2 + g'^2\right)v^2}{2} \tag{2.1}$$

By the first requirement, I actually mean that, at the tree level, the W and the Z bosons are unmixed with the gauge bosons of a possibly larger gauge group (e.g., with an extra U(1) factor). The third condition is dictated by the experimental observation that the ρ-parameter, to be precisely defined later on, equals one within few per mille. In this way, among other things, I certainly include the SM itself, a generic multi-Higgs doublet model, the Minimal Supersymmetric Standard Model (MSSM)[2] and QCD-like TechniColour (TC) models[3] , all of which give the same tree level predictions for the precision observables that we shall consider. In an SU(2)xU(1) gauge theory with any number of Higgs doublets H_i, like the MSSM itself where i =1,2, it is always possible to define the normalized linear combination

$$H = \frac{\sum_i v_i H_i}{\sqrt{\sum_i v_i^2}}, \quad \langle H_i^0 \rangle = v_i \tag{2.2}$$

which gets a vacuum expectation value

$$v = \sqrt{\sum_i v_i^2} \tag{2.3}$$

and plays the same role as the SM Higgs boson in giving rise to the W and Z masses (2.1). In the technicolour case, it is well known that the relations (2.1) hold, in fact to all orders in the technicolour interactions, as a consequence of an extra "custodial" $SU(2)_C$ symmetry[4] which is

only broken by the gauging of the U(1) factor or by the interactions responsible for the top-bottom mass splitting (see Section 13).

In this way I am led to consider a model described by a Lagrangian L(g,g',v;...), which fixes, at the tree level, in terms of g, g' and v only, a series of "precision observables". The dots in L(g,g',v;...) stand for the many other possible parameters, e.g. the top or the Higgs masses. The "precision observables", by definition, can be influenced by these extra parameters only via radiative corrections.

Given this framework, even sticking only to the tree level predictions, three measurements are needed to determine the basic parameters, whereas any extra measurement can be used to check the theory (or determine, via radiative corrections, the remaining parameters). The basic observables that are used to determine the theory (rather than checking it) must be precisely measured on one side, and theoretically calculable in a clean way on the other side. At present, three quantities neatly emerge:

i) The electromagnetic fine structure constant, α, as measured by the Josephson effect or the electron (g-2)

$$\alpha = 137.0359895\ (61)\ . \tag{2.4}$$

ii) The Fermi constant G as determined from the muon lifetime τ_μ and the theoretical formula[5]

$$\frac{1}{\tau_\mu} = \frac{G^2 m_\mu^5}{192\pi^3}\left(1 - 8\frac{m_e^2}{m_\mu^2}\right)\left[1 + \left(\frac{\alpha}{\pi}\right)\left(\frac{25}{4} - \pi^2\right)\left(1 + \frac{2\alpha}{3\pi}\lg\frac{m_\mu}{m_e}\right)\right], \tag{2.5}$$

which includes the QED corrections to the 4-fermion interaction

$$G = 1.16637(2)10^{-5}\ \mathrm{GeV}^{-2} \tag{2.6}$$

iii) The Z-mass [6]

$$M_Z = 91.187 \pm 0.007 GeV \tag{2.7}$$

The uncertainty on M_Z, which is the less precisely known among the basic observables, is far smaller (by more than one order of magnitude at least) than the relative uncertainty in any of the other observables that are currently measured and that we shall consider.

At the tree level, the basic observables are expressed in terms of the parameters g, g', v as

$$\alpha_0 = \frac{g^2 s^2}{4\pi}, \quad G_0 = \frac{\sqrt{2}}{4v^2}, \quad M_{Z0}^2 = \frac{g^2 v^2}{2c^2} \tag{2.8}$$

where[f1]

$$s^2 \equiv \frac{\left(\frac{g'}{g}\right)^2}{\left(1+\left(\frac{g'}{g}\right)^2\right)} \qquad and \qquad c^2 \equiv 1 - s^2 \tag{2.9}$$

3. DERIVED OBSERVABLES. TREE LEVEL VALUES

Equations (2.8) can be solved in favour of g, g' and v as functions of α_0, G_0, M_{Z0}. In this way, at the tree level, the precision observables that we shall use to check the theory can all be expressed in terms of α_0, G_0, M_{Z0}. To this end, one needs:

i) the W-mass

$$M_{W_0}^2 = M_{Z_0}^2 c_0^2, \quad c_0^2 \equiv 1 - s_0^2 \equiv \frac{1}{2}\left(1+\left(1+\frac{4\pi\alpha_0}{\sqrt{2}G_0 M_{Z_0}^2}\right)^{1/2}\right) \tag{3.1}$$

ii) the photon coupling to the fermion f of charge Q_f

$$V_\mu\left(\gamma - f\,\bar{f}\right) = -\sqrt{4\pi\alpha_0}\,Q_f\gamma_\mu; \tag{3.2}$$

iii) the Z coupling to the fermion f of weak isospin T_{3f} and charge Q_f

$$V_\mu\left(Z - f\,\bar{f}\right) = -\left(\sqrt{2}G_0 M_{Z_0}^2\right)^{1/2}\gamma_\mu\left(g_{Vf}^0 - g_{Af}^0\gamma_5\right); \tag{3.3}$$

$$g_{Vf}^0 = T_{3f} - 2Q_f s_o^2, \quad g_{Af}^0 = T_{3f} \tag{3.4}$$

iv) the W-coupling to the fermion doublet (f, f')

$$V_\mu\left(W - f\,\bar{f}'\right) = -\frac{\sqrt{4\pi\alpha_0}}{2\sqrt{2}s_0}\gamma_\mu\left(1-\gamma_5\right). \tag{3.5}$$

[f1] The reader should be aware of the fact that, although keeping the same symbol, the precise meaning of s^2 will change as we progress.

Of special interest to us are the Z widths into a pair of fermions $\Gamma(Z \to f\,\bar{f})$, the forward-backward asymmetries at the Z pole

$$A_{FB}^{f} \equiv \frac{\sigma_F^f - \sigma_B^f}{\sigma_F^f + \sigma_B^f} \tag{3.6}$$

$$\sigma_{F(B)}^{f} = \int_0^1 \left(\int_{-1}^{0} \right) d\cos\theta \frac{d\sigma}{d\cos\theta}\left(e^+e^- \to f\,\bar{f}\right), \tag{3.7}$$

and the τ-polarization asymmetry

$$A_{Pol}^{\tau} \equiv \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} \tag{3.8}$$

$$\sigma_{L(R)} = \sigma\left(e^+e^- \to \tau_{R(L)}^{+}\,\tau_{L(R)}^{-}\right). \tag{3.9}$$

It is a simple matter to obtain, from these definitions, the following tree-level expressions[f2]

$$\Gamma^{(0)}\left(Z \to f\,\bar{f}\right) = N_C \frac{G_0 M_{Z_0}^3}{6\pi\sqrt{2}}\left(g_{Vf}^{0\,2} + g_{Af}^{0\,2}\right) \tag{3.10}$$

where $N_C = 1,3$ for leptons and quarks respectively, and

$$A_{Pol}^{\tau(0)} = 2\eta_e \qquad A_{FB}^{f(0)} = 3\eta_e\eta_f \tag{3.11}$$

where

$$\eta_f = \frac{g_{Vf}^{0}\, g_{Af}^{0}}{g_{Vf}^{0\,2} + g_{Af}^{0\,2}} \tag{3.12}$$

If we do not care at all about radiative corrections, it is already possible to obtain the predictions for some of the "derived observables", starting from the input values of the basic observables given in (2.4-7). This is shown in Table 1, column A.

[f2] For simplicity I do not write the trivial fermion mass corrections, which are however not completely negligible for the τ and the b quark.

Table 1: Comparison of theory (in different approximations) with experiment

Observable	A "Bare" relations	B e.m. corrections	C e.m. + large m_t	Experiment [6,7]
$\Gamma(Z \to l\,\bar{l})$	84.99 Mev	83.56 Mev	86.36 Mev	83.52 ± 28
A_{FB}^{l}	0.0637	0.0168	0.0351	0.0164 ± 21
A_{Pol}^{τ}	0.296	0.150	0.216	0.142 ± 17
A_{FB}^{b}	0.210	0.105	0.152	0.098 ± 9
M_W/M_Z	0.8876	0.8768	0.8935	0.8798 ± 28

4. ELECTROMAGNETIC CORRECTIONS

At the level of precision that is of interest here, it is not possible to doubt the effectiveness of QED in describing the radiative corrections from photon exchanges. Always having in mind the required precision, it is less obvious but nevertheless true, with some provisos, that one can speak of pure QED corrections in isolation from the full electroweak corrections. Let me briefly describe them(f3).

For observables at the Z-pole, the largest effect, by far, from pure QED radiative corrections is the change in the electric charge when going from $q^2 = 0$, where the fine structure constant defined in (2.4) is measured, to $q^2 = M_Z^2$. This change from α to $\alpha(M_Z^2)$ is related to the photon vacuum polarization function

$$\Pi_{\mu\nu}^{\gamma\gamma}(q) = -ig_{\mu\nu}\left[A^{\gamma\gamma}(0) + q^2 F^{\gamma\gamma}(q^2)\right] + q_\mu q_\nu \; terms \tag{4.1}$$

via

$$\alpha(M_Z^2) = \frac{\alpha}{1 + F^{\gamma\gamma}(M_Z^2) - F^{\gamma\gamma}(0)} \equiv \frac{\alpha}{1 - \Delta\alpha} \tag{4.2}$$

In $F^{\gamma\gamma}$ we only include the contributions from the lepton and the light quark loops:

$$\Delta\alpha = \Delta\alpha_l + \Delta\alpha_q \tag{4.3}$$

(f3) I refer to the original work[8] that deals with the QED corrections to Bhabha scattering, an essential tool in determining the luminosity.

that is to say we will conventionally include in the remainder of the corrections both the top quark and the W-boson loops. The lepton loops give $\Delta\alpha_l = -0.0314$, whereas, for the quark contribution, a perturbative calculation is not possible, because of strong interaction effects at low q^2. The way out consists[9] in relating $\Delta\alpha_q$ to the measured, and properly normalized, hadronic cross-section

$$R(s) = \frac{3s}{4\pi\alpha^2}\sigma\left(e^+e^- \to \gamma \to hadrons; s\right) \tag{4.4}$$

via a dispersive representation

$$\Delta\alpha_q = \frac{\alpha}{3\pi} M_Z^2 \int_{4m_\pi^2}^{\infty} \frac{dsR(s)}{s\left(s - M_Z^2\right)} \tag{4.5}$$

This gives[10] $\Delta\alpha_q = -0.0282(9)$, with the error dominated by the uncertainties in the measured cross-section between the charm and the beauty threshold. Putting together the lepton and the quark contributions, one obtains

$$\alpha\left(M_Z^2\right)^{-1} = 128.87 \pm 0.12 \tag{4.6}$$

which replaces Eq.(2.4) as the reference value of the fine structure constant. The uncertainty of this new value is not such that we can obviously neglect it in the following.

The other relevant electromagnetic corrections for the observables at the Z-pole, which we briefly describe in the following, arise from initial and final state photon radiation and from the one photon exchange contribution to the scattering amplitudes.

Bremmstrahlung from the initial particles is the largest source of corrections to the Breit-Wigner shape. It can be accounted by folding the cross section with a "structure function" $F(x,s)$ that describes the probability that the initial electron or positron emit a photon which carries away a fraction x of the initial momentum:

$$\sigma(s) \equiv \int_0^{1-s_0/s} dx\, F(x,s)\sigma_0\left(s(1-x)\right), \tag{4.7}$$

where s_0 is a threshold energy below which the final state does not exist or is not detected and σ_0 is an "uncorrected" cross section that we now define, taking into account of the relativistic phase space and of the one photon exchange amplitude. One has

$$\sigma_0\left(e^+e^- \to f\,\bar{f}; s\right) = \sigma^0_{Z,f\bar{f}}(s) + \sigma^0_{\gamma,f\bar{f}}(s) + \sigma^0_{Z\gamma,f\bar{f}}(s) \tag{4.8}$$

where

$$\sigma^0_{Z,f\bar{f}}(s) = \frac{12\pi\Gamma_e\Gamma_f}{M_Z^2\Gamma_Z^2}\frac{s\Gamma_Z^2}{\left(s-M_Z^2\right)^2 + s^2\,\Gamma_Z^2/M_Z^2} \tag{4.9}$$

and $\sigma^0_{\gamma,f\bar{f}}(s)$, $\sigma^0_{Z\gamma,f\bar{f}}(s)$ are respectively the pure photon and the Z-photon interference contributions to the cross section, as computed in the Standard Model. Equation (4.9) is in fact the very definition of the Z mass and of the Z widths. At the needed level of accuracy, the Z mass so defined coincides with the real part of the pole of the Z propagator.

Finally, photon radiation from the final state $f\,\bar{f}$, is accounted for a factor $\left(1+3\alpha/4\pi Q_f\right)$ multiplying the QED uncorrected width.

The importance of the QED corrections is manifest from column B of Table1. After the deconvolution of the initial state radiation effect, they are largely dominated by the charge renormalization from $q^2=0$ to $q^2 = M_Z^2$. The entries in column B are obtained in the same way as in column A, except for the use of (4.6) instead of (2.4) and the introduction of the final state radiation factor in the Z leptonic width.

5. RENORMALIZATION

If we now want to deal with the full electroweak corrections, we need to go through the renormalization procedure. In principle, this opens the way to many possible different schemes. In a weak coupling theory, however, as the one that we are dealing with, one procedure emerges above all, which avoids useless intermediate steps and speeds up the necessary calculations in a significant way. Technically, it is itself a renormalization scheme.

First, we focus on the basic observables

$$\left(\alpha_0, G_0, M_{Z0}\right) \equiv a_0^i \tag{5.1}$$

and we compute the radiative corrections to them

$$a_0^i \xrightarrow{RC} a_0^i + \delta a^i\left(a_0^i\right) \equiv a^i\left(a_0^i\right) \tag{5.2}$$

In this way we obtain the renormalized basic observables $a^i\left(a_0^i\right)$ as (divergent) power series in $\hbar$, which can be formally inverted to

$$a_0^i \equiv a_0^i(a^i). \tag{5.3}$$

In the same way we have to consider the loop corrections to the derived observables, or rather to the S-matrix elements that allow to compute the derived observables. As an effect of these corrections, any tree level S-matrix element $S_0(a_0^i)$ goes into the renormalized one

$$S_0(a_0^i) \xrightarrow{RC} S_0(a_0^i) + \delta S(a_0^i) \equiv S(a_0^i) \tag{5.4}$$

again as a formal (and divergent) power series in $\hbar$.

Finally we get the desired (finite) connection between the renormalized S-matrix elements, needed to compute the derived observables, and the renormalized basic observables a^i, by considering the expansion in $\hbar$ of $S(a_0^i(a^i))$ both in $S(a_0^i)$ and in the $a_0^i(a^i)$ themselves. For example, at one loop, one will have

$$\begin{aligned} S(a_0^i(a^i)) &\cong S_0(a_0^i) + \delta^{(1)} S(a_0^i) \\ &\cong S_0(a^i) + \delta^{(1)} S(a^i) - \sum_i \frac{\partial S_0}{\partial a^i} \delta^{(1)} a^i \\ &\equiv S_0(a^i) + \Delta S^{(1)}(a^i), \end{aligned} \tag{5.5}$$

which shows explicitly that the finite one-loop shift $\Delta S^{(1)}(a^i)$ gets both a direct contribution $\delta^{(1)} S(a^i)$ and an indirect one, $-\sum_i (\partial S_0 / \partial a^i) \delta^{(1)} a^i$, from the shifts of the basic parameters. I shall follow this procedure consistently throughout these lectures. The fact that, in a two-loop calculation, other parameters come in which need to be renormalized, simply requires an enlargement of the basic observables.

6. RADIATIVE CORRECTIONS IN THE "GAUGE-LESS" LIMIT

We have already seen how significant the electromagnetic corrections are. With the purpose of dealing with the other corrections in order of importance, we now consider the radiative corrections to the various precision observables that grow like powers of the top-quark mass. From the early work of Veltman[11], it is well known that such effects do arise from top/bottom loop corrections to the W and Z vacuum polarization amplitudes, making the ρ-parameter deviate from one. Relatively more recently, an analogous effect has been pointed out[12], which leads to the presence of a GIM-violating $Z \to b\bar{b}$ vertex, also violently growing with

m_t Following Reference 13, one can deal with these effects by means of a Lagrangian that knows nothing about the gauge couplings. The point is that, at least for a very heavy top, the leading corrections both to the ρ-parameter and to the GIM violation $Z \to b\bar{b}$ vertex can be viewed as power series in the "top fine structure constant" $\alpha_t = g_t^2/4\pi$, where g_t is the top Yukawa coupling. These corrections have clearly nothing to do with the gauge couplings.

With this in mind, let us consider the SM Lagrangian for the third generation of quarks, with the vector bosons treated as external classical currents and the bottom Yukawa coupling neglected:

$$L = i\overline{Q}_L D Q_L + i\bar{t}_R D t_R + i\bar{b}_R D b_R + \left|D_\mu \varphi\right|^2 + V(\varphi) + \left(g_t \overline{Q}_L \varphi t_R + h.c.\right) \quad (6.1)$$

where $Q_L = (t, b)_L$ and φ is the Higgs doublet. As usual, to minimize $V(\varphi)$, we set

$$\varphi = \begin{pmatrix} v + \frac{1}{\sqrt{2}}(H + i\chi) \\ i\varphi^- \end{pmatrix}, \quad (6.2)$$

so that we obtain

$$L = \left|\partial_\mu \varphi^- - i\frac{gv}{\sqrt{2}} W_\mu^-\right|^2 + \frac{1}{2}\left|\partial_\mu \chi - i\frac{gv}{\sqrt{2}c} Z_\mu\right|^2 + \frac{1}{2}\left(\partial_\mu H\right)^2 + \\ i\bar{t}\partial t + \left(g_t v \bar{t}_L t_R + h.c.\right) + i\bar{b}\partial b + \ldots \quad (6.3)$$

The dots in (6.3) stand for all the interactions among the Goldstone bosons φ^-, χ, the physical Higgs fields H, and the t,b quarks, dependent upon the top Yukawa coupling g_t and the quartic Higgs coupling λ.

Suppose now that we perform loop corrections with this Lagrangian. We can compactly describe their results in terms of the following effective Lagrangian

$$L^{eff} = Z_2^\varphi \left|\partial_\mu \varphi^- - i\frac{gv}{\sqrt{2}} W_\mu^-\right|^2 + \frac{Z_2^\chi}{2}\left|\partial_\mu \chi - i\frac{gv}{\sqrt{2}c} Z_\mu\right|^2 + \frac{Z_2^H}{2}\left(\partial_\mu H\right)^2 + \\ iZ_{2L}^t \bar{t}_L \partial t_L + iZ_{2R}^t \bar{t}_R \partial t_R + iZ_{2L}^b \bar{b}_L \partial b_L + i\bar{b}_R \partial b_R + \left(g_t v Z_m^t \bar{t}_L t_R + h.c.\right) + \quad (6.4)$$

$$i\frac{Z_1}{v}\left(\partial_\mu\chi - \frac{gv}{\sqrt{2}c}Z_\mu\right)\bar{b}_L\gamma_\mu b_L + \ldots$$

which contains several (divergent) constants $Z_i = Z_i(g_t, \lambda)$. The important points about (6.4) are:

i) the presence of the $\chi\bar{b}b$ coupling, which must be of the derivative type in the χ field, because the b-quark is massless (the b_R does not interact);

ii) the fact that the derivative terms in the Goldstone boson fields keep the covariant form in terms of the classical fields W_μ and Z_μ, as it can explicitly be shown by means of the Ward identities of the SU(2)XU(1) global invariance[13].

In terms of the constants Z_i's appearing in (6.4) we can now express the renormalized observables. The gauge couplings do not get any corrections, so that, in particular

$$\alpha = \frac{g^2 s^2}{4\pi}. \tag{6.5}$$

The Z and the W masses, from (6.4), become

$$M_Z^2 = Z_2^\chi \frac{g^2 v^2}{2c^2}, \quad M_W^2 = Z_2^\varphi \frac{g^2 v^2}{2}. \tag{6.6}$$

The μ-decay, or the Fermi constant, is only affected through the corrections to the W mass, namely

$$G = \frac{\sqrt{2}g^2}{8M_W^2} = \frac{\sqrt{2}}{4v^2 Z_2^\varphi}. \tag{6.7}$$

One may also be interested in the neutral-current Fermi constant G^{NC}, defined in an analogous way to G, for example, from the elastic electron-neutrino amplitude at $q^2 = 0$

$$G^{NC} = \frac{\sqrt{2}g^2}{8M_Z^2 c^2} = \frac{\sqrt{2}}{4v^2 Z_2^\chi} \tag{6.8}$$

Finally, for the GIM-violating $Z \to b\bar{b}$ vertex, always from (6.4) and taking into account the renormalization of the b_L field, one has

$$V_\mu^{GIM}\left(Z \to \bar{b}b\right) = \frac{g}{2c}\frac{Z_1}{Z_2^b}\gamma_\mu \frac{1-\gamma_5}{2} \equiv \frac{g}{2c}\tau\,\gamma_\mu \frac{1-\gamma_5}{2} \tag{6.9}$$

According to the procedure outlined in Section 5, we have to express all the derived observables in terms of α, G and M_Z. We have therefore, from Equations (6.5-8)[14]:

$$G^{NC} = G\frac{Z_2^\varphi}{Z_2^\chi} \equiv G\rho \tag{6.10}$$

$$s^2c^2 = \frac{\pi\alpha}{\sqrt{2}GM_Z^2}\frac{Z_2^\chi}{Z_2^\varphi} = \frac{\pi\alpha}{\sqrt{2}GM_Z^2\rho} \tag{6.11}$$

$$M_W^2 = M_Z^2\rho c^2 \tag{6.12}$$

Knowing $\rho \equiv Z_2^\varphi / Z_2^\chi$ and $\tau \equiv Z_1/Z_2^b$ only, we can in fact express all the radiative effects that we are dealing with in this section. Since, from (6.6-7),

$$\frac{g^2}{2c^2} = \sqrt{2}GM_Z^2\rho, \tag{6.13}$$

for the Z widths into any pair of fermions other than b one has

$$\Gamma\left(Z \to f\bar{f} \neq b\bar{b}\right) = N_C\rho\frac{GM_Z^3}{6\pi\sqrt{2}}\left(g_{Vf}^2 + g_{Af}^2\right) \tag{6.14}$$

where g_{Vf}, g_{Af} have their tree level value with s^2 given by (6.11). On the other hand, for the $Z \to b\bar{b}$ width, taking into account of the GIM-violating contribution (6.9), one has

$$\Gamma\left(Z \to b\bar{b}\right) = \rho\frac{3GM_Z^3}{6\pi\sqrt{2}}\left(g_{Vb}^2 + g_{Ab}^2\right) \tag{6.15}$$

$$g_{Vb} = -\frac{1}{2}\left(1 - \frac{4}{3}s^2 + \tau\right),\ g_{Ab} = -\frac{1}{2}(1+\tau) \tag{6.16}$$

Finally, for the asymmetries at the Z-pole again the tree level expressions hold with s^2 given by (6.11). (The inclusion of τ in A_{FB}^b

according to (6.16) gives a numerically irrelevant correction due to the smallness of g_{Ve}).

7. HOW HEAVY CAN THE TOP BE?

Now that we have seen how the large m_t effects spread out, via ρ and τ, in all the precision observables, we can compute ρ and τ themselves in the SM using the Lagrangian (6.1). At one-loop level, the only diagrams that contribute to ρ and τ, are shown in Figs.1a and b respectively.

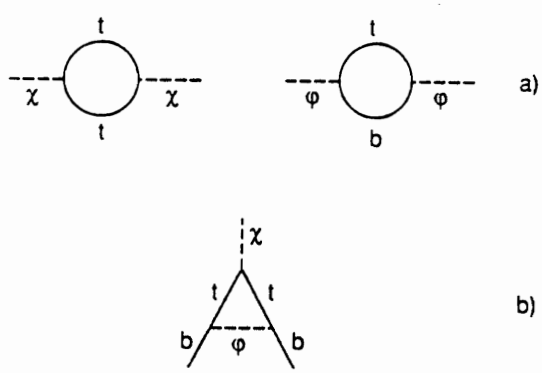

Figure 7.1 Relevant one-loop diagrams contributing to ρ (a) and τ (b).

From them it is immediate to get the well-known leading-order contributions[11,12]

$$\rho - 1 \approx 3x, \quad \tau = -2x \tag{7.1}$$

where we have set

$$x = \frac{Gm_t^2}{8\pi\sqrt{2}}, \tag{7.2}$$

which, in leading order, coincides with $\alpha_t/8\pi$.

Quite a few more diagrams contribute at the two-loop level. Nevertheless, the simplification achieved by working with the Goldstone Lagrangian rather than the gauge Lagrangian itself makes even the two-loop calculation quite manageable[13]. At this order, also the Higgs quartic coupling λ comes in, which can be traded for the Higgs mass m_H by means of the lowest-order relation $m_H^2 = 4\lambda v^2$. In other words, expanding

in x, the second-order coefficients of both ρ and τ are functions of m_H^2/m_t^2. For $m_H^2/m_t^2 << 1$, they are[f4]

$$\rho - 1 \approx 3x\left(1 + x\left(22 - 2\pi^2\right)\right) \tag{7.3}$$

$$\tau \approx -2x\left(1 + x\left(9 - \pi^2/3\right)\right) \tag{7.4}$$

whereas, for $r \equiv m_t^2/m_H^2 << 1$,

$$\rho - 1 \approx 3x\left(1 + \frac{x}{4}\left(61 + 4\pi^2 + 54\lg r + 6\lg^2 r\right)\right) \tag{7.5}$$

$$\tau \approx -2x\left(1 + \frac{x}{144}\left(311 + 24\pi^2 + 282\lg r + 90\lg^2 r\right)\right) \tag{7.6}$$

Needless to say, we have obtained finite second-order coefficients because we have expressed α_t in favour of the basic observables G and m_t. It is

$$\alpha_t = \frac{g_t^2}{4\pi} = \frac{Gm_t^2}{\pi\sqrt{2}} \frac{Z_2^{\varphi} Z_{2L}^t Z_{2R}^t}{\left(Z_m^t\right)^2} \tag{7.7}$$

where the top renormalization constants have the usual meaning, as in (6.4), except that they are computed at the pole of the top propagator, since m_t is defined as the position of the pole itself.

Figures 7.2 show ρ and τ as functions of m_t for different values of m_H[13]. The expansion up to second order is well convergent for all $m_t \leq 300$ GeV, $m_H \leq 2$ TeV. This gives us confidence in the use of the perturbative calculation to get an upper bound on m_t from the comparison with the experimental data, since the bound that one obtains is well inside this region. If we use, say, m_t=300 GeV (and any $m_H \leq 2$ TeV), where ρ and τ give the largely dominating corrections, and plug their values in the expressions for any at the observables given in the previous section, we obtain in all cases striking deviations from the data (see Table 1, column C). To get an accurate determination of the bound

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[f4] Equation (7.3) confirms a previous result obtained in Reference 15.

itself, one actually needs to take into account all the other electroweak corrections that do not grow like powers of m_t (see Section 10).

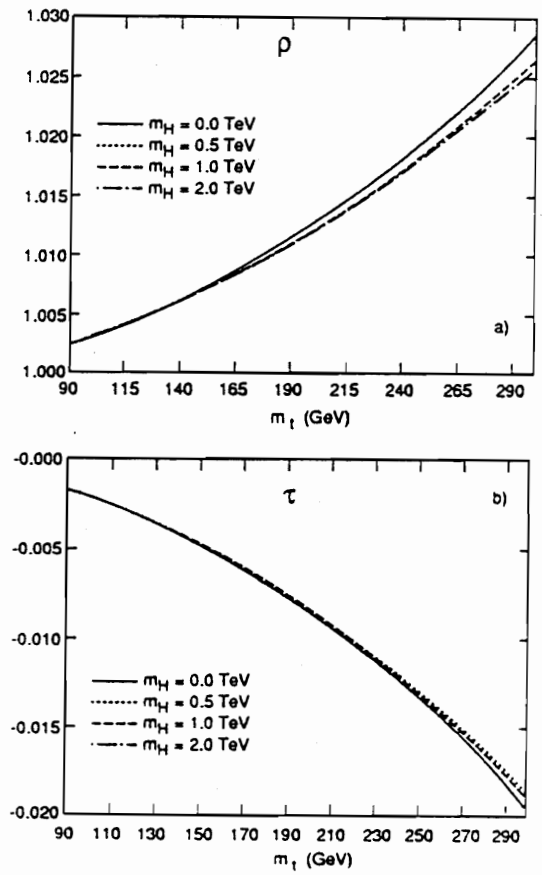

Figure 7.2 The ρ parameter (a) and the GIM-violating $Z \to b\bar{b}$ coupling τ (b) as functions of the top mass, for different values of the Higgs mass.

8. GENERAL ONE-LOOP EXPRESSIONS OF THE ELECTROWEAK RADIATIVE CORRECTIONS

It is time that we discuss the full electroweak radiative correction effects. To this purpose, I give in the following the general expressions of the derived observables in terms of the one-loop corrected Green functions, in such a way that one will no longer have to worry about the renormalization procedure. In doing this, I closely follow the scheme outlined in Section 5.

The relevant quantities, or the needed ingredients, are:

i) the vacuum polarization amplitudes for the W, Z and γ

$$\Pi^{ij}_{\mu\nu}(q) = -ig_{\mu\nu}\left[A^{ij}(0) + q^2 F^{ij}\left(q^2\right)\right] + q_\mu q_\nu \; terms \tag{8.1}$$

where i,j = W,. γ, Z or possibly i,j = 3,0 for the W_3 or the B-boson respectively;

ii) the contributions to the vector and the axial form factors at $q^2 = M_Z^2$ in the $Z \to f\,\bar{f}$ vertex from proper vertex diagrams and fermion self energies only

$$-i\frac{e}{2sc}\bar{v}\gamma_\mu\left(\delta g_{Vf} - \gamma_5 \delta g_{Af}\right)u \tag{8.2}$$

iii) all the one-loop corrections except the vacuum polarization (boxes, vertices and fermion self-energies) to the μ-decay amplitude at zero external momenta

$$-i\delta G_{V,B}\left(\bar{e}\gamma_\mu(1-\gamma_5)\nu_e\right)\left(\bar{\nu}_\mu\gamma_\mu(1-\gamma_5)\mu\right) \tag{8.3}$$

Before proceeding further, several comments are in order. There is, in principle, no special reason to isolate the vector boson vacuum polarization amplitudes (8.1) from the remaining terms: the motivation for doing this will be clear in the following. In the expression (8.2) for the $Z \to f\,\bar{f}$ vertex, also a magnetic form factor might be introduced. We assume that the chiral symmetries associated with the various fermions, in the theory under consideration, are controlled by their masses (or their Yukawa couplings). This assumption makes the magnetic form factors negligible to the present purposes. The various functions defined in (8.1-3) have in general some imaginary parts. In view of the present phenomenological constraints, we assume that possible further contributions to the imaginary parts from new relatively light particles give negligible effects. Finally, there is the obvious comment that any of the individual form factors defined in (8.1-3) is in general neither finite nor gauge-invariant. It is only when their different contributions are grouped together in the derived observables that finiteness and gauge-invariance will be restored.

In terms of the quantities defined in (8.1-3), we can first express the shifts of the basic observables (or the input parameters) as defined in Section 5. They are:

$$\alpha = \alpha_0 + \delta\alpha; \quad \frac{\delta\alpha}{\alpha} = -F^{\gamma\gamma}(0) - 2\frac{s}{c}\frac{A^{\gamma Z}(0)}{M_Z^2} \tag{8.4}$$

$$M_Z^2 = M_{Z0}^2 + \delta M_Z^2; \quad = -A_{ZZ}(0) - M_Z^2 F_{ZZ}\left(M_Z^2\right) \tag{8.5}$$

$$G = G_0 + \delta G; \quad \frac{\delta G}{G} = \frac{A_{WW}(0)}{M_W^2} + \frac{\delta G_{V,B}}{G} \tag{8.6}$$

The derived observables of interest to us are: the Z width into a pair of fermions, $Z \to f\,\bar{f}$, the asymmetries at the Z pole and the W mass. At one loop, these observables receive direct contributions from the amplitudes (8.1-3) as well as corrections due to the shifts (8.4-6) of the input parameters. For the W mass one immediately obtains

$$M_W^2 = M_Z^2 c^2 + \delta M_W^2 - \frac{M_Z^2 c^2}{s^2 - c^2}\left(s^2 \frac{\delta\alpha}{\alpha} - s^2 \frac{\delta G}{G} - c^2 \frac{\delta M_Z^2}{M_Z^2}\right) \tag{8.7}$$

where

$$s^2 = 1 - c^2 = \frac{1}{2}\left[1 - \left(1 - \frac{4\pi\alpha}{\sqrt{2} G M_Z^2}\right)^{1/2}\right] \tag{8.8}$$

and

$$\delta M_W^2 = -A_{WW}(0) - M_W^2 F_{WW}\left(M_W^2\right) \tag{8.9}$$

To obtain the Z widths and the asymmetries, one first has to write down the $e^+e^- \to f\,\bar{f}$ amplitude close to the Z pole

$$\begin{aligned} A\left(e^+e^- \to f\,\bar{f}\right) &= \sqrt{2} G_0 M_{Z0}^2 \left(1 - F_{ZZ}\left(M_Z^2\right) - M_Z^2 F'_{ZZ}\left(M_Z^2\right)\right) \frac{1}{q^2 - M_Z^2} \\ &\times \bar{v}_e \gamma_\mu \left(\left(-\frac{1}{2} + 2s_0^2 + 2scF_{Z\gamma}\left(M_Z^2\right) + \delta g_V\right) - \gamma_5\left(-\frac{1}{2} + \delta g_A\right)\right) u_e \\ &\times \bar{v}_f \gamma_\mu \left(\left(T_{3f} - 2s_0^2 Q_f - 2scQ_f F_{Z\gamma}\left(M_Z^2\right) + \delta g_{Vf}\right) - \gamma_5\left(T_{3f} + \delta g_{Af}\right)\right) u_f \end{aligned} \tag{8.10}$$

from which, using the shifts (8.1-3), one gets

$$\begin{aligned} \Gamma\left(Z \to f\,\bar{f}\right) = \frac{G M_Z^3}{6\pi\sqrt{2}} N_C &\left(1 - \frac{\delta G}{G} - \frac{\delta M_Z^2}{M_Z^2} - F_{ZZ}\left(M_Z^2\right) - F'_{ZZ}\left(M_Z^2\right)\right) \\ &\times \left(\left(g_{Vf} + \Delta g_{Vf}\right)^2 + \left(g_{Af} + \Delta g_{Af}\right)^2\right) \end{aligned} \tag{8.11}$$

$$A_{FB}^l = 3 \frac{\left(g_{Vl} + \Delta g_{Vl}\right)^2 \left(g_{Al} + \Delta g_{Al}\right)^2}{\left(\left(g_{Vl} + \Delta g_{Vl}\right)^2 + \left(g_{Al} + \Delta g_{Al}\right)^2\right)^2} \tag{8.12}$$

where

$$g_{Vf} = T_{3f} - 2s^2 Q_f, \qquad g_{Af} = T_{3f} \tag{8.13}$$

$$\Delta g_{Vf} = 2Q_f \delta s^2 + \delta g_{Vf} - 2scQ_f F_{Z\gamma}(M_Z^2), \qquad \Delta g_{Af} = \delta g_{Af} \tag{8.14}$$

$$\delta s^2 = -\frac{s^2 c^2}{c^2 - s^2}\left(\frac{\delta G}{G} + \frac{\delta M_Z^2}{M_Z^2} - \frac{\delta\alpha}{\alpha}\right) \tag{8.15}$$

Both in the amplitude (8.10) and in the forward-backward asymmetry (8.12) we have neglected possible box diagram contributions, which is legitimate for observables at the Z pole. Analogous expressions hold for A_{Pol}^{τ} and A_{FB}^{b}, with the lepton couplings g_{Vl}, g_{Al} replaced by $g_{Vl} + \Delta g_{Vl}$ and $g_{Al} + \Delta g_{Al}$ as in (8.12). We have already mentioned that the forward-backward b-asymmetry is practically insensitive to (reasonable) corrections to the $Z - b\bar{b}$ couplings. For them one can then simply take the tree level expressions with s^2 given in (8.8).

It is customary, and useful for later purposes, to define three auxiliary dimensionless parameters $\Delta r_W, \Delta\rho, \Delta k'$ which are in direct correspondence with three derived observables $M_W, \Gamma(Z \to l^+ l^-)$ and A_{FB}^{l} via[16,17]

$$\left(1 - \frac{M_W^2}{M_Z^2}\right)\frac{M_W^2}{M_Z^2} \equiv \frac{\pi\alpha}{\sqrt{2} M_Z^2 (1 - \Delta r_W)} \tag{8.16}$$

$$\Gamma(Z \to l^+ l^-) = \frac{G M_Z^3}{6\pi\sqrt{2}}\left(g_V^2 + g_A^2\right)\left(1 + \frac{3}{4}\frac{\alpha}{\pi}\right) \tag{8.17}$$

$$A_{FB}^{l} \equiv \frac{3 g_V^2 g_A^2}{\left(g_V^2 + g_A^2\right)^2} \tag{8.18}$$

where

$$g_A \equiv -\frac{1}{2}\left(1 + \frac{\Delta\rho}{2}\right), \qquad \frac{g_V}{g_A} \equiv 1 - 4s^2(1 + \Delta k') \tag{8.19}$$

and s^2 given in (8.8). Notice that g_V and g_A defined in (8.17,18) have only an auxiliary role, being related to $\Delta\rho, \Delta k'$ through (8.19). By looking at Eqs. (6.14) and (8.17-19), notice also the relation between $\Delta\rho$ and the ρ parameter defined in Section 6, $\rho \approx 1 + \Delta\rho$. In $\Gamma(Z \to l^+ l^-)$

we have inserted the final state radiation factor, whereas by A_{FB}^{l} we mean the asymmetry at the pole, after extraction of the γZ interference effect. From these definitions one obtains[18]

$$\Delta r_W = \frac{\delta G}{G} + \frac{c^2}{s^2}\frac{\delta M_Z^2}{M_Z^2} - \frac{\delta\alpha}{\alpha} + \frac{s^2 - c^2}{c^2}\frac{\delta M_W^2}{M_W^2} \tag{8.20}$$

$$\Delta\rho = -\frac{\delta G}{G} - \frac{\delta M_Z^2}{M_Z^2} - F_{ZZ}\left(M_Z^2\right) - M_Z^2 F'_{ZZ}\left(M_Z^2\right) - 4\Delta g_{Al} \tag{8.21}$$

$$\Delta k' = \frac{1}{2s^2}\left(\Delta g_{Vl} - \left(1 - 4s^2\right)\Delta g_{Al}\right) \tag{8.22}$$

This is as much as one can do in general, without specifying the actual theory that one wants to deal with: the SM or something else.

9. THE ε-PARAMETERS

In any given theory of the type defined in Section 2, it becomes now a rather straightforward matter to compute explicitly from the various equations in Section 8 the one-loop electroweak corrections, without having to worry about the different subtleties of the renormalization procedure, since everything has already been taken care of. Let us take however a further general step, still with the purpose of being able to deal with different models in an efficient, unified way[17,19,20]. The main purpose, apart from avoiding the restriction of the analysis to the SM only, is to try to isolate, in the measurements, effects of some special significance, related to particularly delicate sectors of the theory of the weak interactions. This may help to clarify the physical origin of any deviation from the Standard Model that will possibly be found, or to discard theories that require some specific pattern of radiative effects, which turns out to be incompatible with the data.

To serve this purpose, under suitable hypotheses, I will write any of the precision observables as

$$O_i = O_i^0\left(1 + \sum_{j=1}^{4} a_{ij}\varepsilon_j\right) \tag{9.1}$$

where:

i) O_i^0 is the corresponding prediction of the theory in the Born approximation and ***including*** the QED radiative-corrections effects;

ii) ε_j are four dimensionless parameters containing all the "genuine" electroweak radiative-correction effects, dependent, as such, on m_t or m_H, or on any other parameter of similar nature;
iii) a_{ij} are fixed numerical constants.

Up to now, the only restriction on the possible theories is that they should all give the same O_i^0 as the SM. By now, in view of the present data, this is a very interesting set of theories. To define the ε_j, j = 1,2,3,b, one picks up four observables of particular interest, M_W / M_Z, Γ_l (the Z leptonic width into charged leptons), A_{FB}^l (the forward-backward asymmetry at the Z peak for charged leptons) and Γ_b (the Z width into a $b\bar{b}$ pair) and puts them in suitable correspondence with the four ε_j. Such correspondence is set according to the following principles:
i) Two sectors in the physics of the electroweak interactions are considered as more likely to deviate from the SM and, as such, more interesting: the gauge-boson vacuum polarization amplitudes and the $Z \to b\bar{b}$ vertex.
ii) In defining the ε_j, one should avoid having to specify the top-quark mass, which is at present a main source of uncertainty in the comparison of theory and experiment.

With this in mind, in an effective Lagrangian description, 4 coefficients are of particular interest, three of them related to the masses and the kinetic terms of the gauge bosons

$$L_{VB}^{eff} = -\frac{1}{2}W_{\mu\nu}^{+}W_{\mu\nu}^{-} - \frac{1}{4}(1-e_2)W_{\mu\nu}^{3}W_{\mu\nu}^{3} - \frac{1}{4}B_{\mu\nu}B_{\mu\nu} - \frac{1}{2}\frac{s}{c}e_3 B_{\mu\nu}W_{\mu\nu}^{3}$$
$$-\frac{1}{2}M_Z^2\left(cW_\mu^3 - sB_\mu\right)^2 - M_Z^2c^2(1+e_1)W_{\mu\nu}^{+}W_{\mu\nu}^{-} \qquad (9.2)$$

and a fourth one to the GIM-violating $Z \to b\bar{b}$ vertex

$$V_\mu^{GIM}\left(Z \to \bar{b}b\right) = \frac{g}{2c}e_b\,\gamma_\mu\frac{1-\gamma_5}{2} \qquad (9.3)$$

(In the SM, and keeping only the power law corrections in m_t, e_b coincides with τ, defined in Section 5). The ε_j are in a one-to-one correspondence with these e_j, j = 1,2,3,b, in the sense that

$$\varepsilon_1 = e_1 + \ldots,$$
$$\varepsilon_2 = e_2 + \ldots,$$

$$\varepsilon_3 = e_3 + \ldots,$$
$$\varepsilon_b = e_b + \ldots, \tag{9.4}$$

where the dots stand for other effects, related to higher dimensional vacuum polarization operators, vertex corrections, etc. Their precise definition is as follows.

For ε_1, ε_2 and ε_3, we introduce the following linear combinations of $\Delta\rho$, $\Delta k'$ and Δr_W, given in the previous Section[17,19]:

$$\begin{aligned} \varepsilon_1 &= \Delta\rho \\ \varepsilon_2 &= c^2\Delta\rho + \frac{s^2\Delta r_W}{\left(c^2 - s^2\right)} - 2s^2\Delta k' \\ \varepsilon_3 &= c^2\Delta\rho + \left(c^2 - s^2\right)\Delta k' \end{aligned} \tag{9.5}$$

We further define ε_b from Γ_b, the inclusive partial width for $Z \to b\bar{b}$, according to the relation[20]

$$\Gamma_b = \frac{GM_Z^3}{2\pi\sqrt{2}}\beta\left(\frac{3-\beta^2}{2}g_{Vb}^2 + \beta^2 g_{Ab}^2\right)R_{QCD}\left(1 + \frac{\alpha}{12\pi}\right) \tag{9.6}$$

where $\beta = \sqrt{1 - 4m_b^2/m_Z^2}$, with m_b=4.8 GeV, R_{QCD} is the QCD correction factor given by

$$R_{QCD} = 1 + 1.2a - 1.1a^2 - 13a^3; \quad a = \frac{\alpha_s(m_Z)}{\pi} \tag{9.7}$$

(for $\alpha_s(M_Z) = 0.118$, R_{QCD}= 1.0428) and g_{Vb}, g_{Ab} are specified as follows

$$g_{Ab} = -\frac{1}{2}\left(1 + \frac{\Delta\rho}{2}\right)(1 + \varepsilon_b) \tag{9.8}$$

$$\frac{g_{Vb}}{g_{Ab}} = \frac{1 - 4/3(1 + \Delta k')s^2 + \varepsilon_b}{1 + \varepsilon_b} \tag{9.9}$$

The physical meaning of the ε_i can be understood by looking at their explicit expressions in terms of the amplitudes defined in Section 8. After linearization, one obtains

$$\varepsilon_1 = e_1 - e_5 - \frac{\delta G_{V,B}}{G} - 4\delta g_{Al} \tag{9.10}$$

$$\varepsilon_2 = e_2 - s^2 e_4 - c^2 e_5 - \frac{\delta G_{V,B}}{G} - \delta g_{Vl} - 3\delta g_{Al} \tag{9.11}$$

$$\varepsilon_3 = e_3 + c^2 e_4 - c^2 e_5 + \frac{c^2 - s^2}{2s^2}\delta g_{Vl} - \frac{1+2s^2}{2s^2}\delta g_{Al} \tag{9.12}$$

$$\varepsilon_b = e_b + \tag{9.13}$$

$$\frac{1}{2-\frac{4}{3}s^2}\left[\left(1-\frac{4}{3}s^2\right)\delta g_{Vd} + \delta g_{Ad} + \frac{2}{3}\left(1-\frac{4}{3}s^2\right)\delta g_{Vl} - \frac{2}{3}\left(-5+\frac{8}{3}s^2\right)\delta g_{Al}\right]$$

where, for the contributions of the vacuum polarization amplitudes, we introduce the combinations

$$e_1 = \frac{A_{33}(0) - A_{WW}(0)}{M_W^2} \tag{9.14}$$

$$e_2 = F_{WW}\left(M_W^2\right) - F_{33}\left(M_Z^2\right) \tag{9.15}$$

$$e_3 = \frac{c}{s} F_{30}\left(M_Z^2\right) \tag{9.16}$$

$$e_4 = F_{\gamma\gamma}(0) - F_{\gamma\gamma}\left(M_Z^2\right) \tag{9.17}$$

$$e_5 = M_Z^2 \frac{dF_{ZZ}}{dq^2}\left(M_Z^2\right) \tag{9.18}$$

Notice that, for the first three of these coefficients, we have used the same notations as for the parameters defined in the effective Lagrangian (9.2). Indeed they coincide, which clearly illustrates the connection between the ε_i and the e_i. The ε_i are directly measurable quantities, which receive all sort of different kinds of corrections: constant and first (e_1, e_2, e_3) and higher derivatives (e_4, e_5) of the vacuum polarization amplitudes, as well as vertex and box corrections(f5) .

The relations between the basic observables and the epsilons can be linearised and inverted, leading to the formulae

$$\varepsilon_1 = -0.9882 + 0.011963\frac{\Gamma_l}{MeV} - 0.1511x \tag{9.19}$$

$$\varepsilon_3 = -0.7146 + 0.009181\frac{\Gamma_l}{MeV} - 0.69735x \tag{9.20}$$

(f5) The parameters e_1, e_2, e_3 are proportional to the variables S,T,U, also used in the literature to analyze the electroweak precision tests[21,22]

amount of model dependence is introduced by including other purely leptonic quantities at the Z pole such as A^{τ}_{pol}, A_e (measured[6] from the angular dependence of the τ polarisation) and A_{LR}. At this stage, one is simply relying on lepton universality. Indeed, with the definition of $x = g_V/g_A$ in Equation (8.18) in terms of A^{l}_{FB}, the data on A^{τ}_{pol}, A_e and A_{LR}(f6) can be taken into account by using the simple formulae

$$A^{\tau}_{pol} = A_e = A_{LR} = \frac{2x}{1+x^2} \tag{10.4}$$

with the same x as defined from A^{l}_{FB}, apart from still negligible differences. With essentially the same assumptions one can also include the data on the b-quark forward backward asymmetry A^{b}_{FB}. As mentioned, A^{b}_{FB} is almost unaffected by the $Z \to b\bar{b}$ vertex correction.

As a result of the previous discussion, we can combine the values of $x = g_V/g_A$ obtained from the whole set of asymmetries measured at LEP and we get the value:

$$x = g_V/g_A = 0.0725 \pm 0.0033 \tag{10.5}$$

At this stage the best values of $\varepsilon_1, \varepsilon_2, \varepsilon_3$ and ε_b are modified according to

$$\begin{aligned} \varepsilon_1 &= (0.0 \pm 3.4)\,10^{-3} \\ \varepsilon_2 &= (-8.3 \pm 7.6)\,10^{-3} \\ \varepsilon_3 &= (1.6 \pm 3.5)\,10^{-3}. \\ \varepsilon_b &= (-4.5 \pm 7.0)\,10^{-3} \end{aligned} \tag{10.6}$$

All observables measured on the Z peak at LEP can be included in the analysis provided that we assume that all deviations from the Standard Model are only contained in vacuum polarisation diagrams (without demanding a truncation of the q^2 dependence of the corresponding funtions) and/or the $Z \to b\bar{b}$ vertex. A set of primary measurements at LEP contains Γ_Z, $R = \Gamma_h/\Gamma_l$, the hadronic peak cross section σ_h (with a

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(f6) Recently the SLD Collaboration at SLAC has published the first measurement of A_{LR} with a polarised beam[26]. The result, A_{LR}=0.100 ± 0.044, is not included in the present analysis. It would change the average obtained at LEP from A^{t}_{pol} and A_e, which from table 3 is 0.138 ± 0.014, to 0.135 ± 0.013.

3x3 correlation matrix given in Reference 6), $R_{bh} = \Gamma_b/\Gamma_h$ plus the asymmetries that lead to the value of x in Equation (10.5). The relations between these quantities and the epsilons can be written down in the following linearised form:

$$\begin{aligned}
\Gamma_Z &= \Gamma_Z^0(1+1.35\varepsilon_1 - 0.46\varepsilon_3 + 0.35\varepsilon_b) \\
R &= R^0(1+0.28\varepsilon_1 - 0.36\varepsilon_3 + 0.50\varepsilon_b) \\
\sigma_h &= \sigma_h^0(1-0.03\varepsilon_1 + 0.04\varepsilon_3 - 0.20\varepsilon_b) \\
R_{bh} &= R_{bh}^0(1-0.06\varepsilon_1 + 0.07\varepsilon_3 + 1.79\varepsilon_b) \\
x &= x^0(1+17.6\varepsilon_1 - 22.9\varepsilon_3)
\end{aligned} \tag{10.7}$$

The constant values are functions of $\alpha_s(M_Z)$ and also of $\alpha(M_Z)$. Defining

$$\delta a_s = \left[\alpha_s(M_Z) - 0.118\right]/\pi\,;\ \delta\alpha = \left[\alpha(M_Z) - 1/128.87\right]/\alpha(M_Z)\,; \tag{10.8}$$

we have, from the code of Reference 23:

$$\begin{aligned}
\Gamma_Z^0 &= 2488.1(1+0.73\delta a_s - 0.35\delta\alpha)MeV \\
R^0 &= 20.812(1+1.05\delta a_s - 0.28\delta\alpha) \\
\sigma_h^0 &= 41.425(1-0.41\delta a_s + 0.03\delta\alpha)\,nb \\
R_{bh}^0 &= 0.21818(1+0.15\delta a_s + 0.06\delta\alpha) \\
x^0 &= 0.07526 - 1.32\delta\alpha
\end{aligned} \tag{10.9}$$

As anticipated, at the one-loop level, all the dependence on m_t and m_H of the quantities in Equations (10.7) enters through the epsilons and not in the various numerical coefficients.

One can now make a global fit of the data on M_W/M_Z, Γ_Z, $R = \Gamma_h/\Gamma_l$,, σ_h, $R_{bh} = \Gamma_b/\Gamma_h$, given in Table 4 and the results on the asymmetries, summarised by the value of x in Equation (10.5). The result of the fit is:

$$\begin{aligned}
\varepsilon_1 &= (-0.3 \pm 3.2)\,10^{-3} - 0.1\delta a_s \\
\varepsilon_2 &= (-8.5 \pm 7.6)\,10^{-3} - 0.11\delta a_s + 0.23\delta a \\
\varepsilon_3 &= (1.5 \pm 3.3)\,10^{-3} - 0.04\delta a_s - 0.77\delta a. \\
\varepsilon_b &= (3.1 \pm 4.5)\,10^{-3} - 1.42\delta a_s
\end{aligned} \tag{10.10}$$

	$m_t(GeV)$	$m_H =$ 50 GeV	100	500	1000
$\varepsilon_2 10^3$	100	-5.30	-5.13	-4.82	-4.81
	125	-6.31	-6.04	-5.82	-5.69
	150	-7.15	-6.90	-6.59	-6.40
	175	-8.00	-7.70	-7.23	-7.07
	200	-8.83	-8.57	-8.03	-7.82
	225	-9.89	-9.55	-8.89	-8.61
	250	-11.2	-10.7	-9.90	-9.64
	275	-12.7	-12.1	-11.1	-10.7
	300	-14.6	-13.9	-12.6	-12.0

	$m_t(GeV)$	$m_H =$ 50 GeV	100	500	1000
$\varepsilon_3 10^3$	100	4.75	5.49	6.78	7.12
	125	4.51	5.34	6.47	6.88
	150	4.31	5.08	6.24	6.65
	175	4.07	4.85	6.07	6.42
	200	3.93	4.65	5.79	6.19
	225	3.69	4.45	5.59	5.92
	250	3.42	4.22	5.32	5.57
	275	3.16	3.94	4.98	5.27
	300	2.78	3.57	4.57	4.88

Table 3. Values of $\varepsilon_b\ 10^3$ obtained from Equations (9.19-21) and the programme ZFITTER[23] with $\alpha_s(m_Z) = 0.118$.

$m_t(GeV)$	$\varepsilon_b\ 10^3$
100	-1.12
125	-2.82
150	-4.88
175	-7.13
200	-9.79
225	-12.8
250	-16.1
275	-19.9
300	-23.9

10. DETERMINATION OF THE EPSILONS FROM THE DATA

The experimental results that will be used in the present analysis are collected in Table 4. We start by deriving from the data on the input observables the values of the epsilons that follow by directly applying their definition, with no dynamical assumptions.

By combining the value of M_W/M_Z [7] with the LEP values for the Z mass, the charged lepton partial width and the forward-backward asymmetry[6], assuming $e-\mu-\tau$ universality, given in Table 4, one finds, from Equations (8.16-19)

$$\begin{aligned} \Delta r &= (-16 \pm 16)\, 10^{-3} \\ \Delta\rho &= (-0.3 \pm 3.4)\, 10^{-3} \\ \Delta k' &= (0.9 \pm 5.0)\, 10^{-3} \end{aligned} \tag{10.1}$$

and correspondingly, from Equations (9.5)

$$\begin{aligned} \varepsilon_1 &= (-0.3 \pm 3.4)\, 10^{-3} \\ \varepsilon_2 &= (-7.6 \pm 7.6)\, 10^{-3} \\ \varepsilon_3 &= (0.4 \pm 4.2)\, 10^{-3}. \end{aligned} \tag{10.2}$$

Table 4. List of experimental values used in the text[6,7]

$m_Z(GeV)$	91.187 ± 0.007
m_W/m_Z	0.8798 ± 0.0028
$\Gamma_T(MeV)$	2488 ± 7
$R = \Gamma_h/\Gamma_l$	20.83 ± 0.06
$\sigma_h = 12\pi\Gamma_e\Gamma_h/m_Z^2\Gamma_T^2\,(nb)$	41.44 ± 0.17
$\Gamma_l(MeV)$	83.52 ± 0.28
$\Gamma_h(MeV)$	1739.7 ± 6.3
$\Gamma_b(MeV)$	383 ± 6
$R_{bh} = \Gamma_b/\Gamma_h$	0.220 ± 0.003
A_{FB}^l	0.0164 ± 0.0021
A_{pol}^τ	0.142 ± 0.017
A_e	0.130 ± 0.025
A_{FB}^b	0.098 ± 0.009

Finally from the value of Γ_b listed in Table 4 one finds, by means of Equations (9.6-9) or (9.21):

$$\varepsilon_b = (4.4 \pm 7.0)\, 10^{-3} \tag{10.3}$$

To proceed further, and include other measured observables in the analysis we need to make some dynamical assumptions. The minimum

$$\varepsilon_b = -0.62\varepsilon_1 + 0.24\varepsilon_3 + 0.436\left(\frac{\Gamma_b}{\Gamma_b^0} - 1\right) \tag{9.21}$$

$$\varepsilon_2 = 1.43\varepsilon_1 - 0.86\varepsilon_3 + 0.43\Delta r_W \tag{9.22}$$

where $x = g_V/g_A$ is defined in Equation (8.18) from A_{FB}^l. The quantity Γ_b^0 is the value of Γ_b in the limit when all epsilons are neglected. With $\alpha_S(M_Z) = 0.118$, $\Gamma_b^0 = 379.4\ MeV$ (more in general its value is given by Equations (9.6,7)).

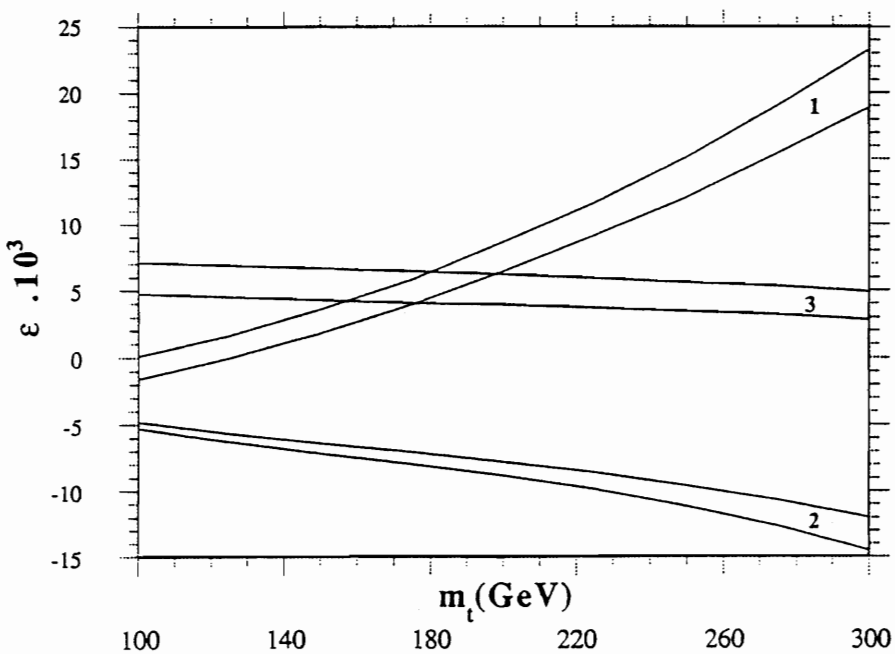

Figure 9.1 The quantities $\varepsilon_1, \varepsilon_2$ and ε_3 as predicted in the SM as functions of m_t. The bands correspond to the dependence on m_H in the range $m_H = 50 - 1000\ GeV$

Equations (9.19-22) are also used to obtain, by definition, the behaviour of the epsilons in the SM as functions of m_t and m_H. By means of the latest version of the programme ZFITTER[23], one can compute Γ_l, Γ_b, x and Δr_W as functions of m_t and m_H. In this way one obtains the results for $\varepsilon_1, \varepsilon_2$ and ε_3 given in Table 2 and shown in figure 9.1, while the result for ε_b , which is practically independent of m_H, is given in Table 3 and plotted in figure 9.2 for $\alpha_S(m_Z) = 0.118$. The above results were also checked by using the new code TOPAZ0[24].

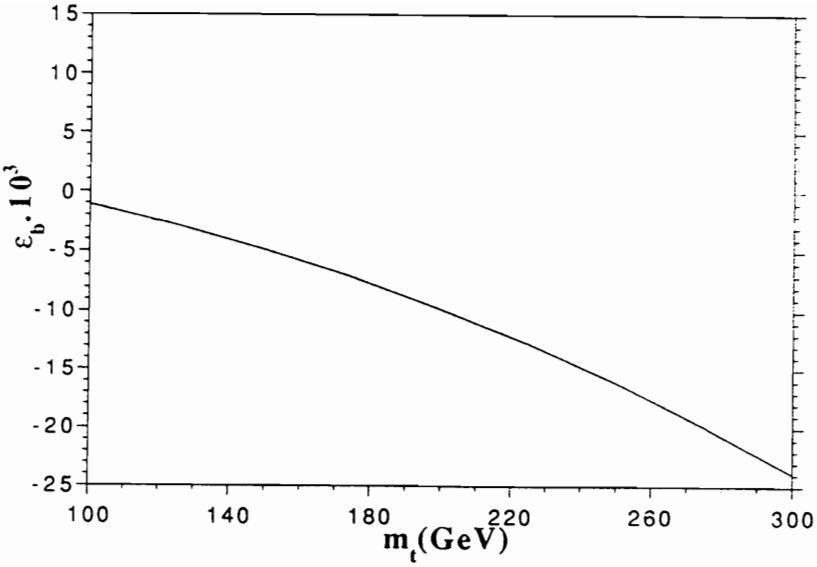

Figure 9.2 The quantity ε_b as predicted in the SM for $\alpha_S(M_Z) = 0.118$.

An important property of these parameters is that, in the SM, the dependence on m_t and m_H of all observables at the Z pole only enters through the epsilons (see Section 10). This is also true in any extension which deviates from the SM in vacuum polarizations and/or in the $Z \to b\bar{b}$ vertex. This is an exact statement if one loop corrections are only considered. At two loops, some residual dependence on m_t is induced in the various observables from terms of order α_S^2 [25] which can be reabsorbed in the epsilons only in an approximate but sufficiently accurate way.

Table 2. Values of $\varepsilon_i.10^3$ obtained from Equations (9.19-22) and the programme ZFITTER[23].

	$m_t(GeV)$	$m_H =$ 50 GeV	100	500	1000
$\varepsilon_1 10^3$	100	0.09	0.05	-0.87	-1.59
	125	1.63	1.71	0.56	-0.05
	150	3.58	3.55	2.41	1.80
	175	5.83	5.80	4.67	3.94
	200	8.59	8.45	7.10	6.39
	225	11.6	11.5	9.95	9.12
	250	15.1	14.9	13.1	12.0
	275	19.0	18.8	16.5	15.4
	300	23.3	23.0	20.2	18.9

where the dependence on the values of $\alpha_s(M_Z)$ and $\alpha(M_Z)$ has also been indicated. If we neglect possible contributions from δa and add in quadrature the effect of a ± 0.007 error on $\alpha_s(M_Z)$ around the central value 0.118, we obtain:

$$\begin{aligned} \varepsilon_1 &= (-0.3 \pm 3.2)\, 10^{-3} \\ \varepsilon_2 &= (-8.5 \pm 7.6)\, 10^{-3} \\ \varepsilon_3 &= (1.5 \pm 3.3)\, 10^{-3}. \\ \varepsilon_b &= (3.1 \pm 6.5)\, 10^{-3} \end{aligned} \tag{10.11}$$

The errors induced from $\alpha_s(M_Z)$ are only significant in the case of ε_b.

Provided that all deviations from the SM are only contained in vacuum polarizations and/or in the $Z \to b\bar{b}$ vertex, Equations (10.7) apply. For that to be true, it is important that we focus on observables at the intermediate gauge boson scale of mass. To include in the analysis lower energy observables as well, a stronger hypothesis needs to be made: only vacuum polarization diagrams are allowed to vary significantly from the SM ones and only in their constant and first derivative terms in a q^2-expansion[21], a likely picture, e.g., in technicolour theories. In such a case, to Equations (10.7) one can add the linearized expressions, in terms of the epsilons, of the ratio R_ν of neutral to charged current processes in deep inelastic neutrino scattering on nuclei[27]

$$R_\nu = 0.3115 + 0.83\varepsilon_1 - 0.27\varepsilon_3 \tag{10.12}$$

or of the "weak charge" Q_W measured in atomic parity violation experiments on Cs [27]

$$Q_W = -72,72 \pm 0.13 - 102\varepsilon_3 \tag{10.13}$$

(with the error due to the theoretical uncertainties in the atomic wave functions).

It is now possible to include these low energy observables in an overall fit. Using, together with the data in Table 3, the present average value from νN scattering[28]

$$\langle R_\nu \rangle = 0.312 \pm 0.003 \tag{10.14}$$

(which takes the recent CCFR result[29] into account) and the result from cesium data[30]

$$Q_W = -71.04 \pm 1.81 \tag{10.15}$$

one obtains the global fit

$$\begin{aligned} \varepsilon_1 &= (-0.1 \pm 2.7)\, 10^{-3} \\ \varepsilon_2 &= (-8.2 \pm 7.5)\, 10^{-3} \\ \varepsilon_3 &= (1.3 \pm 3.1)\, 10^{-3}. \\ \varepsilon_b &= (2.9 \pm 5.4)\, 10^{-3} \end{aligned} \tag{10.16}$$

Here the error induced from $\alpha_S(M_Z)$ was taken into account in the same way as in Equation (10.11).

The results of the various fits are collected in Table 5. For the fit to all LEP data, we also display in figures. (10.1-3) the projected ellipses in the planes $\varepsilon_3 - \varepsilon_1$, $\varepsilon_b - \varepsilon_1$, $\varepsilon_3 - \varepsilon_b$ corresponding to $1\,\sigma$ errors, togheter with the SM predictions.[20]

Table 5. Summary of the different determinations of the epsilon parameters.

	Defining Variables	Defining Variables + Asymmetries	All LEP data	"All data"
$\varepsilon_1 10^3$	-0.3 ± 3.4	0.0 ± 3.4	-0.3 ± 3.2	-0.1 ± 2.7
$\varepsilon_2 10^3$	-7.6 ± 7.6	-8.3 ± 7.5	-8.5 ± 7.6	-8.2 ± 7.5
$\varepsilon_3 10^3$	0.4 ± 4.2	1.6 ± 3.5	1.5 ± 3.3	1.3 ± 3.1
$\varepsilon_b 10^3$	4.4 ± 7.0	4.5 ± 7.0	3.1 ± 5.5	2.9 ± 5.4

Several comments can be made at this point:

i) None of the ε_i is clearly different from zero. This means in turn that no "genuine" electroweak correction has clearly been observed yet. On the other hand, if they are there, which cannot be doubted, they must be small enough. Significantly smaller, for example, than those expected, in the SM, for a very heavy top, of 250 GeV or more, or in some technicolour theories (see below). To see the "genuine" electroweak corrections is a remarkable goal, *per se*, of the program of the precision tets.

ii) At the same time, no clear deviation is seen from the SM expectation. In a sense, however, figures. (10.1-3) show that the specific pattern of radiative corrections in the SM has not quite emerged yet. It will, if it is the correct one, with the foreseen precision of the electroweak precision data at the end of the LEP1 phase.

iii) The central values of ε_3 and ε_b are respectively slightly lower and slightly higher than expected in the SM, for whatever value of the top and the Higgs masses. This observation has, at the moment, little statistical significance. Small ε_3 corresponds, in the SM, to a slight preference for light Higgs masses. High ε_b is an indication that the top cannot be too heavy even in theories where the ρ parameter, or ε_1, deviates significantly from its value in the SM[20,31].

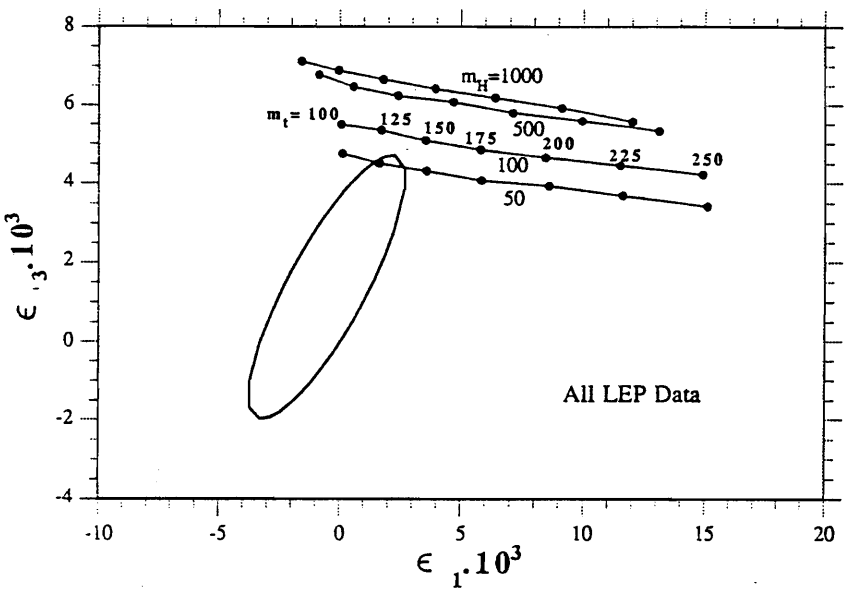

Figure 10.1 Plot of ε_3 versus ε_1 from the fit of all LEP data. The projections of the ellipsis on the axis correspond to the 1 σ errors displayed in Table 5

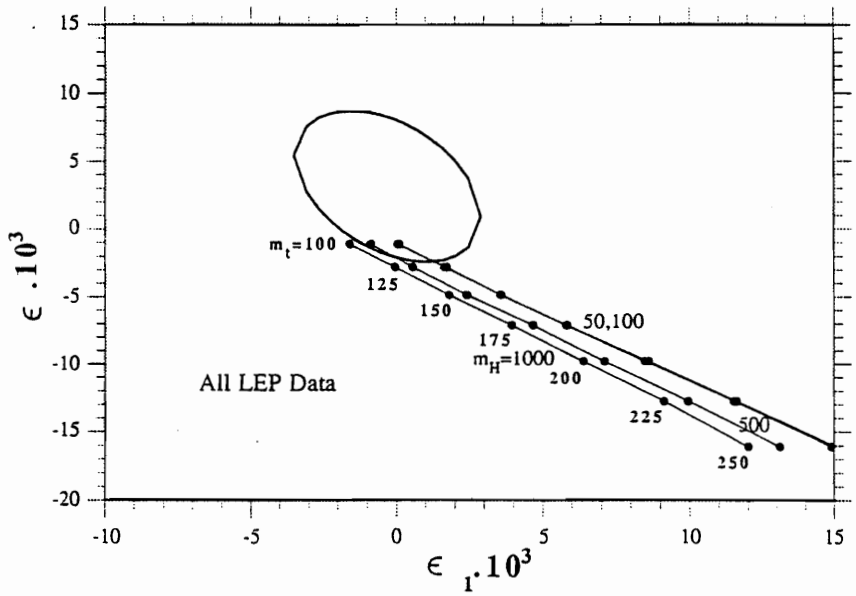

Figure 10.2 Plot of ε_b versus ε_1 as in figure 10.1

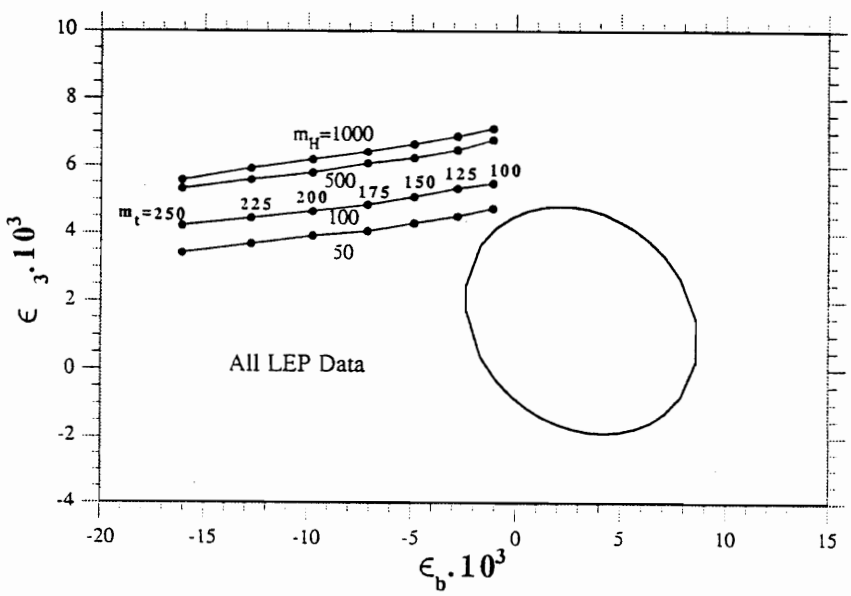

Figure 10.3 Plot of ε_3 versus ε_b as in figure 10.1

11. COMPARISON WITH THE MINIMAL SUPERSYMMETRIC STANDARD MODEL

The Minimal Supersymmetric Standard Model[2] is a well-motivated alternative to the SM. From the point of view of the precision electroweak tests, however, the significant features of the MSSM make it clearly resemble the SM itself: i) in both cases one has a fundamental Higgs; ii) both the MSSM and the SM (with a not too heavy Higgs) are perturbative theories.

The MSSM is really defined by two assumptions: i) it has minimal supersymmetric particle content; ii) the supersymmetry breaking occurs in a hidden sector and is transmitted to standard matter by universal supergravity couplings[32]. (R -parity conservation plays no role in the following considerations.) Two basic properties are in turn implied by the previous assumptions:

i)The MSSM has one dimensionless coupling less then the SM: the quartic Higgs self-coupling is not a free parameter but it is actually a gauge coupling. On the other hand, the MSSM has five parameters with dimension of mass, m_i, in place of the only mass parameter appearing in the quadratic term of the SM scalar potential. In a standard notation[2]

$$m_i = m, M, \mu, m_A, m_B,$$

all of them being related to the scale of effective supersymmetry breaking[(f7)];

ii) By taking $m_i >> m_Z$, the light spectrum of the MSSM is identical to the SM one with a light Higgs. As is by now well known, the exact upper bound on the light Higgs depends critically on the top-quark mass[33]. All the remaining particles predicted by the MSSM can be made arbitrarily heavy, although at the price of an increasingly unnatural fine tuning.

These two properties imply that, in the $m_i >> m_Z$ limit, all the superparticles "decouple" from radiative-correction effects and the MSSM becomes undistinguishable from the SM itself with a light Higgs. Notice that to reach this conclusion, both properties i) and ii) are essential. It is in fact essential that the mass terms of the particles that decouple be compatible with unbroken SU(2)XU(1). Needless to say, the top quark is the best example of a particle that does not "decouple" from

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(f7) Even the so-called μ parameter, which, in the low-energy theory, appears as a supersymmetric term, may very well originate in a more complete theory, as a result of supersymmetry breaking, as many examples show.

radiative corrections: also the top-quark mass can be made large, but, since the mass term is not consistent with unbroken SU(2)XU(1), a heavy top requires a correspondingly large Yukawa coupling.

If, on the other hand, some of the superpartners are light, the pattern of radiative correction effects in the MSSM may deviate from the one of the SM. The relatively most interesting effects occur in vacuum polarization amplitudes and/or in the $Z \to b\bar{b}$ vertex and can therefore be described in term of the epsilon-parameters. They are:

i) a threshold effect in the Z-wave-function renormalization, contributing to a positive e_5-term, Equation (9.18), mostly due to the vector coupling of charginos and (off-diagonal) neutralinos to the Z itself[18]. Notice that this effect on e_5 propagates in ε_1, ε_2 and ε_3 according to Equations (9.10-12) and reduces all of them by a similar amount. Physically, all the Z widths would be reduced without affecting the asymmetries. This effect can be significant, but it requires the lightest chargino to have a mass below 60 GeV.

ii) A positive contribution to e_1, and therefore to ε_1, due to the virtual exchange of the scalar top and bottom partners, analogous to the contribution due to the top-bottom exchange[34]. The needed isospin splitting requires one of the two scalars (in the MSSM the stop) to be light.

iii) A negative contribution to ε_b due to the virtual exchange of a charged Higgs[35]. If one defines, as customary,

$$tg\beta = \frac{v_2}{v_1} \tag{11.1}$$

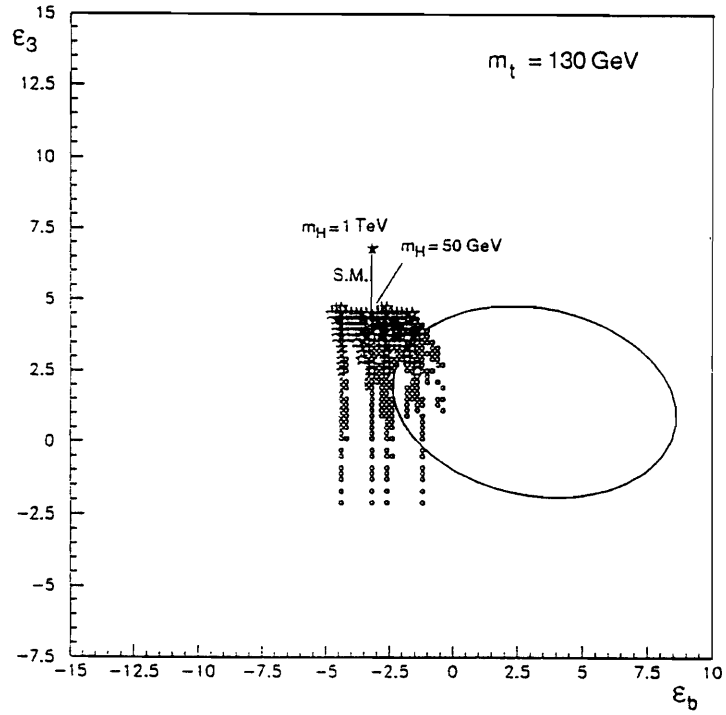
this contribution is proportional to

$$m_t^{\,2} / tg^2\beta .$$

iv) A positive contribution to ε_b due to a virtual chargino-stop exchange[36], this time proportional to

$$m_t^{\,2} / \sin^2\beta .$$

All the above effects are comulatively displayed in figures (11.1-3) where we give the pair correlations $\varepsilon_3 - \varepsilon_1$, $\varepsilon_b - \varepsilon_1$, $\varepsilon_3 - \varepsilon_b$ in the MSSM, for m_t = 130 and 190 GeV, in the form of scatter plots[37]. Also



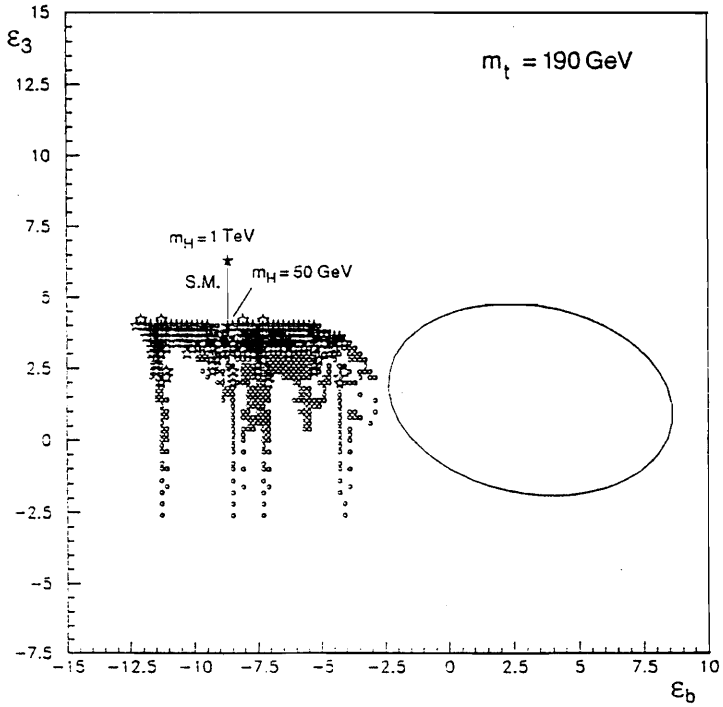


Figure 11.2 ε_3 versus ε_b, aas in figure 11.1

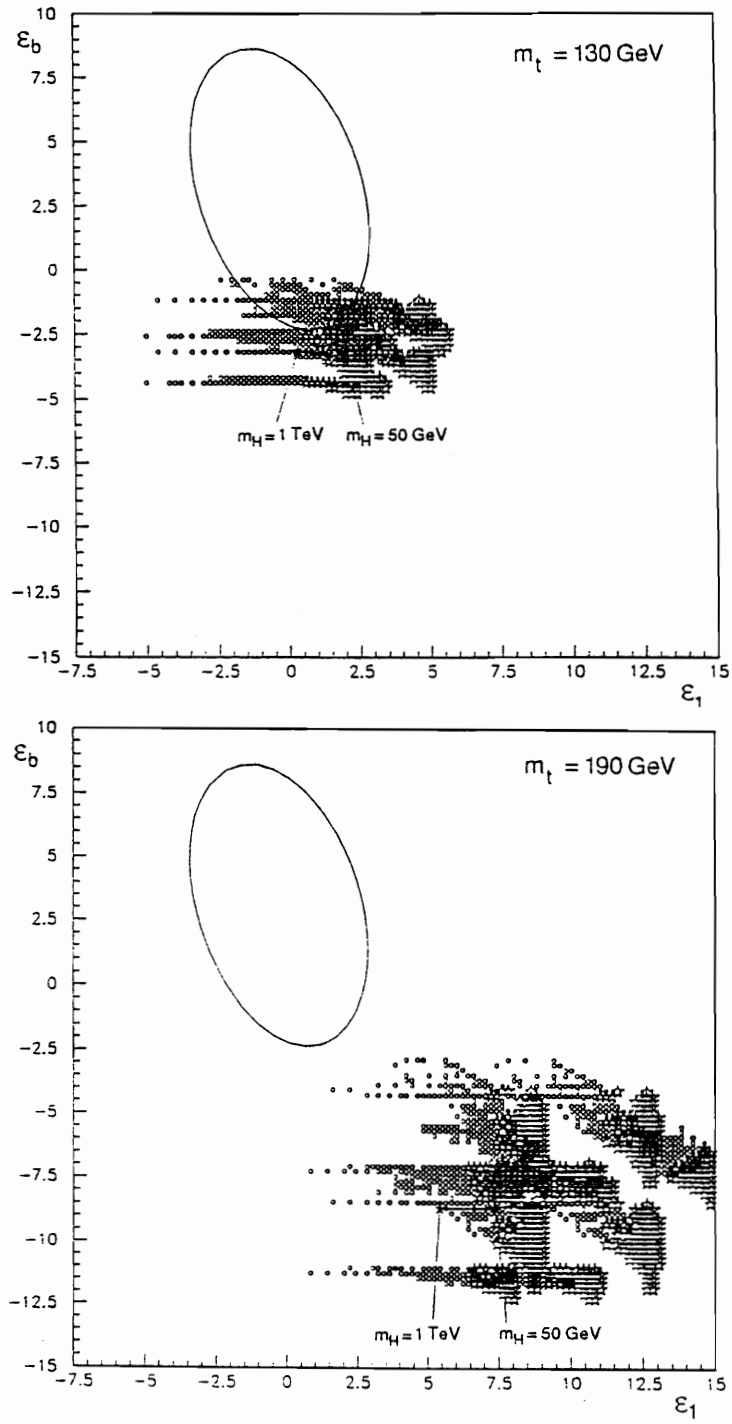


Figure 11.3 ε_b versus ε_1, aas in figure 11.1

shown are the 1 σ contours obtained from present combined LEP data, as well as the SM predictions for m_H ranging between 60 and 1000 GeV. The parameters of the MSSM are allowed to vary in such a way that:
i) The lighest chargino mass is greater than 48 GeV (60 Gev in the darker area). This condition is relevant to effects i) and iv) above.
ii) The charged Higgs mass, relevant to effect iii), is greater than 100 GeV.
iii) $tg\beta > 1$, relevant to iii) and iv).
iv) The lightest stop and the (approximately degenerate) sbottom masses are taken respectively greater than 50 and 150 GeV.

Only this last constraint requires a further qualification. In the mass matrix of the two top superpartners, which has the following approximate form

$$\begin{pmatrix} m(\tilde{b})^2 + m_t^2 & m_t m_A \\ m_t m_A & m(\tilde{b})^2 + m_t^2 \end{pmatrix}$$

we take the supersymmetry breaking mass term $m_A < 3m(\tilde{b})$. This limits the relative stop-sbottom splitting as m($\tilde{b}$) gets large and, in turn, the size of the effect on ε_1.

From the above discussion it should be clear that, for any given m_t: i) the lowest values of ε_2 and ε_3, which are correlated, are obtained for light charginos; ii) the lowest values of ε_1 are obtained for light charginos and heavy stop; iii) the highest values of ε_b are obtained for light charginos and stop, whereas, on the contrary, low ε_b requires a light charged Higgs and small $tg\beta$.

Figures (11.1) make clear that, relative to the current precision of the experiments, the deviations from the SM are not large. However, the situation will change with the increase of the statistic at LEP. One point can be noticed already at present: a top mass as high as 190 Gev, unlikely in the SM, can be made consistent with the precision tests in the MSSM if the chargino is sufficiently light, in fact visible at LEP2[37].

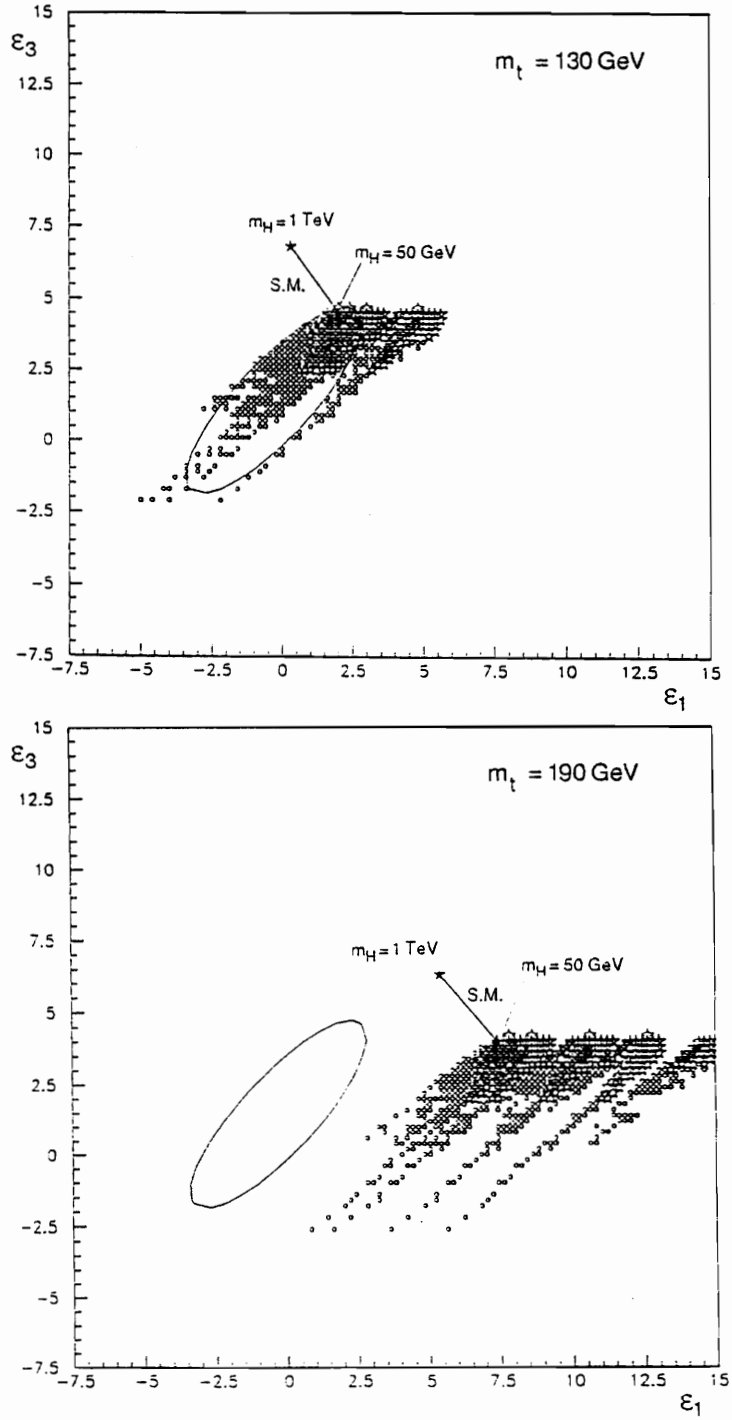


Figure 11.1 ε_3 versus ε_1 at fixed m_t = 130 and 190 GeV in the MSSM, in the form of scatter plot, compared with the data (1 σ ellipse) and with the SM.

12. "HIGGS-LESS" THEORIES.

12.1 THE SM FOR INFINITE HIGGS MASS

In this section we start trying to tackle the hard problem of estimating the size of the radiative corrections in theories without a Higgs particle, that is to say in theories which deviate from the SM in the sector that is supposed to break both the local and the global symmetries of the pure SU(2)XU(1) gauge Lagrangian. Since the local symmetry has to be broken spontaneously, this sector must contain in general, among other things, the charged and neutral Goldstone bosons that provide the longitudinal modes of the W and the Z bosons, respectively called φ and χ in Section 6. These Goldstone bosons could be composite states arising from some new strongly interacting dynamics.

In this situation, it makes sense to assume that all possible deviations from the SM radiative correction effects be confined to the vector boson vacuum polarization amplitudes, in particular to their low derivatives only. We then concentrate on e_1, e_2 and e_3 defined in Equations (9.14-16).

To deviate from zero, both e_1 and e_2 require a breaking of the (custodial) symmetry that is responsible, at the tree level, for $\rho = 1$, or the mass relations (2.1). This can be seen, for example, from the global SU(2) symmetry that is recovered in the effective Lagrangian (9.2) by taking $e_1 = e_2 = 0$ and neglecting mixing effects ($s = 0$ or $g' = 0$). With reference to figure (12.1) we then set

$$e_1 = e_1^{\Delta} + e_1^{B}\,; \qquad e_2 = e_2^{\Delta} + e_2^{B} \tag{12.1}$$

where $e_{1,2}^{B}$ are due to the B-boson exchange and $e_{1,2}^{\Delta}$ to the remaining sources of isospin breaking, like the top-bottom mass difference. The blobs in this figure may have to be computed non-perturbatively in the symmetry breaking sector of the theory. In the case of e_1 or $\Delta\rho$, we have actually seen in Section 6 how it can be viewed as the deviation from 1 of the ratio of the wave function renormalization constants of the charged and neutral Goldstone bosons, as explicitly indicated in figure (12.1). In other words, e_1^{Δ} will be independent from the gauge couplings (in leading order), whereas e_1^{B} will be proportional to g'^2. This is unlike the case for e_2, which requires the couplings to the external gauge bosons: e_2^{B} in particular is proportional to the fourth power of the gauge couplings and is generally small. Unlike e_1 and e_2, e_3 does not need any custodial isospin breaking, which therefore can hopefully be neglected.

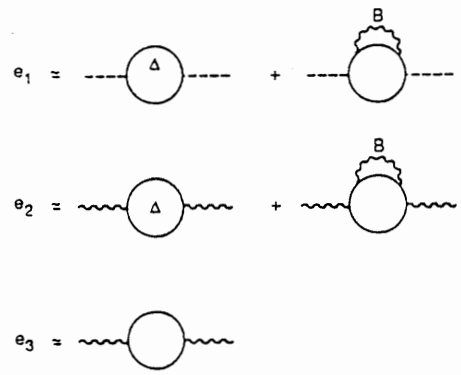


Figure 12.1 Contributions to e_1, e_2, and e_3 in an expansion in custodial isospin-breaking (Δ) and in the gauge couplings. The dotted lines in e_1 stand for the Goldstone bosons φ, χ. The blobs are to all orders in the non-perturbative couplings of the symmetry-breaking sector.

A first step towards a "Higgs-less" theory can be made by considering the various ε_i' s in the SM for an infinitely heavy Higgs[38] namely when the Higgs is never excited. The relevant diagrams at one loop are shown in figure (12.2). In this approximation, there is in fact no diagram at all with a Higgs boson internal line that contributes to e_1^Δ and e_2^Δ; these are therefore independent from the Higgs mass. There are, on the contrary, diagrams with internal Higgs boson lines that contribute to e_1^B and e_3, which would in fact be needed to make the pure Goldstone diagrams of figure (12.2) ultraviolet-convergent. In terms of an ultraviolet and infrared cut-off, Λ and μ respectively, the diagrams of figures (12.2b) and (12.2e), both computed at $q^2 = 0$, immediately give[39]

$$e_1^B \approx -\frac{3\alpha}{8\pi c^2} \lg\frac{\Lambda}{\mu} \approx -1.2\times 10^{-3} \lg\frac{\Lambda}{\mu} \tag{12.2}$$

$$e_3 \approx \frac{\alpha}{24\pi s^2} \lg\frac{\Lambda}{\mu} \approx 0.5\times 10^{-3} \lg\frac{\Lambda}{\mu} \tag{12.3}$$

The result for e_1^B is actually obtained by taking the B-boson propagator in the Landau gauge. This is the gauge that has to be used if one wants to compute e_1 in terms of Goldstone boson properties only; the corresponding gauge-fixing term, although breaking the local symmetry,

respects in fact the global symmetry that is responsible for the existence of the massless Goldstone bosons in the first place.
This is at least a quick way to obtain, in the SM, the leading logarithmic dependence[40] of e_1 and e_3 as the Higgs mass becomes infinitely large, since for that it is enough to replace Λ in Equations (12.2) and (12.3) with the Higgs mass. (The infrared cut-off in a complete calculation is replaced by the gauge-boson masses.) What the calculation actually shows is that, to compute the e-parameters (or the εs) at the per mille level, which is the foreseen precision in their determination , it is necessary to specify how the divergences in Equations (12.2) and (12.3) are regularized in a theory without the Higgs as an explicit degree of freedom. Needless to say, it is quite clear what the counterpart of the divergences (12.2) and (12.3) is in an effective Lagrangian description of the Goldstone boson interactions. Unlike the case of the SM itself, where both the local SU(2)$\times$U(1) and the approximate global custodial SU(2) symmetries are linearly realized in terms of the full Higgs doublet, the same symmetries can be non-linearly realized using the Goldstone bosons only[39]. The point now is that the corresponding (non-renormalizable) effective Lagrangian realizing these symmetries will have terms that contribute directly to e_1 and e_3 with unknown coefficients. Without independent theoretical or experimental inputs, this makes the e-coefficients unpredictable.

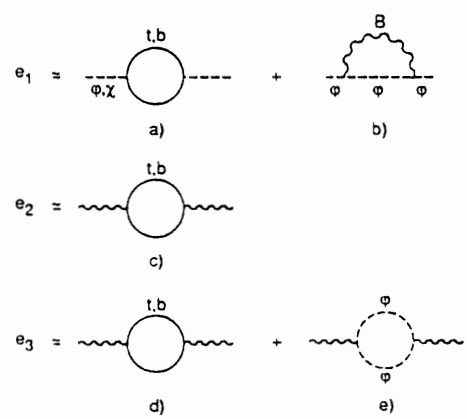


Figure 12.2 One-loop contributions to e_1, e_2, and e_3 in the SM without Higgs boson internal lines in the approximation discussed in the text. The B-boson propagator in (b) is in the Landau gauge.

12.2 VACUUM POLARIZATION EFFECTS IN TECHNICOLOUR MODELS

The divergences due to the Golstone boson exchanges that we have computed in the previous section are certainly cut off in models where the breaking of the gauge symmetry is produced by the condensation of

a fermion bilinear, as in technicolour theories[3]. In such theories, it is therefore a challenge to compute the finite residual contributions to the e-parameters.

Simply for orientation, let us first compute the contributions to e_1, e_2 and e_3 from the virtual exchange of one "technicolour" SU(2) doublet with mean mass $m_T >> M_Z$ and with mass difference, inside the doublet, $\Delta m_T << m_T$. From simple one-loop diagrams, one readily obtains

$$e_1^{\Delta} \approx N_{TC} \frac{G(\Delta m_T)^2}{6\pi^2\sqrt{2}} \approx 10^{-3} N_{TC} \left(\frac{\Delta m_T}{100 GeV} \right)^2 \tag{12.4}$$

$$e_1^{B} = 0 \tag{12.5}$$

$$e_2^{\Delta} \approx -N_{TC} \frac{\alpha}{30\pi s^2} \left(\frac{\Delta m_T}{m_T} \right)^2 \approx -0.3 10^{-3} N_{TC} \left(\frac{\Delta m_T}{m_T} \right)^2 \tag{12.6}$$

$$e_3 \approx N_{TC} \frac{\alpha}{24\pi s^2} \approx 0.5 \times 10^{-3} N_{TC} \tag{12.7}$$

for N_{TC} technicolours. What should be done with these equations is unclear, to say the least. Nevertheless we learn something from them. The heavy fermions do not "decouple" in the large mass limit. Furthermore, to make a numerical estimate of some sort, it is crucial that we know how many heavy fermions there are.

The non-decoupling property of these effects is of course due to the fact that the mass of the technifermions (left-handed SU(2) doublets and right-handed singlets) is not compatible with unbroken SU(2)XU(1). (The discussion of the top-quark-mass effects given in Section 6 is an illustrative example in this sense.) As to the needed number of technifermions, one should not forget the gigantic global symmetry of the SM Lagrangian in the absence of Higgs interactions, which is supposedly broken by extra gauge interactions, shared between the standard fermions and the technifermions (the so-called "extended technicolour"). In actual model building, this is what generally requires a proliferation of technifermions. Finally, the parameter Δm_T hides the problem of generating, in technicolour models, the top-bottom mass difference.

At least this problem of the custodial isospin breaking can be avoided if we stick to the parameters e_3 (or to e_1^B, although this contribution will always appear together with the uncertain e_1^{Δ} contribution). It is therefore interesting to see how an estimate of e_3 can be attempted if one makes the extra assumption that the technicolour dynamics mimics QCD as much as it can[21,41(f8)].

(f8) For an alternative view, see Reference 42.

From the definition in (9.15), one sees that e_3 is related to the Fourier transform of the correlation function (Lorentz and space indices omitted)

$$\langle 0|J_{3L}J_0|0\rangle \tag{12.8}$$

between the current coupled to the neutral W boson

$$J_{3L} = \frac{1}{2}\left(J_{3V} - J_{3A}\right) \tag{12.9}$$

and the current coupled to the B boson

$$J_0 = \frac{1}{2}\left(J_{3V} + J_{3A}\right) + \frac{1}{2}J_Y \tag{12.10}$$

where J_{3V}, J_{3A} and J_Y are the third components of the vector and axial isospin currents and of the (conventional) hypercharge current respectively. Since, as in QCD, technicolour is assumed to conserve parity and isospin, one has therefore

$$\langle 0|J_{3L}J_0|0\rangle = \frac{1}{4}\left(\langle 0|J_{3V}J_{3V}|0\rangle - \langle 0|J_{3A}J_{3A}|0\rangle\right) \tag{12.11}$$

Precisely, taking the proportionality factors into account one has

$$e_3 = \frac{\pi\alpha}{s^2}\left(F_{VV}(0) - F_{AA}(0)\right), \tag{12.12}$$

where, in analogy with (8.1),

$$\begin{aligned}\Pi^{\mu\nu}_{VV,AA} &= \int d^4x\, e^{-iqx}\langle 0|J^{\mu}_{V,A}(x)J^{\mu}_{V,A}(0)|0\rangle \\ &= -ig^{\mu\nu}\left[A_{VV,AA}(0) + q^2 F_{VV,AA}\left(q^2\right)\right] + q^{\mu}q^{\nu}terms\end{aligned} \tag{12.13}$$

In the strict QCD case, the right-hand side of (12.11) can be estimated reliably in several different ways. The quickest one[21], consists in using Vector Meson Dominance, together with the pion pole and current conservation, for the vector and axial vacuum polarization functions

$$-i\Pi^{\mu\nu}_{VV} = \frac{M_V^2}{f_V^2}\frac{1}{q^2 - M_V^2}\left(q^2 g_{\mu\nu} - q_{\mu}q_{\nu}\right) \tag{12.14}$$

$$-i\Pi_{AA}^{\mu\nu} = \frac{M_A^2}{f_A^2}\frac{1}{q^2 - M_A^2}\left(q^2 g_{\mu\nu} - q_\mu q_\nu\right) + F_\pi^2\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) \quad (12.15)$$

and the convergence requirement, at large q^2 [43]

$$\Pi_{\mu\nu}^V - \Pi_{\mu\nu}^A \cong O\left(\frac{1}{\left(q^2\right)^2}\right) \quad (12.16)$$

which give

$$\frac{M_V^2}{f_V^2} = \frac{M_V^2}{f_A^2} + F_\pi^2; \qquad \frac{M_V^4}{f_V^2} = \frac{M_A^4}{f_A^2} \quad (12.17)$$

From (12.12-17), one then gets

$$e_3 = \frac{\pi\alpha}{s^2}\frac{F_\pi^2}{M_V^2}\left(1 + \frac{M_V^2}{M_A^2}\right) \quad (12.18)$$

where F_π, M_V and M_A are the analogues, in the QCD case, of the π decay constant, the ρ mass and the A_1 mass respectively.

How can this QCD result be extrapolated to the technicolour case? Suppose that we continue to make the VMD assumption. The problem is not in the difference between the QCD and the TC scales, since Equation (12.18) is scale-independent. Rather it comes from the generally different numbers of colours and flavours. For N_D doublets and N_{TC} technicolours, planar QCD (or TCD)[44] gives

$$\frac{F_\pi^2}{M_V^2} = N_D \frac{N_{TC}}{3}\frac{f_\pi^2}{m_\rho^2} \quad (12.19)$$

where now we use the lower case letters for the real π decay constant and ρ mass. Other considerations, that give more weight to the threshold behaviour of the vacuum polarization amplitudes, or the Goldstone boson contributions , suggest different estimates, such as[45]

$$\frac{F_\pi^2}{M_V^2} \approx \frac{N_D N}{16\pi^2} \quad (12.20)$$

where N is the total number of technicolour flavours, so $N \geq 2N_D$ Together with

$$\frac{M_V^2}{M_A^2} \approx \frac{m_\rho^2}{m_{A_1}^2} \tag{12.21}$$

and (12.18), the two estimates (12.19) and (12.20) lead respectively to

$$e_3 = \frac{\pi\alpha}{3s^2}\frac{f_\pi^2}{m_\rho^2}\left(1+\frac{m_\rho^2}{m_{A_1}^2}\right)N_D N_{TC} = 0.7\times 10^{-3} N_D N_{TC} \tag{12.22}$$

$$e_3 = \frac{\alpha}{16\pi s^2}\left(1+\frac{m_\rho^2}{m_{A_1}^2}\right)N_D N \ = 0.9\times 10^{-3} N_D N \tag{12.23}$$

The comparison of (12.22) and (12.23) is a bit embarrassing. For a "realistic" technicolour model with N_{TC}=4 and 1 technifamily (N_D=4, N = 8), the two results would differ by about a factor of 2, which is perhaps not an untolerable discrepancy, but is also a rather fortuitous "agreement". On the other hand, only the estimate (12.22) is in accord with the free techniquark estimate, (12.7), and this for quite obvious reasons.

12.3 THE $Z \to b\bar{b}$ VERTEX IN EXTENDED TECHNICOLOUR.

In absence of Higgs Yukawa couplings, the source of the fermion masses can be identified in the couplings to heavy "Extended TechniColour" (ETC) bosons of both the standard (Q_L, t_R) and technicoloured (T_L, U_R) fermions[3]

$$g_{ETC}\left(\xi\overline{Q}_L\gamma_\mu T_L + \frac{1}{\xi}\bar{t}_R\gamma_\mu U_R\right)V_\mu \tag{12.24}$$

At lower scales than the typical ETC vector boson masses, M_{ETC}, ETC vector exchanges give rise to the following (Fiertz transformed) effective 4 fermion-interactions

$$2\frac{g_{ETC}{}^2}{M_{ETC}{}^2}\left(\overline{Q}_L t_R\right)\left(\overline{U}_R T_L\right) \tag{12.25}$$

$$-\frac{1}{2}\xi^2\frac{g_{ETC}{}^2}{M_{ETC}{}^2}\left(\overline{Q}_L\gamma_\mu\tau^a Q_L\right)\left(\overline{T}_L\gamma_\mu\tau^a T_L\right). \tag{12.26}$$

These interactions, as an effect of technicolour condensation, originate respectively the standard fermion masses (e.g. of the top quark)

$$m_t = \frac{g_{ETC}^{\ 2}}{M_{ETC}^{\ 2}}\langle \overline{U}U\rangle \approx \frac{g_{ETC}^{\ 2}}{M_{ETC}^{\ 2}} 4\pi v^3 \tag{12.27}$$

as well as the anomalous $Z \to b\overline{b}$ coupling[46]

$$\frac{1}{2}\xi^2 \frac{g_{ETC}^{\ 2}}{M_{ETC}^{\ 2}} v^2 \frac{g}{c} Z_\mu \left(\overline{Q}_L \gamma_\mu \frac{\tau^3}{2} Q_L \right). \tag{12.28}$$

This coupling results in a modification of g_{Lb}, or in an additional contribution to e_b

$$\Delta e_b^{\ ETC} = -\frac{\xi^2}{2}\frac{g_{ETC}^{\ 2} v^2}{M_{ETC}^{\ 2}} = -\xi^2 1.6\times 10^{-2}\frac{m_t}{100GeV} \tag{12.29}$$

of significantly large size. All of these estimates assume a QCD-like behaviour of the TC dynamics.

12.4 CONCLUSIVE REMARKS.

I draw the following conclusions from these considerations. It is obviously difficult to make a sensible estimate of the radiative correction effects in a technicolour model. The difficulty is partly technical (the necessity of dealing with a strongly interacting field theory) and partly fundamental (related to the undefiniteness of technicolour models in solving the flavour problem). Nevertheless, I believe that the various estimates in the previous subsections make it hard to maintain that, in view of the relatively weak dependence on the ultra-violet cut-off of the radiative correction effects in a "Higgs-less" model [Equations (12.2) and (12.3)], the electroweak precision tests are insensitive to the symmetry-breaking sector of the theory.

In the case of the vacuum polarization parameters, at least in technicolour models, for "realistic" choices of N_D, N and N_{TC}, one finds corrections close to or bigger than the per cent level, to be compared with the per mille level attainable by the experimental determination of the relevant quantities. Furthermore, one can rather clearly identify the source of these potentially large corrections in the way models without the Higgs-Yukawa couplings try to solve the flavour problem, namely by a proliferation of extra degrees of freedom that do not decouple in their radiative correction effects. The situation is similar in the case of the

$Z \to b\bar{b}$ vertex corrections due to ETC vector boson exchanges. In fact, the various estimates discussed indicate a pattern of the corrections in technicolour models clearly disfavoured by present data, both in sign and in size.

13. SUMMARY AND CONCLUSIONS

The following is a summary of the main physics points that I have discussed in the course of the lectures, more or less in order of presentation.

i) The bulk of the radiative correction effects that are clearly seen at LEP is still of pure electromagnetic origin (Section 4).

ii) The dominant top quark radiative correction effects, in an approximation which gets better as the top gets heavier, can be discussed without ever making reference to the gauge theory (Section 6). The two quantities most sensitive to the top mass, the ρ-parameter and the $Z \to b\bar{b}$ vertex, have been studied independently in the SM and both shown not to be compatible with too heavy a top. (Sections 6 and 7).

iii) By introducing suitable parameters, it is possible to make a rather model-independent analysis of the "genuine" electroweak radiative correction effects, not of pure electromagnetic origin (Section 9). The LEP1 programme may show clear evidence for these effects, which at the moment, are constrained to be small enough. The SM is at present compatible with these constraints (Section 10).

iv) All the extra particles introduced in the MSSM fulfil a decoupling property in their virtual contributions to the electroweak precision tests. The SM with a light Higgs is mimicked in most of the MSSM parameter space. Four possibly significant effects have, however, been identified, requiring either a light stop or light gaugino-higgsinos or a light charged Higgs (Section 11).

v) For intrinsic and technical reasons, it is difficult to make a reliable estimate of the radiative correction effects in theories without fundamental scalars, such as technicolour models. In the considered examples, however, these effects attain values that are about one order of magnitude larger than the sensitivity expected at the end of the LEP1 programme (Section 12). To the extent that technicolour models are representative cases, these semi-quantitative considerations indicate no reason why theories with or without a Higgs should show the same pattern of radiative correction effects at the level of precision attainable in the experiments. In view of all this, I believe that the experimental programme that is being carried out at LEP1 may shed light on the symmetry-breaking issue in electroweak physics. To this end, it would be very important to have also a direct determination, with sufficient precision, of the top-quark mass, a goal within reach at Fermilab.

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