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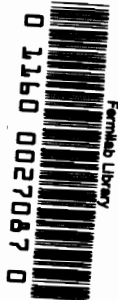
## Nonadiabatic neutrino oscillations revisited

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# Nonadiabatic neutrino oscillations revisited

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## Abstract

Employing the Feynman procedure of ordered exponential operators and the stationary phase method to evaluate the multiple integrals involved, we calculate the level-crossing probability and analyse the role of a resonance in the evolution of a two-level neutrino system. We compare this procedure with more conventional ones, like the Landau's method and the ansatz of Kuo and Pantaleone and Petcov. We verify that our results reproduce the correct extreme nonadiabatic limit and give the standard solutions in the adiabatic regime for any arbitrary matter density distributions. We discuss in particular the case of solar neutrino propagation using the standard solar model predictions for the matter distribution in the Sun.

## I. Introduction

Neutrinos propagating through matter oscillate in a different way than neutrinos propagating in vacuum [1]-[4]. This is because interactions in a medium modify the dispersion relations of particles traveling through. In quantum mechanical language, a different dispersion relation signifies a different Hamiltonian of the system, which gives a different time evolution equation for the corresponding physical system.

Probably the most interesting consequence of the propagation in matter is the possibility of large neutrino mixing occur when neutrinos cross a resonance region. Such effect can be better appreciated observing the neutrino evolution equations in matter, which, for the two generation case, after neglecting irrelevant overall phases, can be written as [2]

$$i \frac{d}{dt} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} 0 & \frac{\Delta}{4E} \sin 2\theta \\ \frac{\Delta}{4E} \sin 2\theta & \frac{\Delta}{2E} \cos 2\theta - \sqrt{2} G_F N_e(t) \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}. \quad (1)$$

$\Delta = m_2^2 - m_1^2$  is the squared mass difference of the vacuum mass eigenstates,  $\theta$  is the neutrino mixing angle in vacuum and  $\sqrt{2} G_F N_e(t)$  is the consequence of electron neutrino coherent forward scattering from electrons in matter which number density at the region reached by neutrinos at instant  $t$  is  $N_e(t)$ . The physical eigenstates in matter  $\nu_1$  and  $\nu_2$  are obtained from the current eigenstates  $\nu_e$  and  $\nu_\mu$  through a convenient two-dimensional rotation which diagonalizes the  $2 \times 2$  evolution matrix appearing in Eq. (1). Such rotation can be parametrized by the mixing angle in matter  $\tilde{\theta}$  and one can write

$$\begin{aligned} \nu_1(t) &= \nu_e(t) \cos \tilde{\theta}(t) - \nu_\mu(t) \sin \tilde{\theta}(t) \\ \nu_2(t) &= \nu_e(t) \sin \tilde{\theta}(t) + \nu_\mu(t) \cos \tilde{\theta}(t), \end{aligned} \quad (2)$$

where  $\tilde{\theta}(t)$  is such that

$$\sin^2 2\tilde{\theta}(t) = \sin^2 2\theta \left[ \left( \frac{2E\sqrt{2}G_F N_e(t)}{\Delta m^2} - \cos 2\theta \right)^2 + \sin^2 2\theta \right]^{-1}. \quad (3)$$

Thus  $\tilde{\theta}$  is substantially modified by the neutrino coherent scattering from the medium. If  $N_e(t) \rightarrow 0$ ,  $\tilde{\theta} \rightarrow \theta$  and we recover vacuum expressions. When the brackets in the denominator of Eq. (3) vanishes, a resonance occurs,  $\tilde{\theta} \rightarrow \pi/4$  and the mixing between flavor eigenstates is maximal. Finally, when  $N_e(t)$  is extremely large,  $\tilde{\theta} \rightarrow \pi/2$ .

It is well known that any physical information about the system of evolution equations (1) can be easily obtained if the following condition is satisfied: the difference of off-diagonal elements of the evolution matrix written in the matter physical eigenstate basis ( $\nu_1, \nu_2$ ) are negligible compared with the difference of diagonal elements of this same matrix. In this case, matter mass eigenstates propagate without mixing and one has the so-called adiabatic propagation. It can be verified [4] that this adiabaticity condition is hardest to be satisfied at the resonance point. That is why one defines the adiabaticity parameter  $\gamma$  [1], on which the evolution equations crucially depend, as the ratio between off-diagonal over diagonal entries of the matter mass eigenstates evolution matrix at the resonance point:

$$\gamma \equiv \frac{\Delta \sin^2 2\theta}{E \cos 2\theta} \left| \frac{d}{dt} \ln N_e(t) \right|_{Res}^{-1}. \quad (4)$$

When  $\gamma \gg 1$ , the propagation is adiabatic everywhere. Nevertheless, if  $\gamma$  is of order 1 or smaller, off-diagonal and diagonal elements can be comparable and transitions between  $\nu_1$  and  $\nu_2$  or vice-versa can occur.

We are typically interested in calculating quantities like the averaged probability of finding an electronic neutrino at an instant  $t$ , if it was created at an instant  $t_0$  as a given combination of  $\nu_1$  and  $\nu_2$  [5]

$$P(\nu_e \rightarrow \nu_e; t, t_0) = \frac{1}{2} [1 + (1 - 2P) \cos 2\tilde{\theta}(t_0) \cos 2\tilde{\theta}(t)], \quad (5)$$

where one has introduced the level-crossing probability of one matter mass eigenstate to be converted into another in the vicinity of a resonance region:

$$P = | \langle \nu_2(t_+) | \nu_1(t_-) \rangle |^2 \quad (6)$$

and  $t_-$  and  $t_+$  refer to two far-away points on either side of the resonance, localized in regions where the propagation is adiabatic. Note that unitarity guarantees that  $P$  is also the crossing probability for  $\nu_2 \rightarrow \nu_1$  transition. One expects that for  $\gamma \gg 1$ , no level-crossing occur, i.e., the propagation is completely adiabatic and, consequently,  $P = 0$ . In this particular case, one easily obtains all the relevant informations about the evolution of the physical system just using adiabatic equations, like Eq. (5) when  $P = 0$ . Nevertheless, if  $\gamma$  is of order 1 or smaller,  $P \neq 0$  and one has to calculate it.

The purpose of this paper is to discuss an alternative way to calculate the level-crossing probability  $P$ . We use the Feynman procedure of ordered

exponential operators [6]-[8] and the stationary phase method [9] to evaluate the multiple integrals involved, to write a solution for the system of equations (1) valid for any parametrization of the matter density distribution  $N_e$ . We compare the obtained result for  $P$  with more conventional ones, like the level-crossing probability from the Landau's method and from the ansatz of Kuo and Pantaleone and Petcov to verify a rather accurate coincidence for a large range of the relevant physical parameters.

## II. Conventional methods of calculating $P$

The usual procedure to obtain the level-crossing probability  $P$  consists in solving exactly the wave equations derived from Eq. (1). One uses the flavor eigenstates  $\nu_e$  and  $\nu_\mu$  to write two coupled first order differential equations. It is possible then to decouple this system of equations eliminating one of the flavor eigenstates from these equations and obtaining a single second order differential equation involving only the resulting flavor eigenstate.

Having the solution of the second order equation, one can calculate, for instance, the electron neutrino survival probability. After making asymptotic approximations around the production and detection points and discarding non-classical oscillating terms, the result, compared with Eq. (6), shows an expression for the level-crossing probability  $P$ .

Nevertheless, exact, analytic solutions of this second order differential equation are not easy to be found in its most general form. It has been found that they exist for only a few functional forms of the matter density distribution: the linear [5][10]-[13], exponential [14]-[17], hyperbolic tangent [18] and  $1/r$  density distributions [19]. These density distributions are not always those ones that appear in Nature. We can quote some examples. Supernova density distributions approach a  $1/r^3$  form which can not be satisfactorily described by any of the above distributions. For the solar case, an exponential function can fit the matter density distribution predicted by the standard solar model [20][21] for a large range of the radial distance. But for the inner 15% of this distance, as well as for regions close to the solar surface this exponential fit does not work well.

The difficulty for obtaining general exact solutions of the evolution equation has motivated the search for alternative methods of calculating the level-crossing probability. In fact, nonadiabatic transition probability between two states was calculated much time ago [11]-[13] in the context of atomic

physics. What is called now the Landau's method was applied to neutrino case [5][10][12][19] to obtain a simple expression for  $P$ :

$$P = \exp(-\gamma F), \quad (7)$$

where the quantity  $F$  depends on how  $N_e(t)$  varies near resonance. However, this equation gives only the leading exponential piece of the level-crossing probability, the dominant part for large  $\gamma$ . In the opposite case, when  $\gamma$  is close to zero and the transition is extremely nonadiabatic, there may be other contributions to  $P$ . In fact, in reference [19], the extreme nonadiabatic limit for  $P$  was evaluated:

$$\lim_{\gamma \rightarrow 0} P = \cos^2 \theta. \quad (8)$$

Obviously this indicates that, at least in this limit case, the Landau's expression (7) fails. The same author developed then an ansatz for  $P$  based on the Landau's formula that evades this problem [22][19]

$$P = \frac{\exp(-\gamma F) - \exp(-\gamma F / \sin^2 \theta)}{1 - \exp(-\gamma F / \sin^2 \theta)}. \quad (9)$$

For  $\gamma$  greater than order  $\sin^2 \theta$  this expression for  $P$  is dominated by the first term in the numerator which coincides with the Landau's expression, Eq. (7).

### III. The Feynman procedure of ordered exponential operators

To obtain the level-crossing probability  $P$ , the wave equations (1) have to be solved. We use the Feynman procedure of ordered exponential operators [6]-[8] to write a formal solution for any parametrization of the evolution matrix elements (1), in particular, for any arbitrary matter density distribution. Using Eqs. (6) and (2), it is easy to calculate the relevant level-crossing amplitude of probability around a resonance

$$\begin{aligned} & \langle \nu_2(t_+) | \nu_1(t_-) \rangle = \\ & = A_R(\nu_e \rightarrow \nu_e) \sin \tilde{\theta}(t_+) \cos \tilde{\theta}(t_-) - A_R(\nu_\mu \rightarrow \nu_e) \sin \tilde{\theta}(t_+) \sin \tilde{\theta}(t_-) + \\ & + A_R(\nu_e \rightarrow \nu_\mu) \cos \tilde{\theta}(t_+) \cos \tilde{\theta}(t_-) - A_R(\nu_\mu \rightarrow \nu_\mu) \cos \tilde{\theta}(t_+) \sin \tilde{\theta}(t_-), \end{aligned} \quad (10)$$

where the subscript  $R$  indicates transitions around resonance.

In order to evaluate the level-crossing probability  $P$  we have to calculate the amplitudes of probability appearing in Eq. (10). Therefore, we want to obtain a solution of the system of equations (1) in a time interval  $[t_-, t_+]$  using an arbitrary boundary condition  $(\nu_e(t_-), \nu_\mu(t_-))$ . Let us assume that electron and muon neutrino states are described by a two-component column spinor  $\phi(t) = (\nu_e(t), \nu_\mu(t))$ . In order to simplify our expressions we will introduce a new spinor  $\psi(t)$  related to  $\phi(t)$  through the following convenient phase transformation:

$$\phi(t) = \exp \left\{ -\frac{i}{2} \int^t dt' \left[ \frac{\Delta}{2E} \cos 2\theta - \sqrt{2} G_F N_e(t') \right] [1 - \sigma_3] \right\} \psi(t), \quad (11)$$

where we have introduced the basis of Pauli matrix  $(1, \sigma_i)$ ,  $i = 1, 2, 3$ . Since we are interested in transition amplitudes around resonance, and the quantity  $[\Delta/2E \cos 2\theta - \sqrt{2} G_F N_e]$  is negligible in this region (see Eq. (3)), the phase factor relating  $\psi$  to  $\phi$  reduces to just an irrelevant phase factor. Consequently, all the relevant informations about the physical system we are interested in are totally given by  $\psi(t)$ . It can be shown, substituting Eq. (11) into (1), that

$$i \frac{d}{dt} \psi(t) = \frac{\Delta}{4E} \sin 2\theta \left[ e^{i\varrho(t)} \sigma_- + e^{-i\varrho(t)} \sigma_+ \right] \psi(t). \quad (12)$$

$\sigma_+$  and  $\sigma_-$  are the lowering and raising operators and

$$\varrho(t) = \int^t dt' \left[ \frac{\Delta}{2E} \cos 2\theta - \sqrt{2} G_F N_e(t') \right]. \quad (13)$$

We can now apply the Feynman procedure to write a formal solution for Eq. (12)

$$\psi(t_+) = \text{Exp} \left\{ -i \int_{t_-}^{t_+} dt' \frac{\Delta}{4E} \sin 2\theta \left[ e^{i\varrho(t')} \sigma_- + e^{-i\varrho(t')} \sigma_+ \right] \right\} \psi(t_-), \quad (14)$$

where Exp indicates an expansion defined as a sum of multiple ordered integrals [6]-[8].

If we now assume that  $\varrho(t)$  is large, as it is the case for a large range of values of the physical interesting matter distributions  $N_e(t)$ , the integrand in Eq. (14) oscillates very quickly leading to vanishing contributions to the integration unless some stationary phase  $t_R$  can be found in the interval of integration  $t_- < t_R < t_+$ . The value of the stationary phase is obtained through the relation [9]

$$\left. \frac{d}{dt} \varrho(t) \right|_{t=t_R} = 0 \implies \frac{\Delta}{2E} \cos 2\theta = \sqrt{2} G_F N_e(t_R). \quad (15)$$

Note that this stationary phase condition coincides with the resonance condition for which the mixing angle in matter  $\tilde{\theta}$ , given by Eq. (3), is maximal. The major contribution to the value of the integrals appearing in Eq. (14) arises from the vicinity of the resonance point, i.e., the point where condition (15) is satisfied. This is the main result from the method of stationary phase and that is why we believe the Feynman procedure of ordered exponential operators together with the stationary phase method is indicated to analyse what happens around the resonance, even if the propagation in this region is in nonadiabatic regime, which is, according to our previous discussion, difficult to analyse through other procedures.

Assuming that the condition (15) for the existence of a stationary phase is satisfied, i.e., there exists a resonance, we can write expressions for the amplitudes of probability which enter in Eq. (10). Thus the amplitude of probability  $A_R(\nu_e \rightarrow \nu_\mu)$  in a resonance region is simply given by the upper off-diagonal term of the  $2 \times 2$  matrix of the Eq. (14) calculated through the stationary phase method [9]. The other relevant amplitudes,  $A_R(\nu_e \rightarrow \nu_e)$ ,  $A_R(\nu_\mu \rightarrow \nu_e)$  and  $A_R(\nu_\mu \rightarrow \nu_\mu)$  are given, respectively, by the upper diagonal, lower off-diagonal and lower diagonal elements of the same matrix.

Let us now calculate the first order contribution of these amplitudes. If we want to evaluate the  $\nu_e \rightarrow \nu_\mu$  transition, terms accompanying  $\sigma_+$  in Eq. (14) have to be considered and we can write

$$A_R(\nu_e \rightarrow \nu_\mu) = -i \sqrt{2\pi \left| \frac{d^2 \varrho(t)}{dt^2} \right|_{t=t_R}^{-1} \frac{\Delta}{4E}} \sin 2\theta \exp \left[ i\varrho(t_R) + i\frac{\pi}{4} \right]. \quad (16)$$

We have applied the stationary phase method [9] to evaluate the relevant integral. Observe that this result is only valid if condition (15) is satisfied somewhere along the neutrino trajectory, otherwise  $A_R(\nu_e \rightarrow \nu_\mu)$  is negligibly small.

Eq. (16) is just a first order approximation. It can be improved analysing the upper order terms in the expansion of the expansional (14). It is not difficult to verify that contributions to  $\nu_e \rightarrow \nu_\mu$  transitions come from terms of order odd in this expansion. Since we are assuming that we have only one stationary phase, the expansion in all orders can be calculated and we obtain:

$$A_R(\nu_e \rightarrow \nu_\mu) = -i2\xi \{1 - \xi^2 + \xi^4 - \xi^6 + \dots\} \exp \left[ i\varrho(t_R) + i\frac{\pi}{4} \right], \quad (17)$$

where

$$\xi = \sqrt{\frac{\pi}{2} \left| \frac{d^2 \varrho(t)}{dt^2} \right|_{t=t_R}^{-1} \frac{\Delta}{4E}} \sin 2\theta. \quad (18)$$

For  $\xi^2 < 1$ , the series appearing in Eq. (17) is convergent [23] and the whole amplitude can be written as

$$A_R(\nu_e \rightarrow \nu_\mu) = -i \frac{2\xi}{1 + \xi^2} \exp \left[ i\varrho(t_R) + i\frac{\pi}{4} \right]. \quad (19)$$

In a completely analogous way we evaluate the opposite  $\nu_\mu \rightarrow \nu_e$  transition amplitude

$$A_R(\nu_\mu \rightarrow \nu_e) = -i \frac{2\xi}{1 + \xi^2} \exp \left[ -i\varrho(t_R) - i\frac{\pi}{4} \right], \quad (20)$$

as well as the survival amplitudes of probability, given now by terms of even order in the expansional

$$A_R(\nu_e \rightarrow \nu_e) = A_R(\nu_\mu \rightarrow \nu_\mu) = \frac{1 - \xi^2}{1 + \xi^2}. \quad (21)$$

Note that the electron and muon neutrino survival amplitudes are equal to each other while  $\nu_e \rightarrow \nu_\mu$  and  $\nu_\mu \rightarrow \nu_e$  transitions differ by one phase factor.

Finally we can put Eqs. (19)-(21) into (10) and (6) to obtain the final expression for the level-crossing probability:

$$P = \left( \frac{1 - \xi^2}{1 + \xi^2} \right)^2 \sin^2 [\tilde{\theta}(t_+) - \tilde{\theta}(t_-)] + \frac{2\xi^2}{(1 + \xi^2)^2} \left\{ \cos^2 [\tilde{\theta}(t_+) - \tilde{\theta}(t_-)] + \cos^2 [\tilde{\theta}(t_+) + \tilde{\theta}(t_-)] \right\}, \quad (22)$$

where we have again neglected oscillating terms. Supposing that the neutrino production and detection positions are far above and below the resonance region, respectively, it is natural to consider

$$\tilde{\theta}(t_-) = \frac{\pi}{2} \quad \text{and} \quad \tilde{\theta}(t_+) = \theta. \quad (23)$$

Furthermore, since the second derivative of  $\varrho(t)$ , Eq. (13), is essentially the first derivative of the matter density distribution  $N_e(t)$ , we compare Eqs. (4) and (18) to conclude, after evaluating the involved quantities at the resonance point through Eq. (15), that

$$\xi^2 = \frac{\pi}{16} \gamma. \quad (24)$$

Interesting enough, we observe that in the extreme nonadiabatic limit, when  $\gamma = (16/\pi)\xi^2 \rightarrow 0$ , we recover from Eqs. (22)-(23) the correct limit for  $P$  presented in Eq. (8), differently from what happens with Landau's expression (7). Without any ansatz, we have obtained the correct nonadiabatic limit.

This result is shown in Fig. 1 where we compare our expression for  $P$ , Eq. (22), with the usually accepted expression for this same quantity, Eq. (9). We assume an electron density distribution decreasing exponentially along the neutrino trajectory [20][21]

$$N_e(t) = 2.4 \times 10^{26} e^{-t/(0.09 \text{ Solar Radius})} \text{ cm}^{-3}, \quad (25)$$

which is in good agreement with the standard solar model predictions for the matter density distribution inside the Sun, except for regions close to the solar center or to the solar surface and implies that  $F$  appearing in Eq. (9) is  $F = 1 - \text{tg}^2\theta$  [14]-[17].

We observe from Fig. 1 that our expression (22)-(23) for  $P$  coincides with the conventional expression (9) for small values of  $\gamma$ , i.e., in the extreme nonadiabatic regime. For  $\gamma$  of order 1, three cases have to be considered. For a large range of values of  $\theta$  ( $\theta$  of order or smaller than 0.2), this agreement is rather accurate everywhere. For  $\theta \approx 0.5$ , a disagreement becomes appreciable in the region where  $\gamma$  is around 1. Finally, for maximal neutrino mixing angle in vacuum,  $\theta \rightarrow \pi/4$ , both expressions (9) and (22) coincide again and are approximately equal to 1/2 independently of  $\gamma$ . Obviously, this is not an interesting physical case and is usually discarded.

It should be noticed that the conventional expression (9) differs from the numerically calculated values of  $P$  [16] in regions where  $\gamma \approx 1$  and  $\theta$  is large ( $\theta \approx 0.4 - 0.6$ ). We believe, furthermore, that our expression (22) does not have a strong dependence with  $\theta$ . Thus we expect the same accuracy of our expression for small or large  $\theta$ . Our main limitation comes from the lack of an expression for  $P$  if  $\xi^2 = (\pi/16)\gamma > 1$ . Nevertheless, this is exactly the beginning of the adiabatic regime and we know that in this region  $P$  goes quickly to zero.

#### IV. Conclusion

Using the Feynman procedure of ordered exponential operators and the stationary phase method to solve the multiple integrals involved, we have

analysed the role of the resonance in a nonadiabatic neutrino propagation in a concise and general way. We have obtained an expression, Eq. (22), for the level-crossing probability valid for  $\gamma$  of order or smaller than 1, which reproduces the correct extreme nonadiabatic limit and coincides with the conventional expression for  $P$ , Eq. (9), for a large range of the involved parameters.

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## Captions

**Figure 1:** Comparison of our expression for the level-crossing probability  $P$ , Eq. (22), (solid line), with the usually accepted expression for this same quantity, Eq. (9), (dashed line), as a function of the adiabaticity parameter  $\gamma$ , Eq. (4), for various values of the vacuum mixing angle  $\theta$  and for  $\xi^2 = (\pi/16)\gamma < 1$ . We have assumed that the electron number density varies exponentially according to Eq. (25) and taken  $\tilde{\theta}(t_-) = \pi/2$ ,  $\tilde{\theta}(t_+) = \theta$ . Note that for the limit case when  $\theta \rightarrow \pi/4$ , both curves coincide presenting a constant value  $1/2$ , independently of  $\gamma$ .



