A Semiclassical Estimate of the Coulomb Excitation of the Giant Dipole Resonance in $^{208}\text{Pb}$

D. Galetti and A. Eiras

*Istituto de Física Teórica*
*Universidade Estadual Paulista*
*Rua Pamplona, 145*
*01405-900 - São Paulo, S.P.*
*Brazil*

and

F.F. de Sousa Cruz, J.R. Marinelli and M.M.W. de Moraes

*Departamento de Física*
*Universidade Federal de Santa Catarina*
*88049 - Florianópolis, S.C.*
*Brazil*

*Supported by CNPq - Brazil*
A Semiclassical Estimate Of The Coulomb Excitation of the Giant Dipole Resonance in $^{208}$Pb

D. Galetti and A. Eiras*
Instituto de Física Teórica
Universidade Estadual Paulista
R. Pamplona 145
01405-900 São Paulo – SP – Brazil

F.F. de Souza Cruz, J.R. Marinelli and M.M.W. de Moraes
Depto de Física
Universidade Federal de Santa Catarina
88049 Florianópolis – SC – Brazil

Abstract

A semiclassical approach to study pure Coulomb excitation of $^{208}$Pb giant dipole isovector resonance is examined. We consider medium energy projectiles and assume the target excitation to be described by a simple Goldhaber-Teller model. It is shown that the main features concerning the angular distribution are obtained in the angular range described by the model and an estimate is made of the pure Coulomb dipole contribution to the measured cross sections.

*Supported by CNPq - Brazil

The use of medium energy heavy ions to excite giant resonances in nuclei has provided new perspectives for the study of that sort of nuclear collective motions. In particular, there have been extensive discussions of the effect of the Coulomb excitation by medium energy projectiles on giant resonance cross sections [1].

The result of the analysis of the Coulomb excitation process in terms of the equivalent virtual photons have pointed out to the very interesting fact that at incident energies around or higher than 50 MeV/nucleon, the giant resonance cross section with excitation energy between 10-20 MeV is dominated by the Coulomb interaction. Thus, within that range of energy the large enhancement of the observed giant resonance cross section is mainly due to the strong increase of the Coulomb excitation.

In this paper we intend to show how one could calculate the giant dipole resonance angular distribution using a semiclassical approach for the pure Coulomb excitation of $^{208}$Pb induced by deuterons or alpha particles with incident energies in the region of interest and how one could estimate the contribution of that sort of giant resonance to the measured cross sections for forward angles. The method [2] makes use of the quantum phase space Wigner function [3] associated to a simple model describing an isovector Goldhaber-Teller dipole vibration degree of freedom for the target nucleus [4], while the projectile is treated as a structureless particle moving under a pure Coulomb force.

Although the present method could be applied to any kind of reactions, for convenience we are considering a nonreactive collision represented as

$$A + BC(n) \rightarrow A + BC(m),$$

where $n$ and $m$ refer to any quantum vibrational numbers to be specified later.

On what concerns the target nuclei, we assume that it is constituted of two masses, $M_n$ (neutrons) and $M_p$ (protons) vibrating along the x-axis. Through an adaptation of a coordinate transformation and an appropriate scaling proposed by Secrest and Johnson [5], the two body oscillator system plus projectile can be replaced by an equivalent one body vibrator plus projectile. Therefore, in the present model, the Coulomb interaction term is considered as the ordinary one associated to only two point charges, $e^2 Z^n Z^p / |\mathbf{Q}_n - \mathbf{r}_p|$, being the whole system described by an appropriate kinematics where $e$ is a
constant [5].

If we assume that the vibrational motion can be described by a harmonic oscillator in a n-state, the corresponding Wigner function is written as [6]

$$W_{\text{target}}^{(n)} = (-1)^n \left( \frac{1}{\pi \hbar} \right)^{3/2} \exp \left( \frac{-2E_{\text{class}}}{\hbar} \right) L_n^{(1)} \left( \frac{2E_{\text{class}}}{\hbar} \right),$$  \tag{2}

where $E_{\text{class}}$ and $\omega$ are the classical energy and frequency of the harmonic oscillator respectively. The last term $L_n^{(1)}(x)$ is the associated Laguerre polynomial. Furthermore, since we want to consider the projectile as a structureless particle, its corresponding phase space distribution function will be taken as a classical distribution:

$$\rho(x, p) = \delta(p - p_{\text{cm}}(t)) \delta(x - x_{\text{cm}}(t)).$$  \tag{3}

This indeed means that the distribution function for the projectile does not introduce any quantum correlation between $x$ and $p$, contrary to (7) where the Wigner function describing the target vibration embodies at least the Heisenberg uncertainty correlation. In this way, at $t = -\infty$, when the $^{208}$Pb target is considered in its ground state, the distribution for the whole system is written as a direct product:

$$\Omega(Q, P, \bar{r}, \bar{p}) = \rho(Q, P) \rho_{\text{target}}(\bar{r}, \bar{p}).$$  \tag{4}

As it is known, the quantum full time evolution of the Wigner function is governed by the Wigner representative of the quantum von Neumann-Liouville equation [7]. In this work, however, we shall restrict ourselves to a semiclassical version of that equation which is equivalent to keep only the lowest order term of the corresponding $\hbar$ power series for the Liouville operator while the quantum character is preserved in the Wigner function. Hence, the semiclassical time evolution of the Wigner function is given as

$$\Omega(t) = \exp(-t L_{\text{class}}) \Omega_0(Q, P, \bar{r}, \bar{p}).$$  \tag{5}

The subscripts in $\Omega$ refer to time at which the system is considered and $L_{\text{class}}$ is the classical Liouville operator which is defined as

$$L_{\text{class}}(Q, P, \bar{r}, \bar{p}) = -\sum_n \left( \frac{\partial H}{\partial Q_n} \frac{\partial}{\partial P_n} - \frac{\partial H}{\partial P_n} \frac{\partial}{\partial Q_n} \right) + \left( \frac{\partial H}{\partial Q} \frac{\partial}{\partial P} - \frac{\partial H}{\partial P} \frac{\partial}{\partial Q} \right).$$  \tag{6}

It is the model Hamiltonian written in terms of the variables defined by the proper kinematics [5],

$$H = \frac{p_x^2}{2m} + \frac{p_y^2}{2m} + \frac{KQ^2}{2} + V_{\text{class}}(\bar{Q} - \bar{Q}'),$$  \tag{7}

where $\mu_{\text{BC}}$ is the target reduced neutron-proton mass and $m$ is the reduced projectile-target mass. The spring constant $K$ is our only free parameter, which was fitted to the 13.5 MeV isovector dipole resonance of $^{208}$Pb.

In order to calculate the time evolution of the Wigner function, we could observe that the classical equations of motion lead to the following identity

$$\Omega(t) = \Omega(0) \Omega(t, \pi(t), \bar{r}(t), \bar{p}(t)) = \Omega_0(Q(t), P(t), \bar{r}(t), \bar{p}(t)),$$  \tag{8}

where $(Q(t), P(t), \bar{r}(t), \bar{p}(t))$ are the phase space points at an arbitrary time obtained by solving classical equations of motion for the Hamiltonian (7) taking $(Q(t_0), P(t_0), \bar{r}(t_0), \bar{p}(t_0))$ as the initial conditions. In fact, the value of the Wigner function at time $t$ at the corresponding phase space point $(Q(t_0), P(t_0), \bar{r}(t_0), \bar{p}(t_0))$ is identical to the value of the Wigner function at time $t_0$ calculated at a phase point generated by the classical equations of motion taking $(Q(t_0), P(t_0), \bar{r}(t_0), \bar{p}(t_0))$ as the initial conditions. In this form, the calculations can be performed in a following way. First, we assume the projectile trajectory to be fixed on the $x$-$y$ cartesian plane and the time propagation is carried out by considering a mesh of points in the relevant domain of the target phase space. Each of these points $(Q(t_0), P(t_0), \bar{r}(t_0), \bar{p}(t_0))$ is evolved according to the six Hamilton differential equations for the projectile, i.e., a set of parameters describing the laboratory energy $E_{\text{lab}}$, momenta $p_{x_0}, p_{y_0}$, impact parameter $b_0 = b_0$, asymptotic position $(x_0)$, and mass of the projectile $m$. In our calculation the set of 30x30 time evolved mesh points characterizing the Wigner function at $t = -\infty$, allows us to calculate the target final Wigner function and the excitation probability as [9]
Calculations were performed for $E_{1u.b} = 218 \text{ MeV}$ and $172 \text{ MeV}$ alpha particles and $E_{1ab} = 108 \text{ MeV}$ deuterons for the one phonon excitation case. The first interesting result can be seen in Fig.(1.a), where in the intermediate range of impact parameters ($b \approx 8 \text{ fm}$), the calculated points for the giant dipole resonance excitation probabilities are fitted by an exponential of the form

$$P_{exc} = Ae^{-Db},$$

(10)

where $A$ and $D$ are two adjustable parameters. The fitting correlation coefficient is 0.999 in all cases.

In this semiclassical picture, the incident particle trajectories can be seen as classical so that we are allowed to use the standard relation for the impact parameter as a good approximation

$$b = \frac{Z_{t}Z_{p}e^{2}}{2E \tan(\frac{\theta}{2})}.$$  

(11)

Hence, the excitation probability can be easily obtained as a function of the projectile scattering angle and the Coulomb excitation differential cross section is the direct product of the Rutherford differential cross section, calculated within the Secrest and Johnson kinematics, and the excitation probability. We may observe that the location of the angular distribution peak within the intermediate range of impact parameters ($b \approx 8 \text{ fm}$), is given by

$$\theta_{peak} = \arccos\left(\frac{M_{NC}Z_{p}e^{2}}{4mc}Z_{t}B\right).$$  

(12)

In particular, $\alpha = (M_{NC}Z_{p}e^{2})/(4mc) = 48.7 \text{ MeV.fm}$ for $^{208}\text{Pb}$. Some results from our model are presented in Table I where they are compared to experimental data.

<table>
<thead>
<tr>
<th></th>
<th>$d - ^{208}\text{Pb}$</th>
<th>$\alpha - ^{208}\text{Pb}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_{exp}$</td>
<td>8.0°</td>
<td>11.0°</td>
</tr>
<tr>
<td>$\theta_{calc}$</td>
<td>8.8°</td>
<td>12.0°</td>
</tr>
</tbody>
</table>

Table I. The values for the experimental and calculated angles refer to the center of mass system. The data for $E=108 \text{ MeV} d - ^{208}\text{Pb}$ is found in Ref.(8), while for $E=172 \text{ MeV} \alpha - ^{208}\text{Pb}$ was extracted from Ref.(10).

In spite of the simplicity of our model, Table I shows the good agreement of the data for the peak. Furthermore, the angular distributions presented in Fig.(2) confirm such agreement within the range for which eq.(10) is applicable.

The calculated $\theta_{peak}$ for the $E = 218 \text{ MeV} \alpha - ^{208}\text{Pb}$ is verified to lie at 8.7° but it is not presented in Table I since the experimental angle cannot be conclusively inferred from data. For large angles more accurate results depend on small values for $b$ and our model is not reliable in this region since, besides the vibration degree of freedom, no other structure was assigned to the target, no nuclear force is present in our Hamiltonian and our projectile is a classical punctual object. Near the calculated peak the Coulomb contribution is expected to be dominant and this may explain the obtained good agreement in the present calculation even though nuclear effects can contribute in that region [10].

Our calculations estimate the pure Coulomb contribution - for the present strongly collective model - of the isovector dipole giant resonance to be of the order of 65% of the experimental cross section for $\alpha - ^{208}\text{Pb}$ and 35% for $d - ^{208}\text{Pb}$ at angles about the peak (Fig. 2). This supports the suggestion that the excitation of that sort of giant resonance is the dominant contribution to the forward angle spectra for large incident energies [8], at least for $\alpha - ^{208}\text{Pb}$, in contrast to the monopole giant resonance whose excitation energy almost coincides with that of the former and also contributes in the same region. Calculations using the Alder and Winther approach [11] for the present model predict a cross section lower than ours in the whole relevant angular domain. A comparison between the Alder and Winther method and ours will appear in a forthcoming publication.

As a conclusion we want to note that, although assuming only an extremely simple collective nuclear Goldhaber - Teller model for a $^{208}\text{Pb}$ vibrating system, the location of the peak is in the intermediate range of the
experimental angular distributions is well described and the above results besides being a relevant argument in favour of the present semiclassical scheme as a method for giving estimates of the pure Coulomb excitation probability for the giant dipole resonance predicts the dominance of this kind of collective motion in the small angle medium energy experiments.

References


Figure Captions

Fig. 1 (1.a) shows the numerical results for the excitation probabilities $P_{01}$ as obtained by our method and the fitting according to Eq.(9) for the $E_{lab}=218$ MeV $\alpha-^{208}$Pb case. (1.b) depicts the fitted excitation probabilities $P_{01}$ for:

i) $E_{lab}=218$ MeV $\alpha-^{208}$Pb; $A = 0.387 \times 10^{-3}$ and $D = 0.322$ fm$^{-1}$;

ii) $E_{lab}=108$ MeV $\alpha-^{208}$Pb; $A = 0.163 \times 10^{-3}$ and $D = 0.328$ fm$^{-1}$.

Fig. 2 (2.a) shows the angular distribution for the $E_{lab}=172$ MeV $\alpha-^{208}$Pb case ($A = 0.487 \times 10^{-3}$ and $D = 0.358$ fm$^{-1}$) and (2.b) $E_{lab}=108$ MeV $\alpha-^{208}$Pb. Experimental data were extracted from ref.(8) and ref.(10).
Figure 2.0: Plot of $d\sigma/d\Omega$ vs. momentum transfer $Q^2$. The data points are labeled as:

- **our results** represented by stars (*).
- **experimental** represented by circles (○).