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Spin 3/2 Fields Non-Minimal Coupling from Linearized Gravity

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Abstract

A non-minimal coupling for spin 3/2 fields is obtained. We use the fact that the Rarita-Schwinger field equations are the square root of the full linearized Einstein field equations in order to investigate the form of the interaction for the spin 3/2 field with gauge fields. We deduce the form of the interaction terms for the electromagnetic and non-abelian Yang-Mills fields by implementing appropiate energy momentum tensors on the linearized Einstein field equations. The interaction found for the electromagnetic case happens to coincide with the dipole term found by Ferrara *et al* by a very different procedure, namely by demanding $g = 2$ at the tree level for the electromagnetic interaction of arbitrary spin particles. The same kind of interaction is found by using the resource of linearized Supergravity N=2. For the case of the Yang-Mills field Supergravity N=4 is linearized, providing also the already foreseen interaction.

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1 Introduction

The standard treatment for elementary free masless spin 3/2 fields is achieved by means of the Rarita-Schwinger (R-S) lagrangian [1,2],

$$L_{RS} = -\frac{1}{2}\epsilon^{\mu\nu\rho\sigma}\bar{\Psi}_\mu\gamma_5\gamma_\nu\partial_\rho\Psi_\sigma\,. \tag{1.1}$$

This lagrangian gives us the field equations

$$\epsilon^{\mu\nu\rho\sigma}\gamma_5\gamma_\nu\partial_\rho\Psi_\sigma = 0\,. \tag{1.2}$$

It is however well known that the usual minimal electromagnetic prescription for the Dirac field does not work adequatelly for this spin 3/2 field. In fact if one couples minimally this field with electromagnetism, then several physical inconsistencies arise of which the most remarkable is the appearance of superluminal speed for the particles [3].

By demanding that the scattering amplitudes should have a good high energy behaviour, Weinberg [4] showed that the gyromagnetic ratio should be $g = 2$ for arbitrary spin particles. Following a consistent procedure for constructing the lagrangians for higher spin massive particles interacting with the electromagnetic field, Ferrara *et al* [5] obtained a gyromagnetic ratio $g = 2$. As a result, their equations of motion contain an extra dipole term that can be implemented at the tree level, thus modifying the usual minimal electromagnetic coupling. A very important feature of this extra dipole term is that, as shown by Ferrara *et al*, it avoids the physical inconsistencies for spin 3/2 particles described in ref. [3].

On the other hand, recently two of the authors have constructed a theory of the classical supersymmetric spin 3/2 particle [6] in analogy with the classical supersymmetric spin 1/2 particle formalism developed by Galvao and Teitelboim [7]. The usual γ matrices were realized as grassman variables and it was shown that the generalized Poisson bracket of the Rarita-Schwinger constraint in flat space-time gives the full linearized Einstein field equation.

If in the spin 3/2 field differential operator we preserve the γ matrices then the same result follows, but instead of the generalized Poisson bracket, the anticommutator of the R-S operator must be used. This is *not* a consequence of the well known result in canonical supergravity [8,9], where it was shown that the supersymmetry constraint is the square root of the usual constraint in canonical general relativity. This last procedure involves only those dynamical equations corresponding to these constraints, in contrast with the relations found in ref. [6] which relates the complete set of linearized Einstein field equations and the Rarita-Schwinger field equations by means of Poisson brackets.

The result of the paper mentioned above showed that the Rarita-Schwinger equation is related with linearized gravity as the Dirac equation is related to the Klein-Gordon equation. Thus, following this analogy and knowing how gravity should couple with matter (in particular gauge fields) one would expect to be able to find out the way that gauge fields (matter) couple with the spin 3/2 field in flat space-time.

The last point is the main guide for this work, because in principle we can add a matter tensor (describing the gauge fields of interest) on the right side of the linearized Einstein field equations and investigate its "square root" in a similar manner to that developed in [6]. As a result of this procedure a modified R-S equation will arise. Obviously, this "square root" must include the terms of interaction with the gauge (and matter fields), and these terms of interaction will give us the information for the coupling of spin 3/2 fields with any kind of gauge fields and matter, particularly electromagnetism or Yang-Mills fields.

This paper is organized as follows: in section 2 we generalize the four indices differential operator representing the linearized general relativity equations [6] in order to include electromagnetism and non-abelian Yang-Mills fields. Based on the particular form of this extra gauge fields terms in the linearized Einstein field equations we search for the corresponding extra terms in the Rarita-Schwinger equation which when squared will produce the implemented gauge fields terms. As a result, we find the interaction for the spin 3/2 field with electromagnetism and Yang-Mills fields. It is to be remarked that this modified R-S equation when squared does not reproduce only the desired extra term in the linearized gravity equations, but there appear extra terms. This is not surprising because the relationship between the R-S equation with interaction and the linearized Einstein field equations with matter is similar to that existing between the Dirac equation with interaction and the Klein-Gordon equation with a potential, where the LS coupling term appears. What we get is a kind of generalized "hamiltonian", for the linearized gravity equations with a four indices generalized gauge field energy-momentum tensor.

Our result shows that the R-S equation is related in a direct manner with linearized gravity. Formally, the latter is the "hamiltonian" and the former one is its corresponding linear differential equation. On the other hand our spin 3/2 equations play the role of some kind of supercharges (not only the equation corresponding to the zero component). It is then natural to think in Supergravity (SG), being the theory that incorporates gravity, the spin 3/2 field and gauge and matter fields.

We expect that by linearizing Supergravity we will be able to obtain the same kind of results that in the case without gauge fields (matter) [6], and also when the interaction is present. As mentioned, the relation we want to get is between the full linearized R-S equations and linearized gravity and not between the canonical constraints of supergravity [8,?]. The free case (without interaction) should correspond to linearized Supergravity N=1. This is performed in section 3.

In the next two sections we also linearize Supergravity N=2 (section 4) and N=4 (section 5). We show that the interaction found in section 2 for the electromagnetic field is similar to that obtained from supergravity and is essentially also the same that Ferrara *et al* [5] found by a very different proccedure. For the case of N=4 by imposing the scalar field equal to zero and neglecting the spin 1/2 field, we get the interaction term for a non-abelian Yang-Mills field which has the same structure as that announced in section 2 . However, in these cases there are correspondingly two and four spin 3/2 fields. The appearance of more spin 3/2 fields is directly related with the fact that in these last two cases we are treating with an enlarged supersymmetry.

The R-S equations resulting from supergravity are a very similar alternative to that of taking the square root of linearized gravity in section 2. However, they are very much more complicated to deal with.

Very probably, the existing relation between the full R-S and the linearized Einstein field equations with gauge fields had not been discovered from SG because of its complicated structure when one includes matter and also because of its necesary linearization. Given that gravity couples with all kinds of matter, and being in our approach the spin 3/2 field the square root of its linearized limit, it is possible to extend our procedure in order to include in the interaction with the R-S field any kind of gauge and matter fields, obtaining in this way much more simple models than those obtained from supergravity. However, both proccedures result essentially in the same kind of non-minimal interaction.

Following this prescription, it is in principle possible to construct particular models considering interactions of phenomenological interest including the mass of the spin 3/2 field. This is the subject of future work.

2 Non-Minimal Coupling for Spin 3/2 Fields from Linearized Gravity with Gauge Fields

As mentioned in the introduction, we have on one hand the linearized Einstein field equations and on the other hand, we have the Rarita-Schwinger equations as their square root. Thus it is natural to think that we can put an energy momentum tensor for these Einstein field equations and obtain its square root in order to investigate the possible coupling of the spin 3/2 field with gauge fields.

It has been shown in ref. [6] that if one associates to the R-S equation the constraint

$$\hat{\mathcal{S}}^{\alpha\mu} \equiv \epsilon^{\alpha\mu\rho\sigma}\hat{\theta}_\rho \hat{P}_\sigma = 0\,, \tag{2.1}$$

where $\hat{\theta}_\rho = \frac{1}{\sqrt{2}}\gamma_5\gamma_\rho$ and $\hat{P}_\sigma = -i\partial_\sigma$. Then the square of this constraint turns out to be

$$\hat{\mathcal{H}}^{\alpha\beta}_{\mu\nu} = \epsilon^{\alpha}{}_{\mu}{}^{\rho\sigma}\epsilon^{\beta}{}_{\nu}{}^{\lambda\gamma}\eta_{\rho\lambda}\hat{P}_\sigma\hat{P}_\gamma\,, \tag{2.2}$$

and this term is the "hamiltonian" operator that acts over $h_{\alpha\beta}$ in standard linearized gravity, *i. e.*,

$$\hat{\mathcal{H}}^{\alpha\beta}_{\mu\nu} h_{\alpha\beta} = 0\,. \tag{2.3}$$

We first note that in the hamiltonian (2.2) the momenta $\hat{P}_\sigma$ appear quadratically. Now we want to add a potential term to $\hat{\mathcal{H}}^{\alpha\beta}_{\mu\nu}$, obviously, this must also be a four indices tensor $\mathcal{T}^{\alpha\beta}_{\mu\nu}$, it should also have units of energy (same as P^2_σ) and it should be possible to take its square root in terms of the fields characterizing the gauge and matter fields under consideration.

In particular, for the electromagnetic case, there are two tensors at our dispossal, $F_{\mu\nu}$ and its dual $\tilde{F}_{\mu\nu}$. We know that with them it is possible to construct expressions with units of energy in order to get a potential term. On the other hand, the mathematical structure of Eq. (2.2) dictate us to accompany the F^2 terms by two Levi-Civitta tensors, *i. e.*,

$$\mathcal{T}^{\alpha\beta}_{\mu\nu} \sim \epsilon^{\alpha}{}_{\mu}{}^{\rho\sigma}\epsilon^{\beta}{}_{\nu}{}^{\lambda\gamma}(AF_{\rho\sigma}F_{\lambda\gamma} + BF_{\rho\sigma}\tilde{F}_{\lambda\gamma} + C\tilde{F}_{\rho\sigma}\tilde{F}_{\lambda\gamma})\,, \tag{2.4}$$

where $\tilde{F}_{\rho\sigma}$ is the usual dual of $F_{\rho\sigma}$, A, B and C are constants products of γ matrices (perhaps of the Grassman type). This expression fits the requeriments of being quadratic in the electromagnetic field and its dual. Besides the mathematical structure of (2.4) is analogous to that of (2.2), so the combined system of expressions allow us to factorize the Levi-Civitta symbols in the computation of the square root of the resulting generalized hamiltonian containing the free plus the potential parts $\hat{\mathcal{H}}^{\alpha\beta}_{\mu\nu} + \mathcal{T}^{\alpha\beta}_{\mu\nu}$.

The square root of this hamiltonian plus potential terms must give us the R-S equations with the interaction term, and it is obvious that when we square this modified R-S equation, it must give the full "hamiltonian" that will result in the hamiltonian (2.2) plus the potential $\mathcal{T}^{\alpha\beta}_{\mu\nu}$, but also new interaction terms as happens in the Dirac and Klein-Gordon equations.

Nevertheless, γ matrices should appear in the generalized R-S equation, analogous to the $\hat{\theta}$'s in Eq. (2.1) since we are computing the squares also by means of anticommutators, besides we are applying these constraints to four spinor fields of four components.

Then the desired interaction term in the 3/2 field operator ought be of the form

$$F^{\mu\nu} + \Gamma \tilde{F}^{\mu\nu}, \tag{2.5}$$

where Γ is in general a product of Dirac matrices. For convenience we will take this Γ as γ_5 because of the algebra we are working with [6] and the fact that when we square this term, it will give us directly quadratic terms in the electromagnetic field tensor and its dual.

A natural generalization of this result to the case of non-abelian Yang-Mills would be the tensor

$$\mathcal{T}^{\alpha\beta}_{\mu\nu} \sim \epsilon^{\alpha}{}_{\mu}{}^{\rho\sigma} \epsilon^{\beta}{}_{\nu}{}^{\lambda\gamma} (A F^a_{\rho\sigma} F^a_{\lambda\gamma} + B F^a_{\rho\sigma} \tilde{F}^a_{\lambda\gamma} + C \tilde{F}^a_{\rho\sigma} \tilde{F}^a_{\lambda\gamma}), \tag{2.6}$$

where $F^a_{\rho\sigma}$ is the Yang-Mills field tensor. The corresponding interaction term will also be of the form of (2.5).

We are now in a position to generalize the R-S constraint (2.1) by implementing the interaction (2.5). For the electromagnetic case it turns out to be

$$\epsilon^{\alpha\beta\mu\nu}[\hat{\theta}_\mu \hat{P}_\nu + g_I\{\tilde{F}_{\mu\nu} + \gamma_5 F_{\mu\nu}\}] = 0\,, \tag{2.7}$$

where we have factorized the Levi-Civitta symbol and introduced a coupling constant g_I which should be related to the gravitational constant κ since this accompanies the generalized energy momentum tensor in linearized gravity.

As already mentioned, we expect that the square of expression (2.7) should give us the generalized hamiltonian containing the free plus the potential parts $\hat{\mathcal{H}}^{\alpha\beta}_{\mu\nu} + \mathcal{T}^{\alpha\beta}_{\mu\nu}$, besides it must give us an extra interaction term $\hat{\mathcal{I}}^{\alpha\beta}_{\mu\nu}$.

The free hamiltonian is clearly the one given by (2.2), the next two terms are given by

$$\mathcal{T}^{\alpha\beta}_{\mu\nu} = 2{g_I}^2 \epsilon^{\alpha}{}_{\mu}{}^{\rho\sigma} \epsilon^{\beta}{}_{\nu}{}^{\lambda\gamma} \Big(F_{\rho\sigma}F_{\lambda\gamma} + \tilde{F}_{\rho\sigma}\tilde{F}_{\lambda\gamma} + \frac{1}{\sqrt{2}}\hat{\theta}_5(F_{\rho\sigma}\tilde{F}_{\lambda\gamma} + \tilde{F}_{\rho\sigma}F_{\lambda\gamma})\Big)\,, \tag{2.8}$$

and

$$\hat{\mathcal{I}}^{\alpha\beta}_{\mu\nu} = g_I\Big(\hat{\theta}_\rho(-i\tilde{F}_{\lambda\gamma,\sigma} + F_{\lambda\gamma}\hat{P}_\sigma - \sqrt{2}i\hat{\theta}_5 F_{\lambda\gamma,\sigma}) + \hat{\theta}_\lambda(-i\tilde{F}_{\rho\sigma,\gamma} + F_{\rho\sigma}\hat{P}_\gamma - \sqrt{2}i\hat{\theta}_5 F_{\rho\sigma,\gamma})\Big)\,. \tag{2.9}$$

Here we have made the identification $\hat{\theta}_5 \equiv \frac{1}{\sqrt{2}}\gamma_5$. Because of the appearance of the terms of the type $F_{\lambda\gamma,\sigma}$ in (2.9), we identify these terms as the analogous to the extra term appearing in the Dirac equation when minimal electromagnetic coupling is introduced, namely the spin-orbit interaction for the magnetic moment of the spin 1/2 particles. This analogy may lead to interesting physical interpretation and it is the subject of future work. Besides it is interesting to comment that $\mathcal{T}^{\alpha\beta}_{\mu\nu}$ may be understood as part of a total energy-momentum tensor which contains also $\hat{\mathcal{I}}^{\alpha\beta}_{\mu\nu}$.

When considering the Yang-Mills case we will necesarily handle some internal symmetries proper of the non-abelian field under consideration, namely

$$F^{\alpha\beta} = A^{\alpha\beta}_k \alpha^k\,, \tag{2.10}$$

where the α_k are the generators of the symmetry under consideration and

$$A^k_{\rho\sigma} = \partial_\rho A^k_\sigma - \partial_\sigma A^k_\rho + e_A \epsilon^{ijk} A^i_\rho A^j_\sigma \,, \tag{2.11}$$

is the non-abelian Yang-Mills field and $f^{ijk}_A = e_A \epsilon^{ijk}$ are the structure constants of the group. Then the R-S corresponding constraint must have a similar structure to that of (2.7), namely

$$\epsilon^{\alpha\beta\mu\nu}[\hat{\theta}_\mu \hat{P}_\nu + g_2\{\tilde{F}_{\mu\nu} + \Gamma F_{\mu\nu}\}] = 0 \,, \tag{2.12}$$

where also g_2 must be related to the gravitational constant. The same previous procedure is followed in order to find out the corresponding generalized hamiltonian. In this case the potential and the interaction terms turn out to be essentially the same than in the electromagnetic case, only we have to replace g_1 by g_2 and the electromagnetic field for the Yang-Mills field (2.10), that is

$$\mathcal{T}^{\alpha\beta}_{\mu\nu} = 2{g_2}^2 \epsilon^{\alpha\ \ \rho\sigma}_{\ \mu} \epsilon^{\beta\ \ \lambda\gamma}_{\ \nu} \left(F_{\rho\sigma} F_{\lambda\gamma} + \tilde{F}_{\rho\sigma} \tilde{F}_{\lambda\gamma} + \frac{1}{\sqrt{2}} \hat{\theta}_5 (F_{\rho\sigma} \tilde{F}_{\lambda\gamma} + \tilde{F}_{\rho\sigma} F_{\lambda\gamma}) \right) \,, \tag{2.13}$$

and

$$\hat{\mathcal{I}}^{\alpha\beta}_{\mu\nu} = g_2 \left(\hat{\theta}_\rho (-i\tilde{F}_{\lambda\gamma,\sigma} + F_{\lambda\gamma}\hat{P}_\sigma - \sqrt{2} i \hat{\theta}_5 F_{\lambda\gamma,\sigma}) + \hat{\theta}_\lambda (-i\tilde{F}_{\rho\sigma,\gamma} + F_{\rho\sigma}\hat{P}_\gamma - \sqrt{2} i \hat{\theta}_5 F_{\rho\sigma,\gamma}) \right) \,, \tag{2.14}$$

notice that the internal index a of (2.6) does not appear in these expressions, since it is already taken into account in (2.10). As in the electromagnetic case, the coupling terms in $\hat{\mathcal{I}}^{\alpha\beta}_{\mu\nu}$ are to be investigated in future work.

In the next three sections we will show that linearized Supergravity N=1, 2 and 4 provide similar results for the free case, the electromagnetic one and the non-abelian Yang-Mills field correspondingly. However, the enlarged supersymmetries involved in the last two cases enforces the appearance of more spin 3/2 fields and the rising of much more complicated expressions than those obtained in this section. Both proccedures provide, however, the same kind of non-minimal interaction and deserve further study in view of possible phenomenological implications for specific gauge fields of interest.

3 Spin 3/2 and Linearized Einstein Fields from Supergravity N=1

We begin by using the langrangian for *Supergravity N=1* given in ref. [10]

$$\mathcal{L} = -\frac{e}{2}\mathcal{R} - \frac{1}{2}\bar{\Psi}_\mu \epsilon^{\mu\nu\rho\sigma}\gamma_5\gamma_\nu D_\rho \Psi_\sigma \,, \tag{3.1}$$

where e is the determinant of the tetrad, $\mathcal{R}$ is the curvature, Ψ_μ is the gravitino field (spin 3/2), and D_ρ is the covariant derivative including the spin connection.

We linearize the field equations by dropping quadratic and higher terms since the spin connection in D_ρ contains Ψ^2 and $h_{\alpha\beta}$ we find that $D_\rho\Psi_\mu$ goes to $\partial\rho\Psi_\mu$. After that we find once more

$$\epsilon^{\mu\nu\rho\sigma}\gamma_5\gamma_\nu\partial_\rho\Psi_\sigma = 0\,. \tag{3.2}$$

We can associate to Eq. (3.2) the operator

$$\hat{\mathcal{S}}^{\mu\nu} \equiv \epsilon^{\mu\nu\rho\sigma}\hat{\theta}_\rho\hat{P}_\sigma = 0\,, \tag{3.3}$$

where $\hat{\theta}_\rho$ and $\hat{P}_\sigma$ are given in the preceding section.

Then, by using the algebra of the anticommutators of $\hat{\theta}_\mu$ [6] namely

$$\{\hat{\theta}_\mu, \hat{\theta}_\nu\} = \eta_{\mu\nu}\,, \tag{3.4}$$

we obtain

$$\{\hat{\mathcal{S}}^\alpha_\mu, \hat{\mathcal{S}}^\beta_\nu\} = \hat{\mathcal{H}}^{\alpha\beta}_{\mu\nu}\,, \tag{3.5}$$

where

$$\hat{\mathcal{H}}^{\alpha\beta}_{\mu\nu} = \epsilon^{\alpha\ \ \rho\sigma}_{\ \mu}\epsilon^{\beta\ \ \lambda\gamma}_{\ \nu}\eta_{\rho\lambda}\hat{P}_\sigma\hat{P}_\gamma\,, \tag{3.6}$$

is the "hamiltonian" operator that acts over $h_{\alpha\beta}$ in standard linearized gravity as described in the preceding section and in ref. [6].

As claimed before, the Rarita-Schwinger equations in flat space-time turn out be the square root of the linearized Einstein field equations. Obviously, we have no contribution of any matter or gauge field.

4 Electromagnetic Interaction for Spin 3/2 Fields from Supergravity N=2

We present the electromagnetic coupling of spin 3/2 fields by using *Supergravity N=2* [10], since it naturally incorporates the graviton, the electromagnetic field and two gravitinos.

The lagrangian for Supergravity N=2 is

$$\begin{aligned} \mathcal{L} &= -\frac{e}{2}\mathcal{R} - \frac{1}{2}\bar{\Psi}^i_\mu \epsilon^{\mu\nu\rho\sigma}\gamma_5\gamma_\nu D_\rho \Psi^i_\sigma - \frac{e}{4}F_{\alpha\beta}F^{\alpha\beta} \\ &+ \frac{\kappa}{4\sqrt{2}}\bar{\Psi}^i_\mu[e(F^{\mu\nu} + \hat{F}^{\mu\nu}) + \gamma_5(\tilde{F}^{\mu\nu} + \tilde{\hat{F}}^{\mu\nu})]\Psi^j_\nu \epsilon^{ij}, \end{aligned} \tag{4.1}$$

where the elements of this lagrangian are the same that in the precceding section and $\tilde{F}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}$ is the dual of the usual electromagnetic fields strenght tensor and $\hat{F}_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - \frac{\kappa}{2\sqrt{2}}[\Psi^i_\mu \Psi^j_\nu - \Psi^i_\nu \Psi^j_\mu]\epsilon^{ij}$ is the supercovariant curl. The appearance of the ϵ^{ij} in the lagrangian and in the above equation has to do with the fact that, as mentioned in the introduction, we are leading with a larger supersymmetry. This is reflected in the above expression, in which the ϵ^{ij} mix up the interaction of the two gravitinos.

After linearizing the resulting field equations by following the same proccedure as in the previous section, we find

$$\epsilon^{\alpha\mu\nu\beta}\gamma_5\gamma_\mu\partial_\nu\Psi^i_\beta - \frac{\kappa\epsilon^{ij}}{\sqrt{2}}[F^{\alpha\beta} + \tfrac{1}{2}\gamma_5\epsilon^{\alpha\beta\rho\sigma}F_{\rho\sigma}]\Psi^j_\beta = 0\,. \tag{4.2}$$

Notice that the term in squared brackets $F^{+\alpha\beta} \equiv F^{\alpha\beta} + \frac{1}{2}\gamma_5\epsilon^{\alpha\beta\rho\sigma}F_{\rho\sigma}$ is exactly the interaction previously found in section 2. This term has been also deduced from a consistent gauge field theory and is precisely the dipole term found by Ferrara *et al* [5] following a lagrangian procedure based on demanding $g = 2$ for arbitrary spin paricles. In their article they have shown that this term cancells divergences and avoids superluminal velocities in systems of spin 3/2 particles. For these reasons the solutions associated to (4.2) and the previously proposed interaction (2.7) should not have any physical inconsistencies.

Equation (4.2) can be shown to be the generalized Rarita-Schwinger equation

$$\epsilon^{\alpha\beta\mu\nu}[\delta_{ij}\hat{\theta}_\mu \hat{P}_\nu + g_I\, \epsilon_{ij}(\tilde{F}_{\mu\nu} + \sqrt{2}\hat{\theta}_5 \tilde{F}_{\mu\nu})]\Psi^j_\beta = 0\,, \tag{4.3}$$

where $g_I = \frac{i\kappa}{8}$ is the coupling constant, and as mentioned in section 2, it is related to the gravitational constant κ.

Thus, we can associate to Eq. (4.3) the constraint

$$\hat{\mathcal{S}}^{\alpha\beta}_{ij} = \epsilon^{\alpha\beta\mu\nu}[\delta_{ij}\hat{\theta}_\mu \hat{P}_\nu + g_I\, \epsilon_{ij}(\tilde{F}_{\mu\nu} + \sqrt{2}\hat{\theta}_5 \tilde{F}_{\mu\nu})] = 0\,, \tag{4.4}$$

which is essentially the same constraint deduced in (2.7) but here δ's and ϵ's factors appear because of the enlarged supersymmetry we are working with.

By using the anticommutators of section 3 and

$$\{\hat{\theta}_5, \hat{\theta}_5\} = 1\,, \tag{4.5}$$

we get the algebra

$$\begin{aligned}
\{\hat{\mathcal{S}}^{\alpha\ ij}_{\ \mu}, \hat{\mathcal{S}}^{\beta\ kl}_{\ \nu}\} &= \epsilon^{\alpha\ \rho\sigma}_{\ \mu}\epsilon^{\beta\ \lambda\gamma}_{\ \nu}\Big[\eta_{\rho\lambda}\hat{P}_\sigma\hat{P}_\gamma\delta^{ij}\delta^{kl} \\
&+ 2{g_I}^2\Big(F_{\rho\sigma}F_{\lambda\gamma} + \frac{1}{\sqrt{2}}\hat{\theta}_5(F_{\rho\sigma}\tilde{F}_{\lambda\gamma} + F_{\lambda\gamma}\tilde{F}_{\rho\sigma}) + \tilde{F}_{\rho\sigma}\tilde{F}_{\lambda\gamma}\Big)\epsilon^{ij}\epsilon^{lk} \\
&+ g_I\Big(\delta^{ij}\epsilon^{lk}\hat{\theta}_\rho\Big(- i\tilde{F}_{\lambda\gamma,\sigma} + 2\tilde{F}_{\lambda\gamma}\hat{P}_\sigma - \sqrt{2}i\hat{\theta}_5 F_{\lambda\gamma,\sigma}\Big) \\
&+ \epsilon^{ij}\delta^{lk}\hat{\theta}_\lambda\Big(- i\tilde{F}_{\rho\sigma,\gamma} + 2\tilde{F}_{\rho\sigma}\hat{P}_\gamma - \sqrt{2}i\hat{\theta}_5 F_{\rho\sigma.\gamma}\Big)\Big)\Big]\,.
\end{aligned} \tag{4.6}$$

The first term in the last equation is the hamiltonian for linearized gravity discussed before up to delta factors

$$\hat{\mathcal{H}}^{\alpha\beta}_{\mu\nu}\delta^{ij}\delta^{lk} = \epsilon^{\alpha}{}_{\mu}{}^{\rho\sigma}\epsilon^{\beta}{}_{\nu}{}^{\lambda\gamma}\eta_{\rho\lambda}\hat{P}_{\sigma}\hat{P}_{\gamma}\delta^{ij}\delta^{lk}\,. \tag{4.7}$$

The second of these terms is precisely the generalized energy-momentum tensor for the electromagnetic field found in section 2 up to epsilon factors, that is

$$\mathcal{T}^{\alpha\beta}_{\mu\nu}\epsilon^{ij}\epsilon^{kl} = 2{g_I}^2\Big(F_{\rho\sigma}F_{\lambda\gamma} + \frac{1}{\sqrt{2}}\hat{\theta}_5(F_{\rho\sigma}\tilde{F}_{\lambda\gamma} + F_{\lambda\gamma}\tilde{F}_{\rho\sigma}) + \tilde{F}_{\rho\sigma}\tilde{F}_{\lambda\gamma}\Big)\epsilon^{ij}\epsilon^{lk}\,. \tag{4.8}$$

This result contributes to corroborate the proposed generalized energy momentum tensors (2.4) and (2.8). However, there is an extra term of interaction, this term is

$$\begin{aligned}\hat{\mathcal{I}}^{\alpha\beta ijkl}_{\mu\nu} \quad &\equiv \quad g_I\,\epsilon^{\alpha}{}_{\mu}{}^{\rho\sigma}\epsilon^{\beta}{}_{\nu}{}^{\lambda\gamma}\Big(\delta^{ij}\epsilon^{lk}\hat{\theta}_{\rho}\Big(-i\tilde{F}_{\lambda\gamma,\sigma} + 2\tilde{F}_{\lambda\gamma}\hat{P}_{\sigma} - \sqrt{2}i\hat{\theta}_5F_{\lambda\gamma,\sigma}\Big)\\ &+ \quad \epsilon^{ij}\delta^{lk}\hat{\theta}_{\lambda}\Big(-i\tilde{F}_{\rho\sigma,\gamma} + 2\tilde{F}_{\rho\sigma}\hat{P}_{\gamma} - \sqrt{2}i\hat{\theta}_5F_{\rho\sigma,\gamma}\Big)\Big),\end{aligned} \tag{4.9}$$

as already mentioned in section 2 this expression contains the information of terms like the spin coupling and it deserves further study.

5 Yang-Mills Interaction for Spin 3/2 Fields from Supergravity N=4

Now we are in a position to explore the coupling of a Yang-Mills field with Rarita-Schwinger field. As a Supergravity model that involves a non-abelian Yang-Mills field we take *Supergravity N=4* [10,?].

In the philosophy of the preceding calculations we can associate a constraint to the field equations for the gravitinos, after eliminating the scalar and the spin 1/2 fields, it turns out to be

$$\hat{\mathcal{S}}_{ij}^{\alpha\beta} \equiv \epsilon^{\alpha\beta}{}_{\mu\nu}\left(\delta_{ij}\hat{\theta}^{\mu}\hat{P}^{\nu} + g_2(\tilde{F}^{\mu\nu}_{(ij)} - \sqrt{2}i\hat{\theta}_5 F^{\mu\nu}_{(ij)})\right) + \frac{i\delta_{ij}}{2\kappa}(e_A + \sqrt{2}ie_B\hat{\theta}_5)\sigma^{\alpha\beta}\,, \tag{5.1}$$

where now the coupling constant $\frac{i\kappa}{4}$ and the contribution of the non-abelian field is given by

$$\mathcal{F}^{\rho\sigma}_{(ij)} = \alpha^{k}_{(ij)}A^{\rho\sigma}_{k} + \sqrt{2}i\hat{\theta}_5\beta^{k}_{(ij)}B^{\rho\sigma}_{k}\,, \tag{5.2}$$

and

$$\begin{aligned} A^{k}_{\rho\sigma} &= \partial_\rho A^k_\sigma - \partial_\sigma A^k_\rho + e_A\epsilon^{ijk}A^i_\rho A^j_\sigma, & (5.3)\\ B^{k}_{\rho\sigma} &= \partial_\rho B^k_\sigma - \partial_\sigma B^k_\rho + e_B\epsilon^{ijk}B^i_\rho B^j_\sigma, & (5.4)\end{aligned}$$

are the non-abelian Yang-Mills fields. In Eqs. (5.1) and (5.2) the indices i and j refer to the four gravitinos and the components of the matrices alpha and beta that generate the $SU(2)\times SU(2)$ symmetry of supergravity N=4. Notice that in the constraint (5.1) we have preserved the notation of ref. [11] and the difference in the definition of γ_5 is obvious with respect to that of the preceding sections. In order to recover the notation used before we have to perform the sustitution $-i\hat{\theta}_5 \rightarrow \hat{\theta}_5$, thus the constraint (5.1) becomes

$$\hat{\mathcal{S}}_{ij}^{\alpha\beta} \equiv \epsilon^{\alpha\beta}{}_{\mu\nu}\left(\delta_{ij}\hat{\theta}^{\mu}\hat{P}^{\nu} + g_2(\tilde{F}^{\mu\nu}_{(ij)} + \sqrt{2}\hat{\theta}_5 F^{\mu\nu}_{(ij)})\right) + \frac{i\delta_{ij}}{2\kappa}(e_A - \sqrt{2}e_B\hat{\theta}_5)\sigma^{\alpha\beta}\,, \tag{5.5}$$

By using anticommutators as in the preceding sections, we get the algebra

$$\begin{aligned} \{\hat{\mathcal{S}}^{\alpha}{}_{\mu ij}, \hat{\mathcal{S}}^{\beta}{}_{\nu kl}\} &= \epsilon^{\alpha}{}_{\mu\rho\sigma}\epsilon^{\beta}{}_{\nu\lambda\gamma}\Big[\eta^{\rho\lambda}\hat{P}^{\sigma}\hat{P}^{\gamma}\delta_{ij}\delta_{kl} \\ &+ 2{g_2}^2\Big(F^{\rho\sigma}_{(ij)}F^{\lambda\gamma}_{(kl)} + \frac{1}{\sqrt{2}}\hat{\theta}_5(\tilde{F}^{\rho\sigma}_{(ij)}F^{\lambda\gamma}_{(kl)} + F^{\rho\sigma}_{(ij)}\tilde{F}^{\lambda\gamma}_{(kl)}) + \tilde{F}^{\rho\sigma}_{(ij)}\tilde{F}^{\lambda\gamma}_{(kl)}\Big) \\ &+ g_2\Big(\delta_{ij}\hat{\theta}^{\rho}(- i\tilde{F}^{\lambda\gamma,\sigma}_{(kl)} + 2\tilde{F}^{\lambda\gamma}_{(kl)}\hat{P}^{\sigma} - \sqrt{2}i\hat{\theta}_5 F^{\lambda\gamma,\sigma}_{(kl)})\end{aligned}$$

$$
\begin{aligned}
&+ \quad \delta_{lk}\hat{\theta}^{\lambda}\big(- i\tilde{F}^{\rho\sigma,\gamma}_{(ij)} + 2\tilde{F}^{\rho\sigma}_{(ij)}\hat{P}^{\gamma} - \sqrt{2}i\hat{\theta}_5 F^{\rho\sigma,\gamma}_{(ij)})\Big)\Big] \\
&+ \quad \frac{i\delta_{kl}}{2\kappa}\epsilon^{\alpha}{}_{\mu\rho\sigma}\Big[\delta_{ij}(\sqrt{2}\epsilon^{\rho\beta}{}_{\nu\tau}\hat{\theta}_5\hat{\theta}^{\tau}\hat{P}^{\sigma} - \sqrt{2}e_B\hat{\theta}_5(\delta^{\rho}{}_{\nu}\hat{\theta}^{\beta} - \delta^{\rho\beta}\hat{\theta}_{\nu})\hat{P}^{\sigma}) \\
&+ \quad 2g_2(e_A\tilde{F}^{\rho\sigma}_{(ij)}\sigma^{\beta}{}_{\nu} - \sqrt{2}e_B\hat{\theta}_5\tilde{F}^{\rho\sigma}_{(ij)}\sigma^{\beta}{}_{\nu} - e_B F^{\rho\sigma}_{(ij)}\sigma^{\beta}{}_{\nu})\Big] \qquad (5.6) \\
&+ \quad \frac{i\delta_{ij}}{2\kappa}\epsilon^{\beta}{}_{\nu\lambda\gamma}\Big[\delta_{kl}(\sqrt{2}\epsilon^{\lambda\alpha}{}_{\mu\tau}\hat{\theta}_5\hat{\theta}^{\tau}\hat{P}^{\gamma} - \sqrt{2}e_B\hat{\theta}_5(\delta^{\lambda}{}_{\mu}\hat{\theta}^{\alpha} - \delta^{\lambda\alpha}\hat{\theta}_{\mu})\hat{P}^{\gamma}) \\
&+ \quad 2g_2(e_A\tilde{F}^{\lambda\gamma}_{(kl)}\sigma^{\alpha}{}_{\mu} - \sqrt{2}e_B\hat{\theta}_5\tilde{F}^{\lambda\gamma}_{(kl)}\sigma^{\alpha}{}_{\mu} - e_B F^{\lambda\gamma}_{(kl)}\sigma^{\alpha}{}_{\mu})\Big] \\
&- \quad \frac{\delta_{ij}\delta_{kl}}{4\kappa^2}\Big[(e_A^2 + e_B^2)(\eta^{\alpha\beta}\sigma_{\mu\nu} + \eta_{\mu\nu}\sigma^{\alpha\beta} + \delta^{\alpha}{}_{\nu}\sigma^{\beta}{}_{\mu} + \delta^{\beta}{}_{\mu}\sigma_{\nu}{}^{\alpha}) + 2e_B^2\sigma^{\alpha}{}_{\mu}\sigma^{\beta}{}_{\nu}\Big].
\end{aligned}
$$

We can identify terms and the first of them is essentially the same tensor found in (2.2), it gives us the linearized operator for the Einstein field equations up to delta factors. That is

$$
\hat{\mathcal{H}}^{\alpha\beta}_{\mu\nu}\delta_{ij}\delta_{lk} = \epsilon^{\alpha}_{\ \mu}{}^{\rho\sigma}\epsilon^{\beta}_{\ \nu}{}^{\lambda\gamma}\eta_{\rho\lambda}\hat{P}_{\sigma}\hat{P}_{\gamma}\delta_{ij}\delta_{lk}\,. \qquad (5.7)
$$

The second term is the Yang-Mills field energy momentum tensor obtained in section 2, corresponding to the one deduced in (2.13) by taking the square root of linearized gravity. This term is:

$$
\begin{aligned}
\mathcal{T}^{\alpha\beta}_{\mu\nu(ijkl)} \quad &= \quad 2{g_2}^2\epsilon^{\alpha}_{\ \mu}{}^{\rho\sigma}\epsilon^{\beta}_{\ \nu}{}^{\lambda\gamma}\Big(F_{\rho\sigma(ij)}F_{\lambda\gamma(kl)} \qquad (5.8) \\
&+ \quad \frac{1}{\sqrt{2}}\hat{\theta}_5(\tilde{F}_{\rho\sigma(ij)}F_{\lambda\gamma(kl)} + F_{\rho\sigma(ij)}\tilde{F}_{\lambda\gamma(kl)}) + \tilde{F}_{\rho\sigma(ij)}\tilde{F}_{\lambda\gamma(kl)}\Big),
\end{aligned}
$$

in this case the latin indices refer to the components of the α and β matrices mentioned before, these indices apply over the different gravitinos. The following term is the corresponding to the interaction term (2.14) for the Yang-Mills field in comparison to that obtained in section 2 (4.9)

$$
\hat{\mathcal{I}}^{\alpha\beta}_{\mu\nu(ijkl)} \quad \equiv \quad \epsilon^{\alpha}{}_{\mu\rho\sigma}\epsilon^{\beta}{}_{\nu\lambda\gamma}\Big[g_2\Big(\delta_{ij}\hat{\theta}^{\rho}\big(- i\tilde{F}^{\lambda\gamma,\sigma}_{(kl)} + 2\tilde{F}^{\lambda\gamma}_{(kl)}\hat{P}^{\sigma} - \sqrt{2}i\hat{\theta}_5 F^{\lambda\gamma,\sigma}_{(kl)})
$$

$$
\begin{aligned}
&+ \quad \delta_{lk}\hat{\theta}^{\lambda}\big(- i\tilde{F}^{\rho\sigma,\gamma}_{(ij)} + 2\tilde{F}^{\rho\sigma}_{(ij)}\hat{P}^{\gamma} - \sqrt{2}i\hat{\theta}_5 F^{\rho\sigma,\gamma}_{(ij)}\big)\Big)\Big] \\
&+ \quad \frac{i\delta_{kl}}{2\kappa}\epsilon^{\alpha}{}_{\mu\rho\sigma}\Big[\delta_{ij}(\sqrt{2}\epsilon^{\rho\beta}{}_{\nu\tau}\hat{\theta}_5\hat{\theta}^{\tau}\hat{P}^{\sigma} - \sqrt{2}e_B\hat{\theta}_5(\delta^{\rho}{}_{\nu}\hat{\theta}^{\beta} - \delta^{\rho\beta}\hat{\theta}_{\nu})\hat{P}^{\sigma}) \\
&+ \quad 2g_2(e_A\tilde{F}^{\rho\sigma}_{(ij)}\sigma^{\beta}{}_{\nu} - \sqrt{2}e_B\hat{\theta}_5\tilde{F}^{\rho\sigma}_{(ij)}\sigma^{\beta}{}_{\nu} - e_B F^{\rho\sigma}_{(ij)}\sigma^{\beta}{}_{\nu})\Big] \qquad (5.9) \\
&+ \quad \frac{i\delta_{ij}}{2\kappa}\epsilon^{\beta}{}_{\nu\lambda\gamma}\Big[\delta_{kl}(\sqrt{2}\epsilon^{\lambda\alpha}{}_{\mu\tau}\hat{\theta}_5\hat{\theta}^{\tau}\hat{P}^{\gamma} - \sqrt{2}e_B\hat{\theta}_5(\delta^{\lambda}{}_{\mu}\hat{\theta}^{\alpha} - \delta^{\lambda\alpha}\hat{\theta}_{\mu})\hat{P}^{\gamma}) \\
&+ \quad 2g_2(e_A\tilde{F}^{\lambda\gamma}_{(kl)}\sigma^{\alpha}{}_{\mu} - \sqrt{2}e_B\hat{\theta}_5\tilde{F}^{\lambda\gamma}_{(kl)}\sigma^{\alpha}{}_{\mu} - e_B F^{\lambda\gamma}_{(kl)}\sigma^{\alpha}{}_{\mu})\Big] \\
&- \quad \frac{\delta_{ij}\delta_{kl}}{4\kappa^2}\Big[(e_A^2 + e_B^2)(\eta^{\alpha\beta}\sigma_{\mu\nu} + \eta_{\mu\nu}\sigma^{\alpha\beta} + \delta^{\alpha}{}_{\nu}\sigma^{\beta}{}_{\mu} + \delta^{\beta}{}_{\mu}\sigma_{\nu}{}^{\alpha}) + 2e_B^2\sigma^{\alpha}{}_{\mu}\sigma^{\beta}{}_{\nu}\Big].
\end{aligned}
$$

Obviously these interaction terms are much more complicated than those obtained in (2.14). They provide however a different approach to our square root procedure developed in section 2, and they should be the subject of future research.

6 Conclusions

We have discussed the problem of the non-minimal electromagnetic and Yang-Mills coupling for the spin 3/2 field. We took advantage of a precceding work where we showed the fact that the Rarita-Schwinger equations in flat space-time are the square root of the linearized Einstein field equations. This fact was used in order to find out the way gauge fields should couple with spin 3/2 fields by implementing appropriate generalized energy momentum tensors on the linearized Einstein field equations. In particular for the electromagnetic and Yang-Mills fields, these generalized energy momentum tensors were build up and inserted in the linear gravity equations. This proccedure allowed us to take the square root of the system consisting of linearized gravity plus matter (gauge fields), thus obtaining modified Rarita-Schwinger equations containing the pursued coupling with gauge fields. Similar R-S equations were obtained by linearizing Supergravity which enforces our proposal and at the same time provides an alternate scheme. When we square these equations, linearized gravity plus the generalized energy momentum arise, but also expected interaction terms analogous to the spin-orbit coupling terms of the Dirac and Klein-Gordon equations. These interesting terms deserve future study.

In the case of electromagnetic interaction, the coupling obtained by us coincides with the one found by Ferrara *et al* [5] by a very different procedure, namely by demanding $g = 2$ at the tree level for arbitrary spin particles. They obtained an extra dipole term in the equations for these fields. This dipole term avoids the bad energy behavior in systems of spin 3/2 particles and the physical inconsistencies discussed in ref. [3]. It is to be remarked that with our proccedure it was also possible to find out the interaction with Yang-Mills fields.

The electromagnetic and non-abelian Yang-Mills fields interaction terms are both of the form

$$Field + \gamma_5 Dual\ Field\,. \tag{6.1}$$

In our square root approach it is always possible to add to the above interaction any other gauge and matter fields. The fact that we were able to obtain the interaction with Yang-Mills besides the electromagnetic field for spin 3/2 fields allows us to construct a variety of models with possible interesting phenomenological implications. This is the subject of future work.

It is interesting to mention that a term of similar structure was implemented by Cucchieri, Porrati and Deser [12] by analyzing the gravitational coupling for higher spin massive fields, where the *Field* of the above expression (6.1)is the Riemman tensor itself. This happens, in particular, when spin 5/2 is considered.

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